# BURST OF TURBOENGINE DISKS

## P.R.C. D.D.V.

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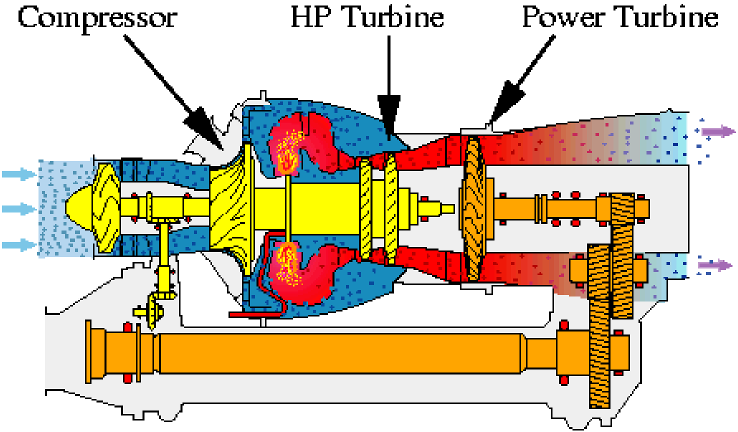
21. November 2007

# ENGINE MECHANISM

## ARRIEL1 HELICOPTER ENGINE - TURBOMÉCA



# PHENOMENA RESPONSIBLE FOR DISK FAILURE

**Limit load**

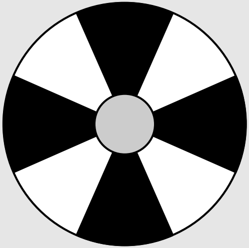
Robinson (1944), Tvergaard (1978)

# Phenomena responsible for disk failure

**Limit load**

Robinson (1944), Tvergaard (1978)

**Non-axisymmetric bifurcation modes**



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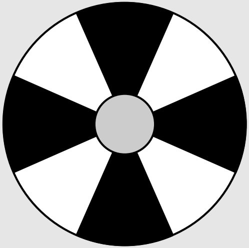

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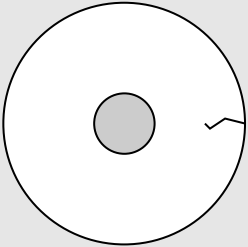
### Fatigue crack propagation



Park *et al.* (2002), Bhaumik *et al.* (2004)

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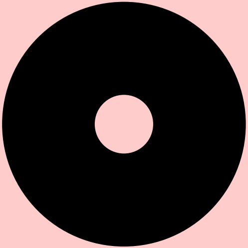

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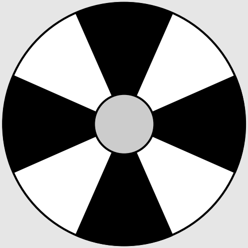

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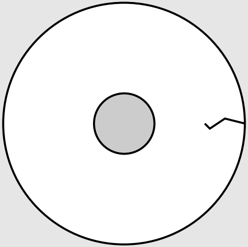

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# TURBINE DISK

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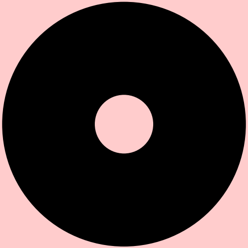

Robinson (1944), Tvergaard (1978)

- Material : Udimet 720
- Temperature $\sim$ 500˚C
- Rotation rate $\sim$ 60 000 $\mathrm{RPM}$
- Mass $\sim$ 5 $\mathrm{Kg}$

(at room temperature)

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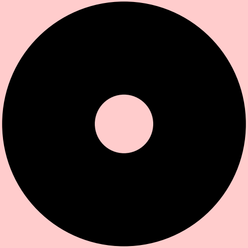

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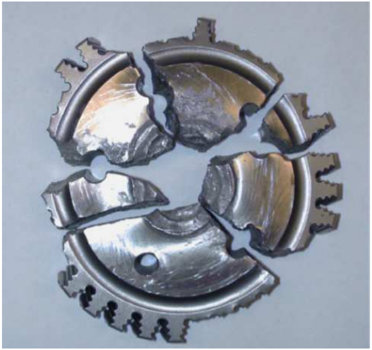

(at room temperature)

# REGULATION RULES

- *"For each fan, compressor, and turbine rotor, it must be established by test, analysis, or combination thereof, that a rotor [...] will not burst when it is operated in the Engine for five minutes at 120% of the maximum permissible rotor speeds [...]"*
- *"An analytical modelling method based on representative test data may be acceptable provided that the model has been validated by comparison with results from specimen and rotor tests"*
- *"The necessary speed adjustment for temperature and material properties will normally be established on the basis of appropriate ratios of material properties"*

[Certification Specifications for Engines (CSE-E 840) - European Aviation Safety Agency - 2003]

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# Overspeed experiments

## Spin Pit

# OVERSPEED EXPERIMENTS

## Spin Pit

+

## Disk

# Overspeed experiments

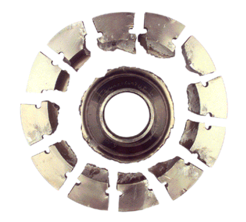
**Spin Pit**

$+$

**Disk**

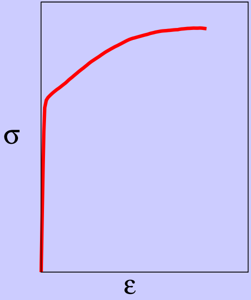
$\Rightarrow$

**$\omega_{EXP}$**



# NUMERICAL PREDICTION AT ROOM TEMPERATURE

## Constitutive Behavior



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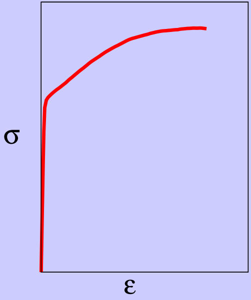


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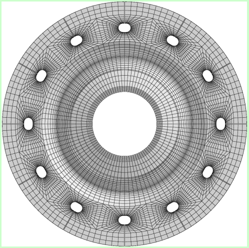
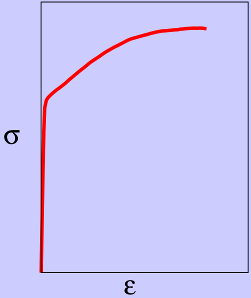
**F.E. Model**



# Numerical prediction at room temperature

**Constitutive Behavior**

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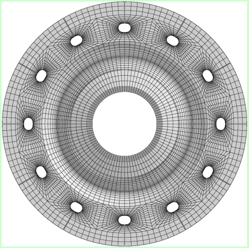


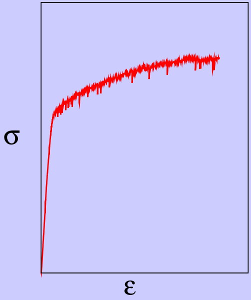
$\Rightarrow$

**$\omega_{NUM}$**

- Burst criterion
- Estimation of confidence interval

# Numerical prediction at 500˚C

**Constitutive Behavior at 500˚C**



# Numerical prediction at 500°C

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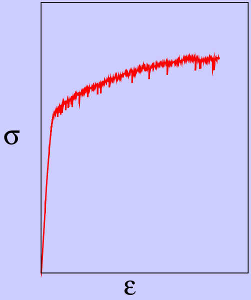


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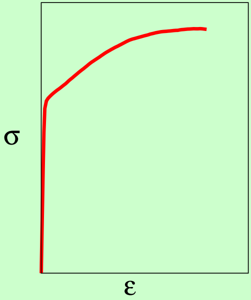
**Constitutive Behavior at room temperature**



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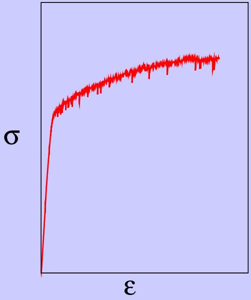


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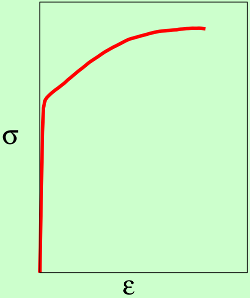
**Constitutive Behavior at room temperature**




⇒

**Adjustment of $\omega_{NUM}$**

- Effect of material strain-rate dependency
- Influence of PLC effect on fracture

# Outline

## Burst prediction at room temperature

- Mechanical behavior of Udimet720 at room temperature
- Material parameter identification and validation
- Mechanical problem of a rotating disk - Existing criteria
- Numerical methods
- Results

## Simulation of the Portevin - Le Chatelier effect

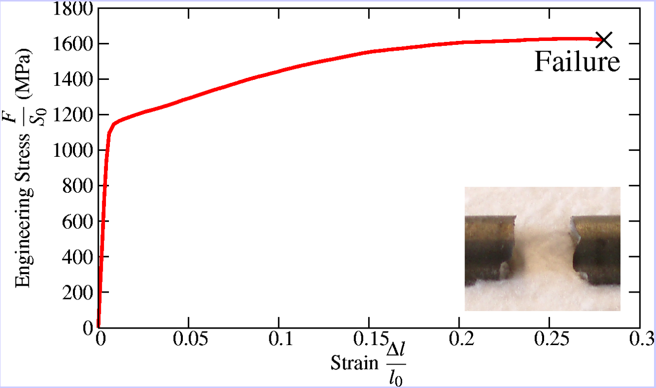
- Mechanical behavior of Udimet720 at 500˚C
- Material parameter identification
- Analysis of strain rate localization phenomena
- Objectivity of simulations
- Large scale simulations of 3D specimens
- Application to rotating disks

