Cosserat Media

Conventional continuum mechanics approaches cannot incorporate any intrinsic material length scale. However real materials often have a number of important length scales, which must be included in any realistic model (e.g., grains, particles, fibers, and cellular structures). So-called nonlocal theories can be used to account for size effects in the mechanical behavior of materials. The departure from local theories begins with the Cosserat continuum. A Cosserat (or micropolar) medium is a continuous collection of particles that behave like rigid bodies. Accordingly, each material point is endowed with translational and rotational degrees of freedom, that describe its displacement and the rotation of an underlying microstructure. The material may then oppose couple stresses to the development of curvature i.e., gradient of microrotation. Although the idea of introducing couple stresses in the continuum modeling of solids goes back to Voigt and the Cosserat brothers (1909), the mechanics of generalized continua really culminated in the late 1950s and 1960s (Kröner 1968). The recent renewal of Cosserat mechanics is due, on the one hand, to the dramatic increase of computational capabilities and, on the other hand, to the development of local strain field measurement methods (Bertram and Sidoroff 1998). For, Cosserat effects can arise only if the material is subjected to nonhomogeneous straining conditions.

1. *Mechanics of Cosserat Media*

1.1 Kinematics

The Cartesian components of the displacement and rotation vectors are respectively denoted by u_i and φ_i. The Cosserat theory is presented here within the framework of small perturbations. The Cosserat microrotation R_{ij} relates the current state of a triad of orthonormal directors attached to each material point to the initial one:

$$R_{ij} = \delta_{ij} - \varepsilon_{ijk}\varphi_k \tag{1}$$

where δ_{ij} is Kronecker's symbol and ε_{ijk} the signature of the permutation (i, j, k). The associated Cosserat deformation e_{ij} and torsion-curvature (or wryness) tensor κ_{ij} read:

$$e_{ij} = u_{i,j} + \varepsilon_{ijk}\varphi_k, \quad \kappa_{ij} = \varphi_{i,j} \tag{2}$$

where subscript i denotes partial derivation with respect to space variable x_i.

1.2 Balance Equations

It is assumed that the transfer of the interaction between two particles of the body through a surface element $n_j \mathrm{d}S$ occurs not only by means of a traction vector $t_i \mathrm{d}S$ but also by means of a moment vector $m_i \mathrm{d}S$ (Fig. 1). Surface forces and couples are then represented by the generally nonsymmetric force–stress and couple–stress tensors σ_{ij} and μ_{ij} (units MPa and MPa m) respectively:

$$t_i = \sigma_{ij} n_j, \quad m_i = \mu_{ij} n_j \tag{3}$$

The force and couple stress tensors must fulfill the equations of balance of momentum and of balance of moment of momentum:

$$\sigma_{ij,j} + f_i = \rho \ddot{u}_i \tag{4}$$

$$\mu_{ij,j} - \varepsilon_{ikl}\sigma_{kl} + c_i = I\ddot{\phi}_i \tag{5}$$

where volume forces f_i, volume couples c_i, mass density ρ, and isotropic rotational inertia I have been introduced.

1.3 Constrained Cosserat Medium

In the often used couple–stress or Koiter theory, the Cosserat microrotation is constrained to follow the material rotation given by the skew-symmetric part of the deformation gradient:

$$\phi_i = -\frac{1}{2}\varepsilon_{ijk} u_{j,k} \tag{6}$$

The associated torsion-curvature and couple stress tensors are then traceless. The antimetric force stresses degenerate into reaction forces directly given by Eqn. (5). If a Timoshenko beam is regarded as a one-dimensional Cosserat medium, constraint (6) then is the counterpart of the Euler–Bernoulli condition.

1.4 Constitutive Equations

The resolution of the previous boundary value problem requires constitutive relations linking the deformation and torsion-curvature tensors to the force- and couple-stresses. The example of linear isotropic elasticity is given here:

$$\sigma_{ij} = \lambda e_{kk}\delta_{ij} + 2\mu e_{(i,j)} + 2\mu_c e_{\{i,j\}} \tag{7}$$

$$\mu_{