## Conclusion

# OUTLINE

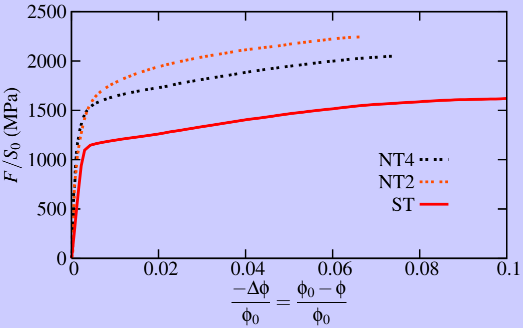
# Constitutive tensile behavior at room temperature

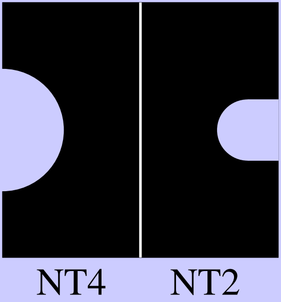
## Tensile curve at room temperature ($\dot{\varepsilon} = 10^{-3}s^{-1}$)



# Simulation on notched specimens

## Tensile curves at room temperature ($\dot{\varepsilon} = 10^{-3}s^{-1}$)




Ductile fracture

Independent from prescribed strain rate

# Constitutive equations

## Elastoplastic model

- Strain tensors : $\dot{\underset{\sim}{\mathbf{e}}} = \dot{\underset{\sim}{\mathbf{e}}}_e + \dot{\underset{\sim}{\mathbf{e}}}_p$
- Yield criterion : $f(\underset{\sim}{\mathbf{s}}, p) = s_{eq} - R(p)$
- Hardening law : $R(p) = R_0 + Q_1\left(1 - e^{-b_1 p}\right) + Q_2\left(1 - e^{-b_2 p}\right)$
- Flow rule : $\dot{\underset{\sim}{\mathbf{e}}}^p = \dot{p}\dfrac{\partial f}{\partial \underset{\sim}{\mathbf{s}}}, \quad \dot{p} \geq 0$

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## Local corotational frame

- Stress tensor : $\underset{\sim}{\mathbf{s}} = \underset{\sim}{\mathbf{Q}}.\underset{\sim}{\mathbf{T}}.\underset{\sim}{\mathbf{Q}}^T$
  where $\underset{\sim}{\mathbf{T}}$ is the Cauchy stress tensor
- Strain tensor : $\dot{\underset{\sim}{\mathbf{e}}} = \underset{\sim}{\mathbf{Q}}.\underset{\sim}{\mathbf{D}}.\underset{\sim}{\mathbf{Q}}^T$
  where $\underset{\sim}{\mathbf{D}}$ is the strain rate tensor

($\underset{\sim}{\mathbf{Q}}$ such as $\dot{\underset{\sim}{\mathbf{Q}}}^T.\underset{\sim}{\mathbf{Q}} = \underset{\sim}{\mathbf{\Omega}} \rightarrow$ corotational)

# Outline

# Identification at room temperature

## Elasticity

Isotropic coefficients :

- $E = 207\ 500\ MPa$
- $\nu = 0.3$
- $\rho = 8\ 080\ kg.m^{-3}$

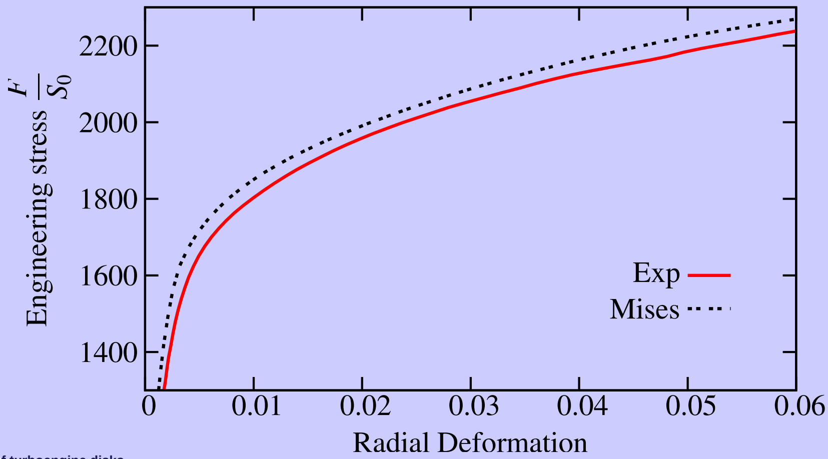
## Plasticity

Isotropic hardening :

- $R_{p0.2} = 1211\ MPa$
- $R_m = 1660\ MPa$
- $p_{crit} = 0.2$

# Identification at room temperature

## Simulation and experiment of a notched specimen (NT2)



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## Yield criterion

- von Mises criterion
- Tresca criterion
- Hosford criterion

$$s_{eq} = \left[\frac{(s_1 - s_2)^n + (s_2 - s_3)^n + (s_1 - s_3)^n}{2}\right]^{1/n}$$

where $s_1 \geq s_2 \geq s_3$

# IDENTIFICATION AT ROOM TEMPERATURE

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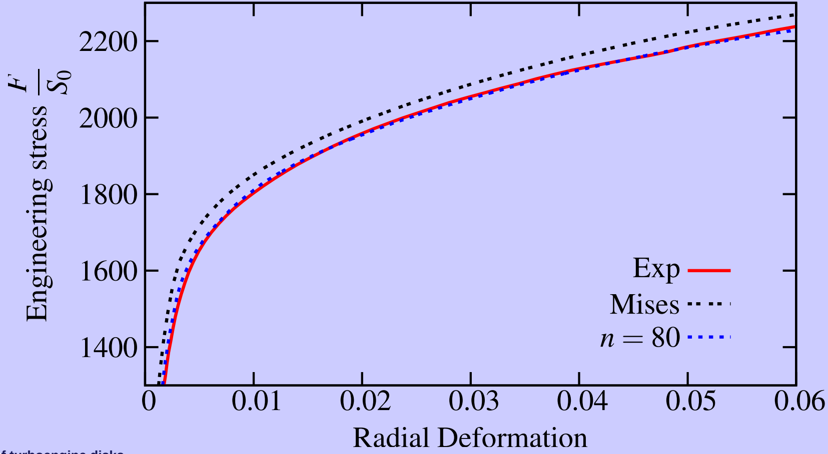


# Outline

## Burst prediction at room temperature



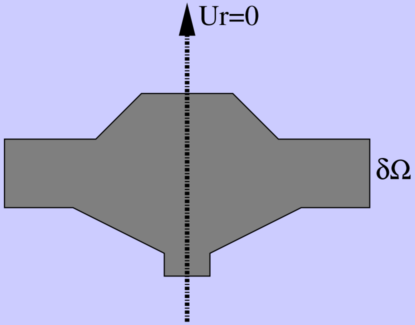
## Simulation of the Portevin - Le Chatelier effect



## Conclusion

# Mechanical problem of a rotating disk

## Model



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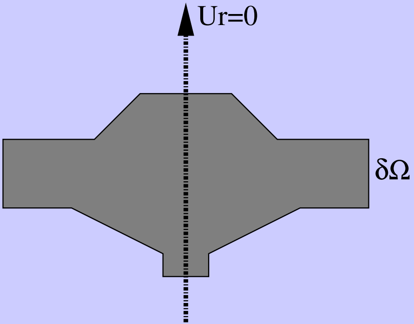
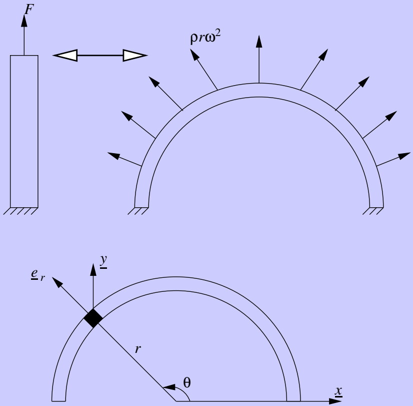



## Equations

- Material Behavior
- Equilibrium :
  $\mathrm{div}_X \underset{\sim}{\boldsymbol{S}} + \rho_0 \omega^2 \underline{\boldsymbol{r}} = 0$ on the whole disk
  ($\underset{\sim}{\boldsymbol{S}} \rightarrow PK1$ , $X$ : Initial Configuration)
- Boundary Conditions :
  $u_r = 0$ at $r = 0$,
  $\underset{\sim}{\boldsymbol{S}}.\underline{\boldsymbol{N}} = 0$ on $\partial\Omega$

# ROTATING RING [1]

## Model




[1] Tvergaard (1978)

# Rotating ring [1]

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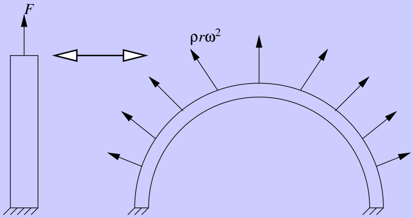
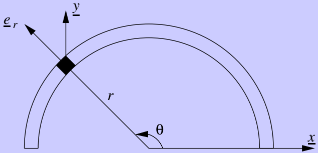
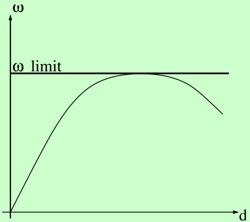



## Equations

- Separating Stress :
  $\sigma_s = \rho r^2 \omega^2$
  evaluated on the initial configuration
- Limit rotation rate $\omega_{LIM}$ such that $\sigma_s = R_m$ (tensile strength)
  $\rightarrow \omega_{LIM} = \sqrt{\dfrac{R_m}{\rho r^2}}$

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**Empirical local criterion (■)**

Burst occurs when a point of the structure reaches the critical plastic strain.

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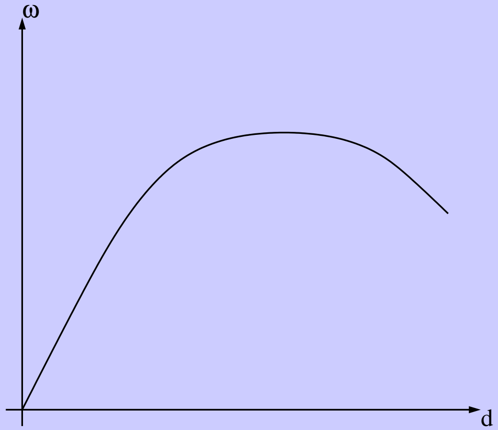
**Multiaxial global criterion - Limit analysis (▲)**

Burst occurs at the maximum of the global equilibrium load/displacement curve

# Outline

# Load control

## Global load/displacement equilibrium curve



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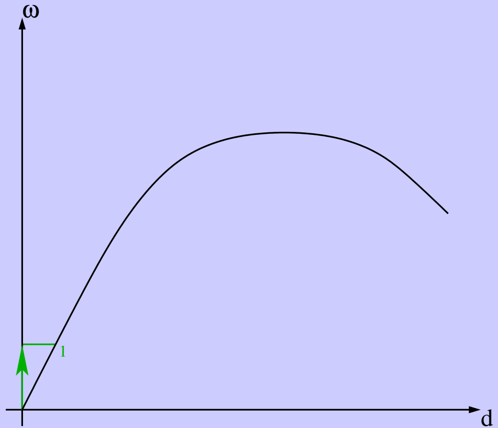

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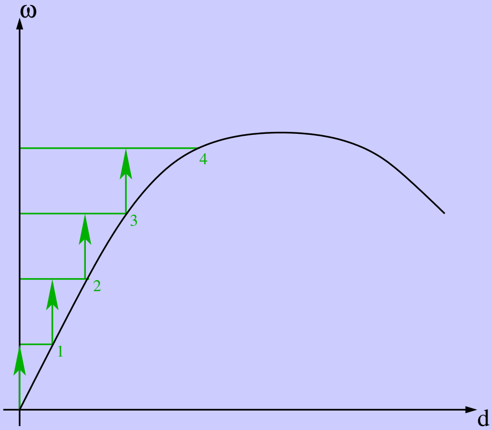

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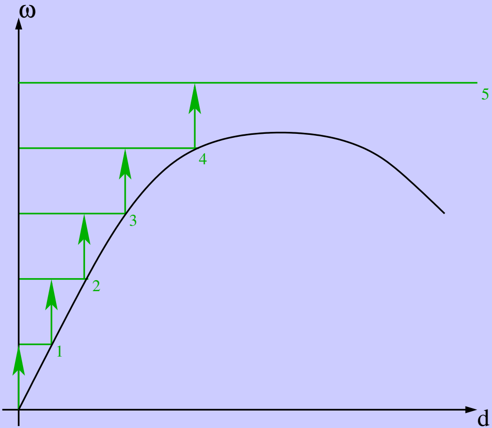

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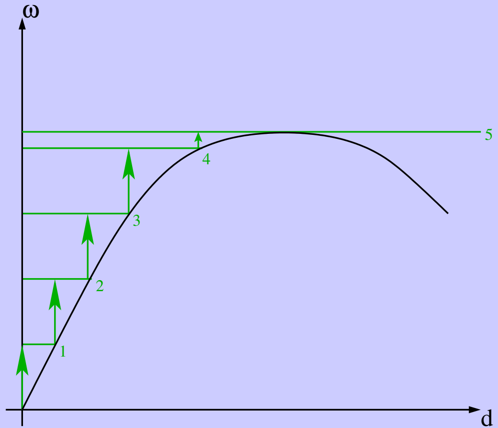

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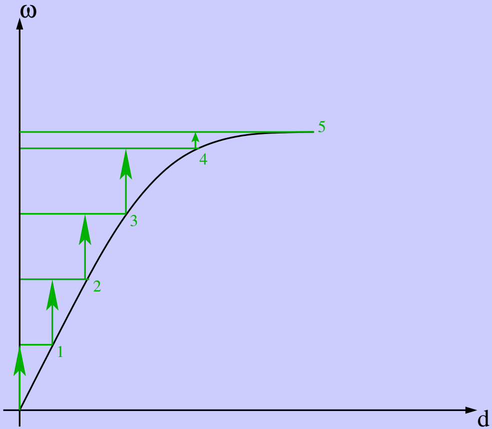

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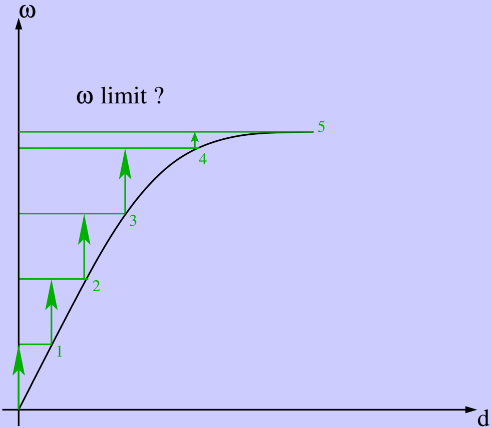
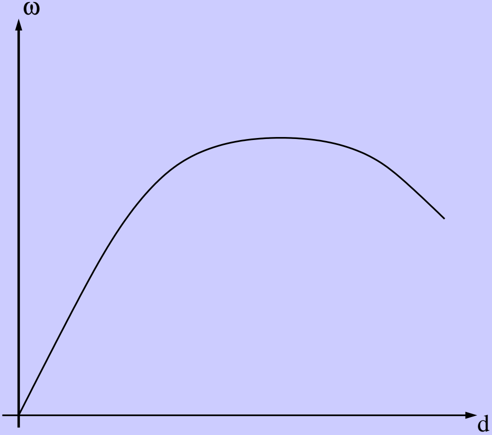


# ARC-LENGTH CONTROL [2]

## Global load/displacement equilibrium curve

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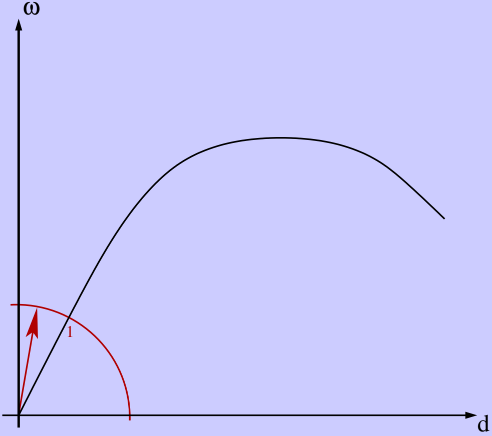


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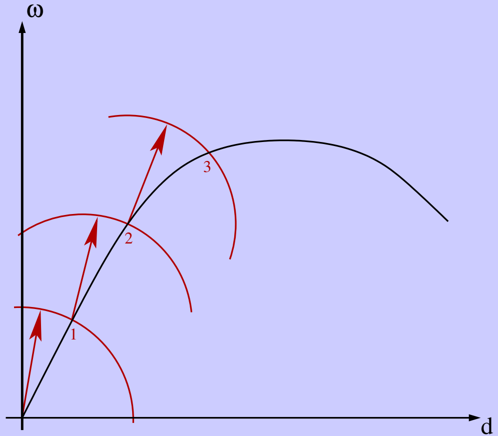
# ARC-LENGTH CONTROL [2]

## Global load/displacement equilibrium curve




[2]Riks (1979)

# Arc-length control [2]

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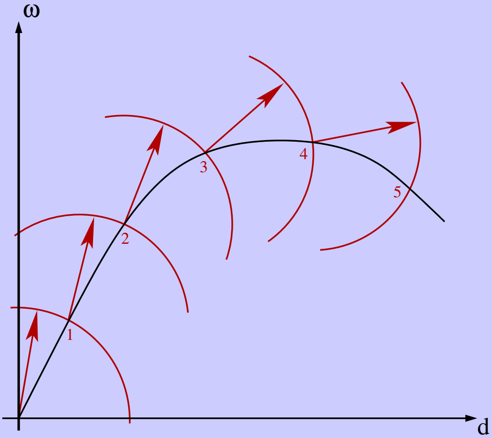


[2] Riks (1979)

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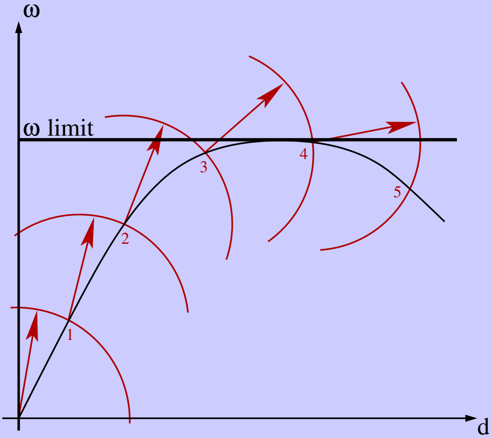



[2] Riks (1979)

# Outline

## Burst prediction at room temperature



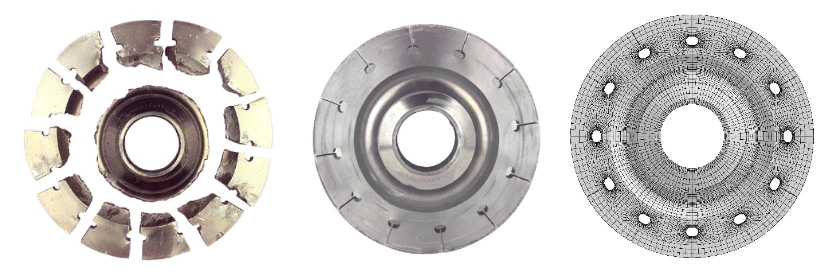
## Simulation of the Portevin - Le Chatelier effect



## Conclusion

# EXPERIMENTAL DISKS

## Photographs of S-disk and B-disk after experiments

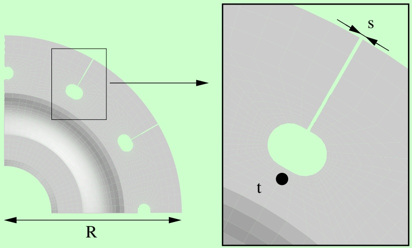


B-Disk (burst at $\omega_{exp}$)   S-Disk (loaded up to 97% of $\omega_{exp}$)   FE model

# Evaluation of residual deformations

## (from the S-disks)

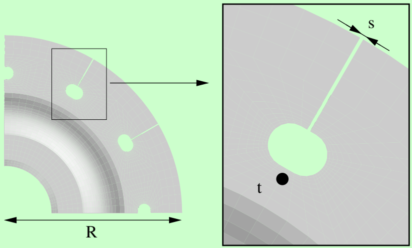
### Location of measured quantities on the S-disk



# EVALUATION OF RESIDUAL DEFORMATIONS

## (FROM THE S-DISKS)

### Location of measured quantities on the S-disk




### Experimental and numerical values

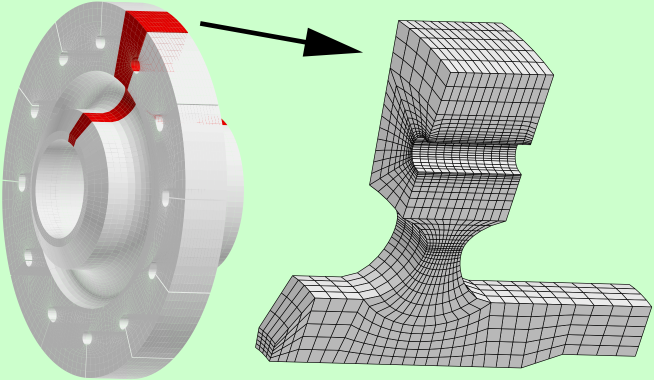
| | $R$ | $s$ | $t$ |
|---|---|---|---|
| Exp. | 1.2% | 137% | 3.3% |
| Mises | 0.22% | 21% | 0.63% |
| $n = 80$ | 1.27% | 119% | 3.06% |

# EVALUATION OF LIMIT ROTATION RATE

## (FROM THE B-DISK)

### Finite element simulation of B-Disk burst

■ : local criterion, ♦ : average hoop stress criterion, ▲ : limit analysis



# Limit rotation rate at 500°C

## Material properties

- $R_m^{RT} = 1660 \text{ MPa}$
- $R_m^{500°C} = 1484 \text{ MPa}$
- $i = \frac{R_m^{500°C}}{R_m^{RT}} = 0.89$

$\rightarrow \omega_{LIM}^{500°C} = 0.945\ \omega_{NUM}^{RT} \simeq 0.945\ \omega_{EXP}$

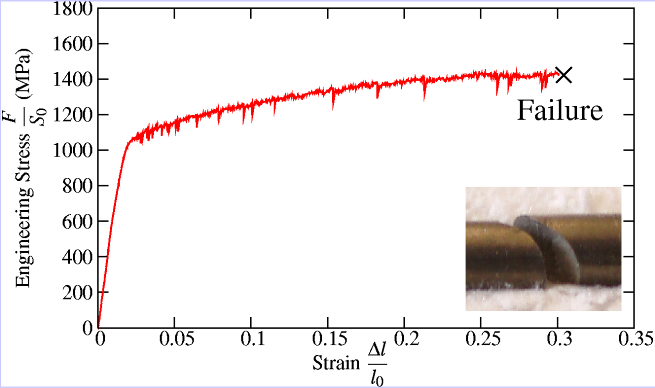
## Equivalent FE simulation

- Elastoviscoplastic behavior at 500˚C
- Same finite element simulation

$\rightarrow \omega_{NUM}^{500°C} = 0.945\ \omega_{NUM}^{RT}$

# Behavior at 500°C

## Tensile curve at 500°C ($\dot{\varepsilon} = 10^{-3}s^{-1}$)



# Outline

## Burst prediction at room temperature

- Mechanical behavior of Udimet720 at room temperature
- Material parameter identification and validation
- Mechanical problem of a rotating disk - Existing criteria
- Numerical methods
- Results

## Simulation of the Portevin - Le Chatelier effect

- Mechanical behavior of Udimet720 at 500˚C
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- Objectivity of simulations
- Large scale simulations of 3D specimens
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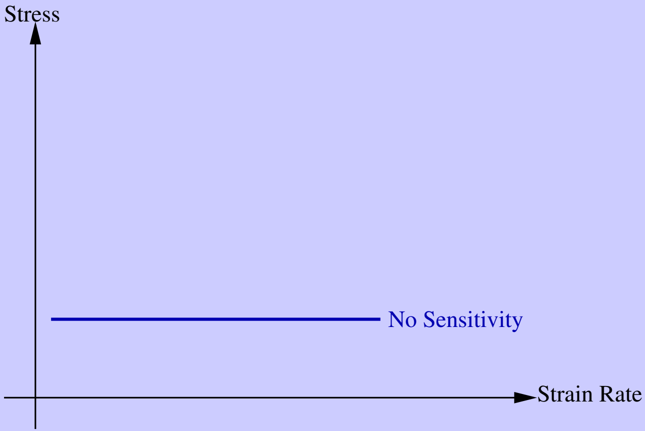
## Conclusion

# Strain rate sensitivity

## Stress/Strain rate curve



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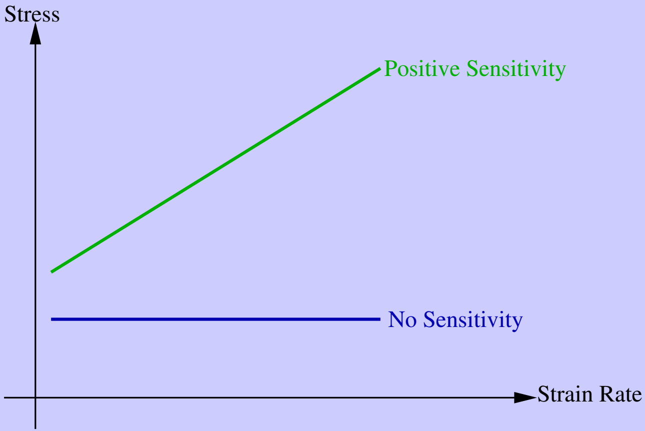

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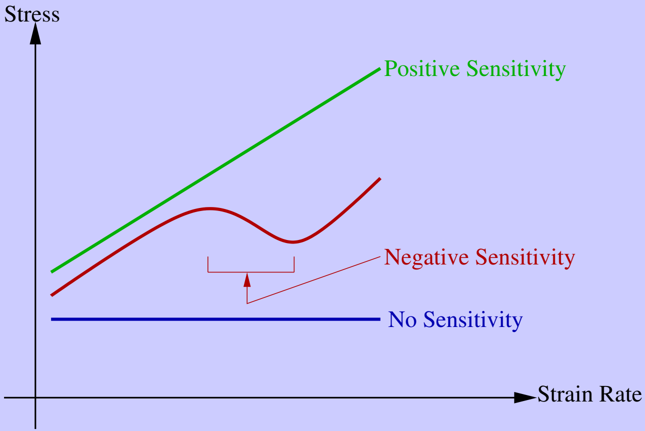

# Constitutive equations

## Strain ageing model Kubin - Estrin - McCormick [3]

- Yield criterion : $f(\underset{\sim}{\mathbf{s}}, p, t_a) = s_{eq} - R(p) - P_1 C_s(p, t_a)$
- Hardening law : $R(p) = R_0 + Q\left(1 - e^{-bp}\right)$
- Flow rule : $\dot{p} = g(f) = \dot{p}_0 \sinh\left(\frac{\langle f \rangle}{K}\right)$
- Extra-hardening : $C_s(p, t_a) = C_m\left(1 - e^{-P_2 p^{\alpha} t_a^m}\right)$
- Kinetics of ageing time : $\dot{t}_a = \frac{t_w - t_a}{t_w}, \qquad t_w = \frac{w}{\dot{p}}$

[3] Zhang, McCormick, Estrin (2001)

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## 1D solution

$$\sigma^{1D}(p) = R_0 + Q\left(1 - e^{-bp}\right) + K \sinh^{-1}\left(\frac{\dot{p}}{\dot{p}_0}\right) + P_1 C_m\left(1 - e^{-P_2 p^\alpha t_a^m}\right)$$

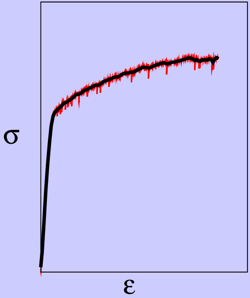
[3] Zhang, McCormick, Estrin (2001)

# Outline

# METHOD OF IDENTIFICATION AT 500°C

**Elastoplastic parameters**
**$\longrightarrow R_0,\ Q,\ b$**

Smoothed stress/strain curve



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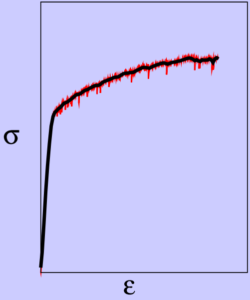
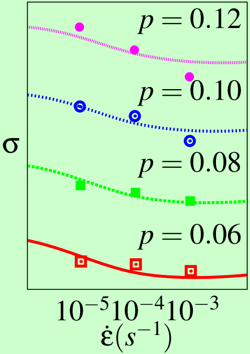



**Time dependent parameters $\rightarrow \dot{p}_0,\ K,\ P_1 C_m,\ P_2$**

Stress/strain rate curve



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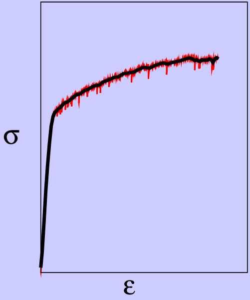


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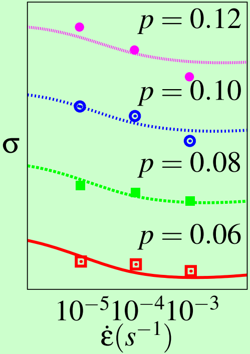
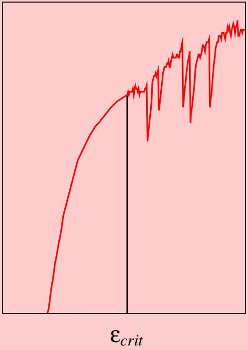



**Strain ageing parameters**
$\rightarrow \alpha$

Critical plastic strain



# Critical plastic strain

## Analytical estimation of the critical plastic strain [4]

- Matrix of a perturbation $e^{\lambda t}$

$$\begin{pmatrix} \delta \dot{p} \\ \delta \dot{t}_a \end{pmatrix} = [\mathbf{M}(p, t_a)] . \begin{pmatrix} \delta p \\ \delta t_a \end{pmatrix}$$

- Eigenvalue $\lambda$

$$h(\lambda) = \lambda^2 + 2\Phi\lambda + \lambda^2 = 0$$

- Critical strain

$$\varepsilon_{crit} \text{ when } \exists \lambda \in \mathbb{R}^+ \text{ such that } h(\lambda) = 0$$

- Significant model parameter

$$\alpha$$

[4] McCormick (1989)

# MC MODEL PARAMETERS

## Elasticity

Isotropic coefficients :

- $E = 180\ 000\ MPa$
- $\nu = 0.3$
- $\rho = 8\ 080\ kg.m^{-3}$

[5] Herring (1950)

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### Plasticity

Isotropic hardening :

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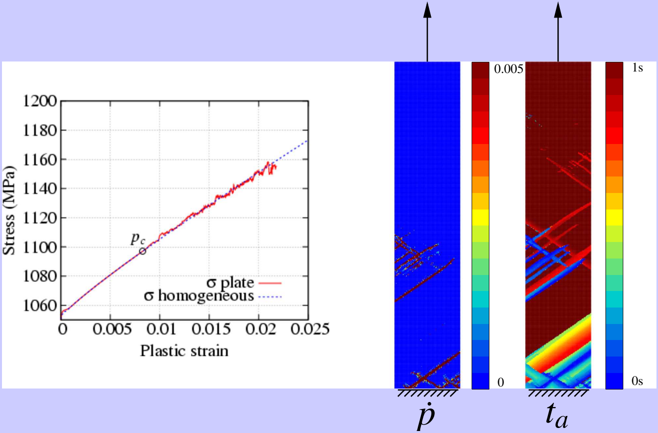
### Strain ageing

- $P_1 C_m = 96\ MPa$
- $P_2 = 4.1\ s^{-n}$
- $\alpha = 0.55$
- $m = 0.33$ (Diffusion[5])
- $w = 10^{-4}$

[5] Herring (1950)

# Simulation of a plate

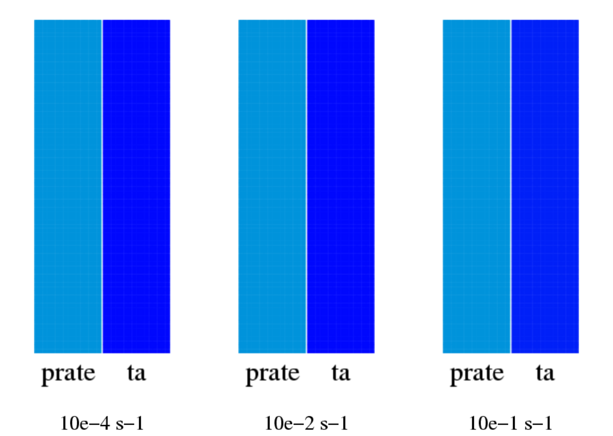
## Simulation of a plate in tension
Udimet 720 at 500°C and $\dot{\varepsilon} = 10^{-4}s^{-1}$



# Outline

# Strain rate sensitivity

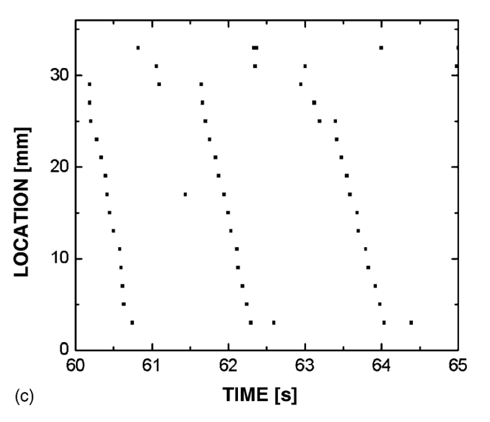
## Localized bands at different strain rates



# Strain rate localization

## Type of bands - Chmelik et al. (2002,2007)

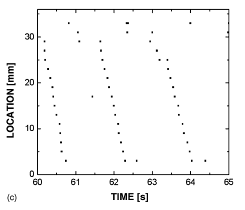
### Band location - Acoustic emission measurements



# Strain rate localization

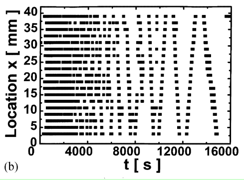
## Type of bands - Chmelik et al. (2002,2007)

### Type A



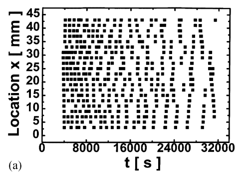

Repetitive continuous propagation

### Type B




Hopping propagation with reflections

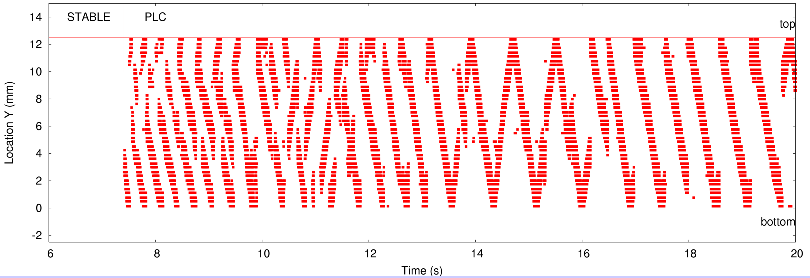
### Type C




Random nucleation without propagation

# Band Location Indicator (BLI tool)

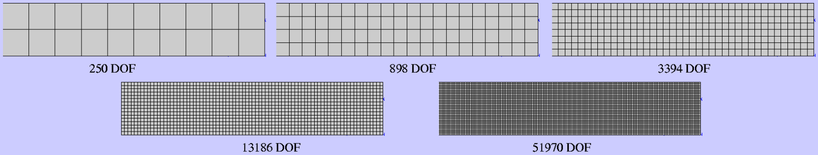
## Location of bands as a function of time ($\dot{\varepsilon} = 10^{-2}s^{-1}$)



# Outline

# MESH SENSITIVITY (1)

## Models



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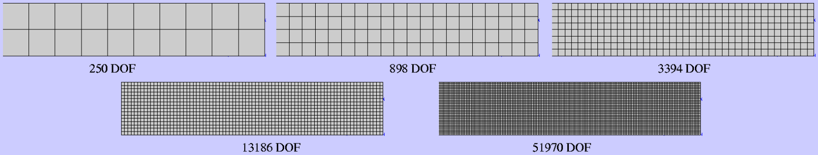
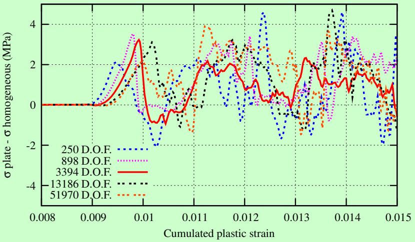
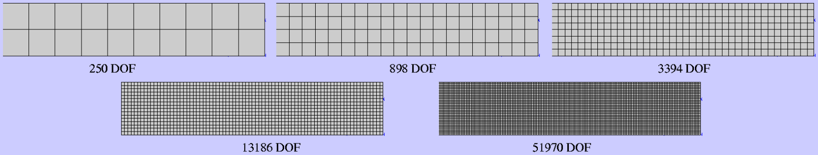



## Constant strain rate

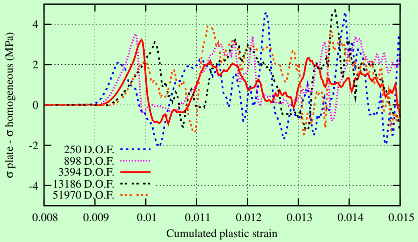


# Mesh sensitivity (1)

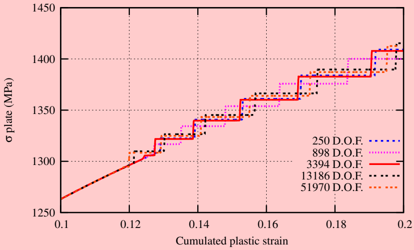
## Models




## Constant strain rate

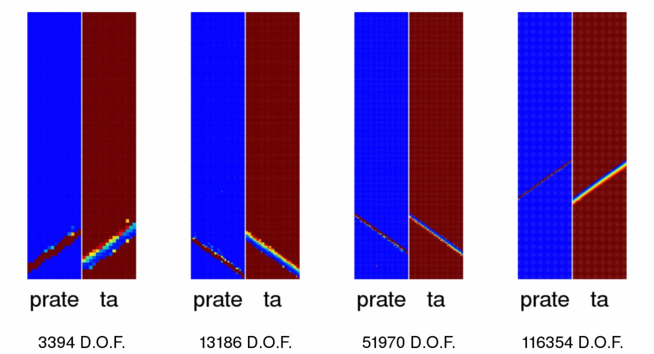



## Constant stress rate



# Mesh sensitivity (2)

## Localized bands on different meshes



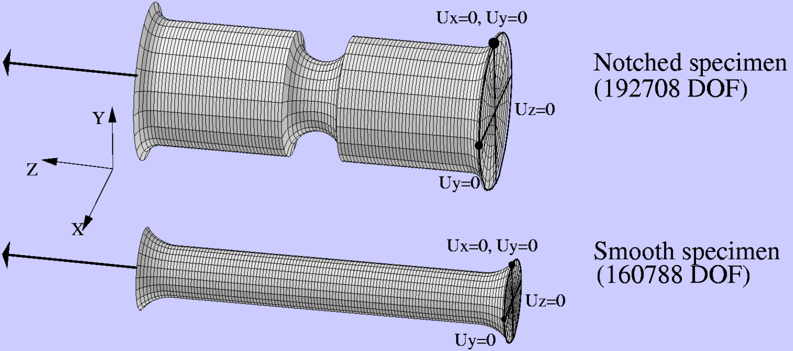

Mesh dependent parameters : band width, maximum plastic strain rate

Mesh quasi-independent parameters : band velocity, band strain

# Outline

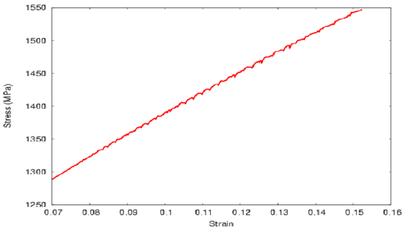
# Simulation of 3D specimens (1)

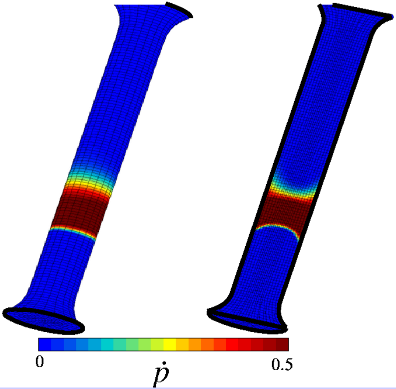
## Model of 3D specimens



# SIMULATION OF 3D SPECIMENS (2)

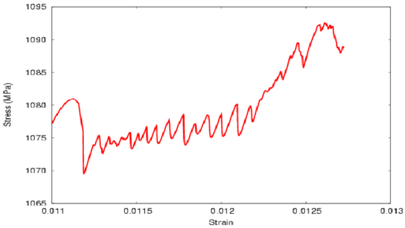
## Smooth axisymmetric specimen at $10^{-1}s^{-1}$

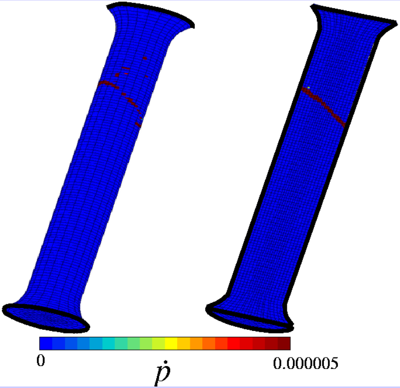


# SIMULATION OF 3D SPECIMENS (3)

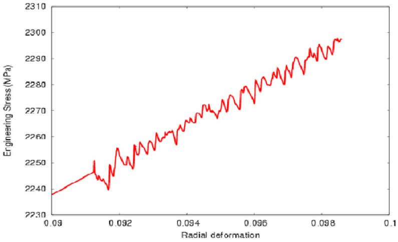
## Smooth axisymmetric specimen at $10^{-6}\text{s}^{-1}$

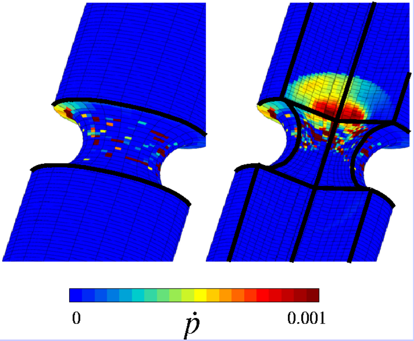


# Simulation of 3D specimens (4)

## Notched axisymmetric specimen at $10^{-3}\mathrm{s}^{-1}$



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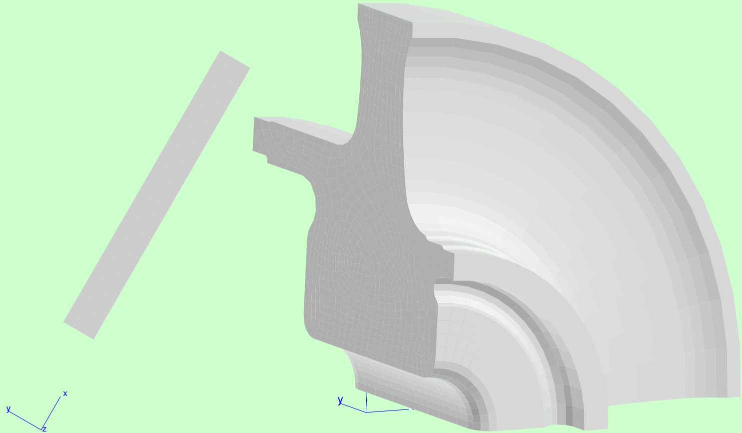
Application to rotating disks

## Conclusion

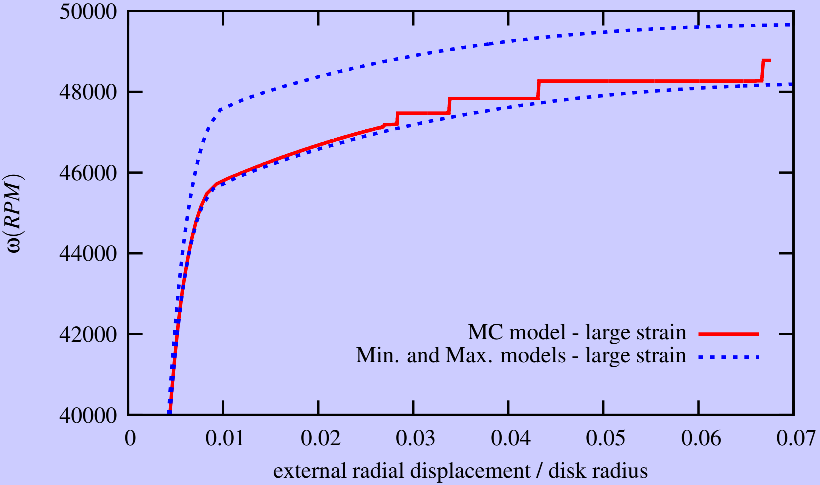
# ROTATING DISK AT 500°C

## Geometries of disks



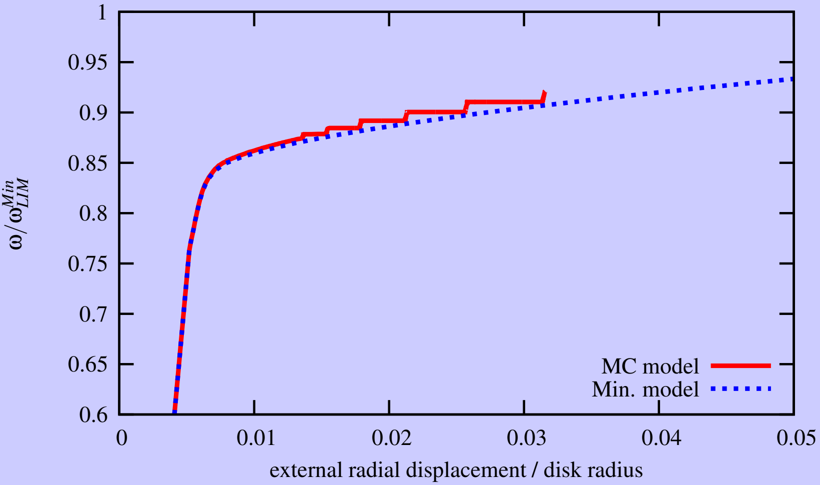
# LIMIT ROTATION RATE (1)

## Global equilibrium curve - Simplified disk



# Limit rotation rate (2)

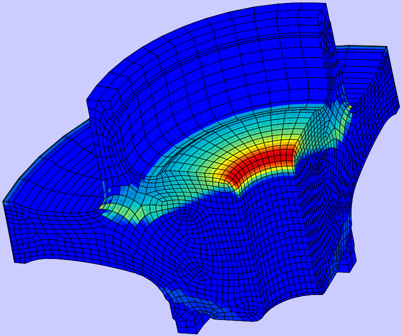
## Global equilibrium curve - Actual disk



# Strain rate localization (2)

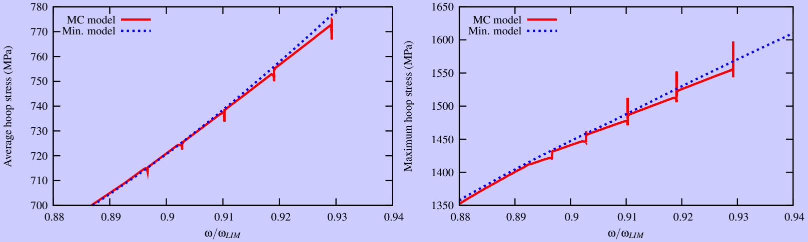
## Symmetry breaking

### Band shape



# Stress analysis

## Average and maximal PK1 hoop stress



# Outline

## Burst prediction at room temperature

- Mechanical behavior of Udimet720 at room temperature
- Material parameter identification and validation
- Mechanical problem of a rotating disk - Existing criteria
- Numerical methods
- Results

## Simulation of the Portevin - Le Chatelier effect

- Mechanical behavior of Udimet720 at 500˚C
- Material parameter identification
- Analysis of strain rate localization phenomena
- Objectivity of simulations
- Large scale simulations of 3D specimens
- Application to rotating disks

## Conclusion

# Conclusion

## At room temperature

- Limit load of rotating disk can be evaluated from large strain finite element simulation.
- The yield criterion is essential.
- Burst rotation rate of an experimental disk is accurately predicted from a limit plastic analysis.

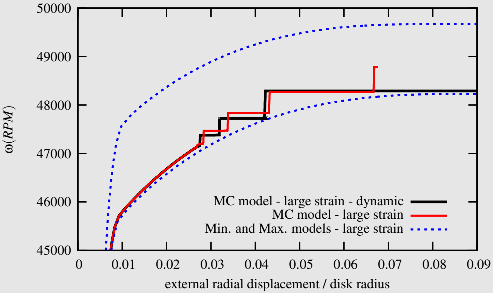
## At 500 ˚C

- An elastoviscoplastic model taking into account strain ageing is used to simulate the PLC effect.
- The sensitivity of this model towards mesh density is considered.
- Simulations are performed on 3D geometries on a large range of strain rate.
- The min. disk response at 500˚C provides a min. value for the limit rotating rate.

# PROSPECTS (1)

## ABOUT DISKS

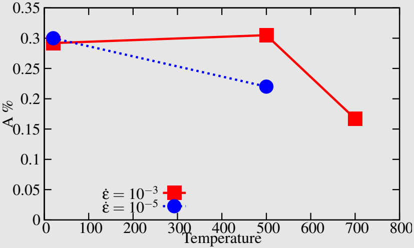
- Thermal effects
- Non-linear dynamic simulations on disks
- FE simulations taking into account PLC - Implicit Dynamic - Finite Strain
- Burst tests at operating conditions

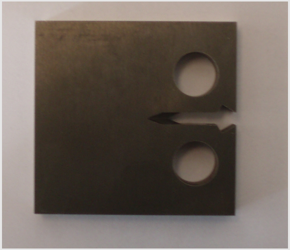


# PROSPECTS (2)

## ABOUT FAILURE AND PLC EFFECT

- Observation of fracture faces at various temperature
- Loss of ductility
- Test on CT specimens



# Prospects (3)

## about simulations of PLC effect

- Non-local strain ageing model
- Large scale computation using strain ageing model
- Prediction of elastoviscoplastic instabilities (PLC) in complex structures

# Composition and Microstructure

## Chemical composition

| Element | Mini (weight %) | Maxi (weight %) |
|---|---|---|
| C | 0.008 | 0.025 |
| Cr | 15.5 | 16.5 |
| Mo | 2.75 | 3.25 |
| W | 1.0 | 1.5 |
| Al | 2.25 | 2.75 |
| Co | 14 | 15.5 |
| Ti | 4.75 | 5.25 |
| Fe | | 0.5 |
| B | 0.01 | 0.02 |
| Zr | 0.025 | 0.050 |
| Ni | Rem. | Rem. |

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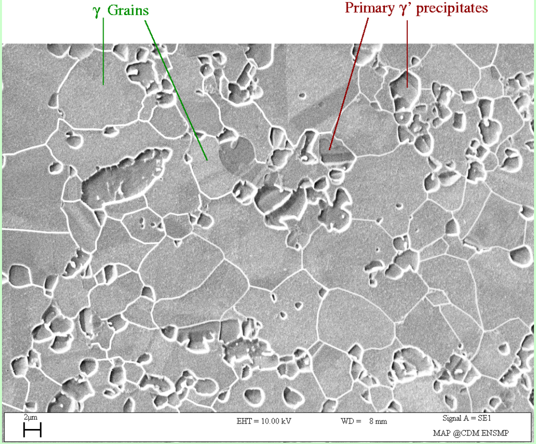
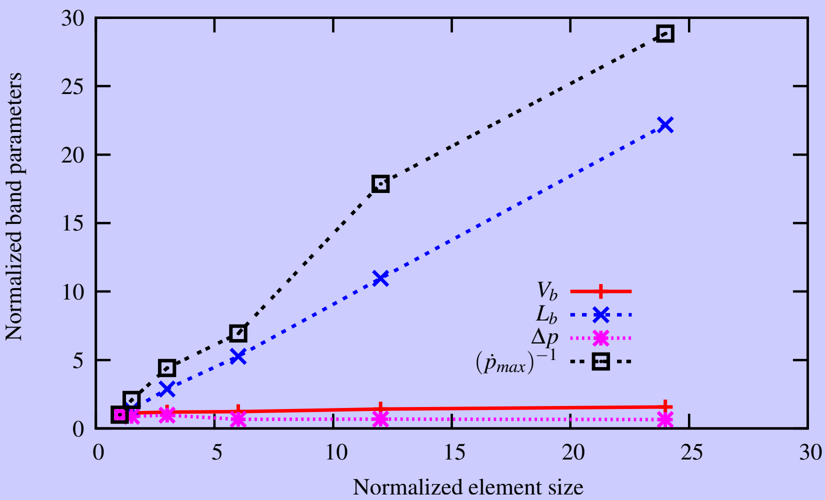
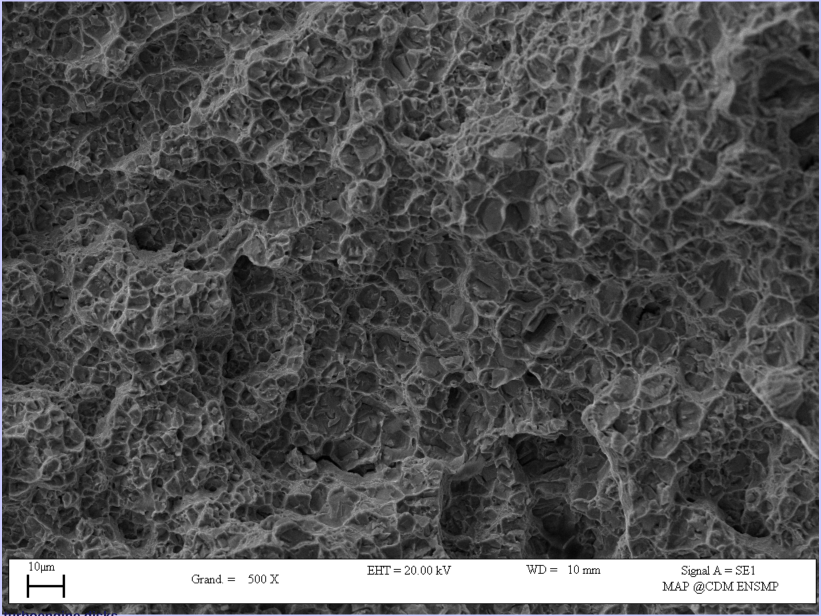
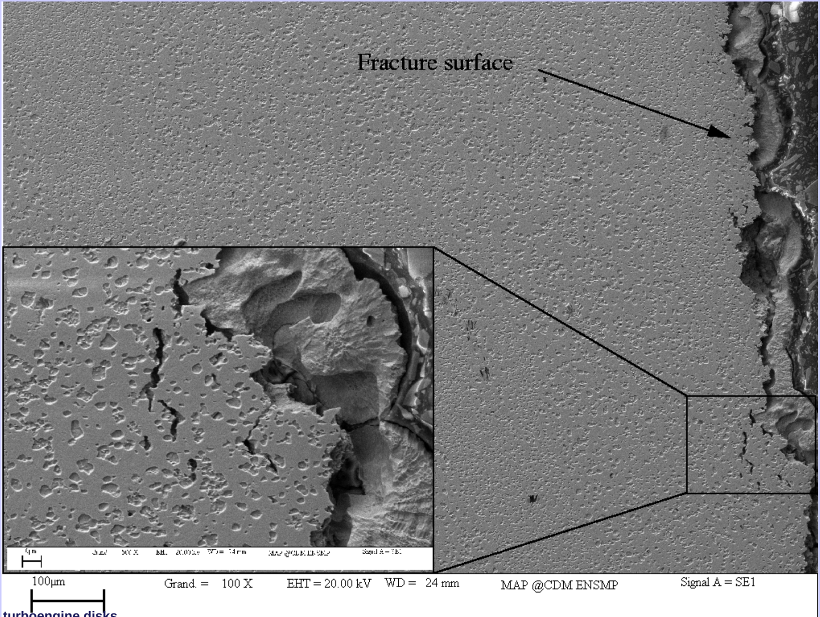
## Micro-structure of Udimet720

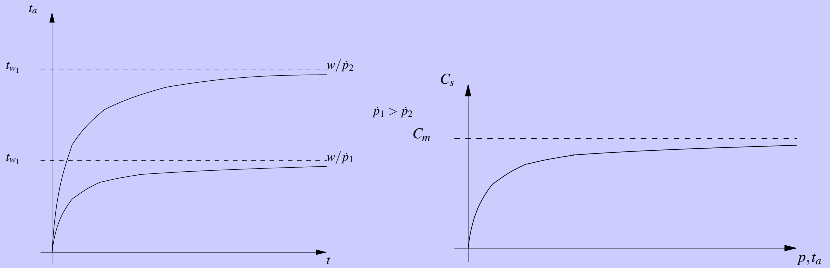


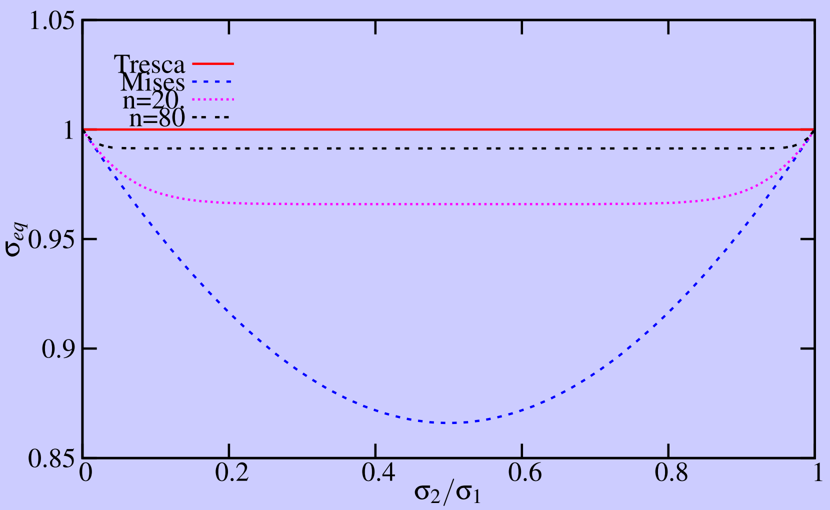


30
25
20
15
10
5
0
0
5
10
15
20
25
30
Normalized band parameters
Normalized element size
$V_b$
$L_b$
$\Delta p$
$(\dot{p}_{max})^{-1}$


10µm
Grand. = 500 X
EHT = 20.00 kV
WD = 10 mm
Signal A = SE1
MAP @CDM ENSMP


Fracture surface
100µm
Grand. = 100 X
EHT = 20.00 kV
WD = 24 mm
MAP @CDM ENSMP
Signal A = SE1


$t_a$
$t_{w_1}$
$w/\dot{p}_2$
$t_{w_1}$
$w/\dot{p}_1$
$t$
$\dot{p}_1 > \dot{p}_2$
$C_s$
$C_m$
$p, t_a$


1.05
1
0.95
0.9
0.85
0
0.2
0.4
0.6
0.8
1
Tresca
Mises
n=20.
n=80
σeq
σ2/σ1

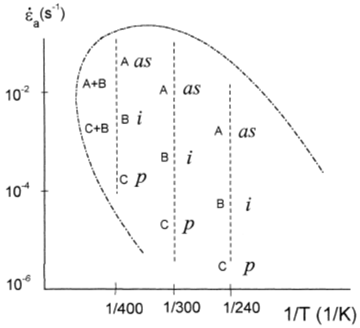



Fig. 3. Classification of the types of serrations and stress drop statistics in an $(\dot{\varepsilon}_a, 1/T)$ map. The boundaries between various regions are drawn in a semi-schematic way. The area where the PLC effect occurs is delineated by the horseshoe-shaped curve.

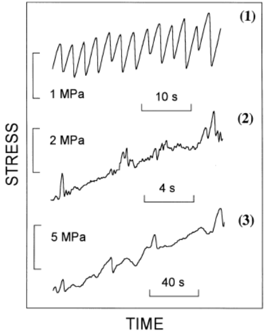



Fig. 4. Fragments of deformation curves for a recrystallized polycrystal at different conditions. From top to bottom: (1) $\dot{\varepsilon}_a = 2.7 \times 10^{-5}$/s, $T = 25$°C (type B); (2) $\dot{\varepsilon}_a = 5.3 \times 10^{-4}$/s, $T = 25$°C (type A); (3) $\dot{\varepsilon}_a = 8.4 \times 10^{-5}$/s, $T = -20$°C (type A).

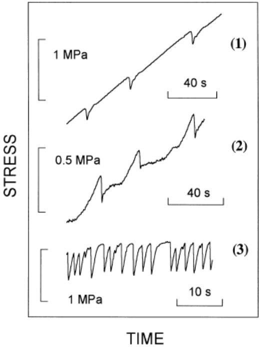



Fig. 6. Fragments of deformation curves for a polycrystal deformed at 100°C: (1) for $\dot{\varepsilon}_a = 4.7 \times 10^{-5}$/s, $\varepsilon \sim 4.3\%$; (2) for an increased strain rate ($\dot{\varepsilon}_a = 9.4 \times 10^{-5}$/s, $\varepsilon \sim 6\%$); (3) for a higher strain ($\dot{\varepsilon}_a = 9.4 \times 10^{-5}$/s, $\varepsilon \sim 14\%$).

## Uniqueness and stability

- Uniqueness :

*Uniqueness is ensured if* $\forall(\underline{\boldsymbol{V}}_1, \underline{\boldsymbol{V}}_2)$ *kinematically admissible* ($\underline{\boldsymbol{V}}_1 = \underline{\boldsymbol{V}}_2 = \underline{\boldsymbol{V}}_i$ on $\partial\Omega_0^1$),

$$\int_{\Omega_0} \Delta\dot{\underset{\sim}{\boldsymbol{S}}} : \Delta\dot{\underset{\sim}{\boldsymbol{F}}} dv_0 - \left( \int_{\Omega_0} \rho_0 \Delta\underline{\boldsymbol{V}} \, . \frac{\partial \underline{\boldsymbol{f_i}}}{\partial \underline{\boldsymbol{u}}} . \Delta\underline{\boldsymbol{V}} \, dv_0 + \int_{\partial\Omega_0^2} \Delta\underline{\boldsymbol{V}} \, . \frac{\partial \underline{\boldsymbol{T_i}}}{\partial \underline{\boldsymbol{u}}} . \Delta\underline{\boldsymbol{V}} \, ds_0 \right) > 0 \quad (1)$$

$$\textit{with } \Delta(.) = (.)_1 - (.)_2$$

- Stability :

*Equilibrium is stable if* $\forall\underline{\boldsymbol{V}}$ *such that* $\underline{\boldsymbol{V}} = \underline{\boldsymbol{0}}$ on $\partial\Omega_0^1$,

$$\int_{\Omega_0} \dot{\underset{\sim}{\boldsymbol{S}}} : \dot{\underset{\sim}{\boldsymbol{F}}} dv_0 - \left( \int_{\Omega_0} \rho_0 \underline{\boldsymbol{V}} \, . \frac{\partial \underline{\boldsymbol{f_i}}}{\partial \underline{\boldsymbol{u}}} . \underline{\boldsymbol{V}} \, dv_0 + \int_{\partial\Omega_0^2} \underline{\boldsymbol{V}} \, . \frac{\partial \underline{\boldsymbol{T_i}}}{\partial \underline{\boldsymbol{u}}} . \underline{\boldsymbol{V}} \, ds_0 \right) > 0 \qquad (2)$$

## Small strain - Simple tension

- Elastoplastic case : $\dot{\underset{\sim}{\sigma}} = \underset{\approx}{\boldsymbol{D}}\dot{\underset{\sim}{\varepsilon}}$
- Indicator of uniqueness : $\Delta\dot{\underset{\sim}{\sigma}} : \Delta\dot{\underset{\sim}{\varepsilon}} = 0 \;\rightarrow\; \det(\underset{\approx}{\boldsymbol{D}}) = 0 \;\rightarrow\; H = 0$
- Indicator of stability : $\dot{\underset{\sim}{\sigma}} : \dot{\underset{\sim}{\varepsilon}} = 0$
- Indicator of ellipticity : $\det(\underline{\boldsymbol{n}}\,\underset{\approx}{\boldsymbol{D}}\,\underline{\boldsymbol{n}}) = 0$ (Rice76)

  $\rightarrow\; H = 0$ (plane stress)

  $\rightarrow\; H = -E/4$ (3D)

## Viscoplasticity - Linear perturbation method

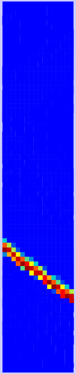
$$\Delta \underline{\boldsymbol{u}} = \delta \underline{\boldsymbol{U}} \exp(iq\underline{\boldsymbol{n}}.\underline{\boldsymbol{x}} + \lambda t) \quad (3)$$

$$\Delta \underset{\sim}{\boldsymbol{\sigma}} = \underset{\approx}{\boldsymbol{H}}_p(\lambda) : \Delta \underset{\sim}{\boldsymbol{\varepsilon}} \quad (4)$$

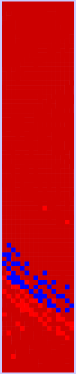
$$\exists \underline{\boldsymbol{n}} \text{ such that } \det\left(\underline{\boldsymbol{n}}.\underset{\approx}{\boldsymbol{H}}_p(\lambda).\underline{\boldsymbol{n}}\right) = 0 \text{ for } \lambda \gg \lambda_h \quad (5)$$

## Application to MC model

$\dot{p}$



$\dot{\underset{\sim}{\sigma}} : \dot{\underset{\sim}{\varepsilon}}$



$\mathrm{MIN}\left[\left(\det(\underline{\boldsymbol{n}}\,\underset{\approx}{\boldsymbol{D}}\,\underline{\boldsymbol{n}})\right)\right]$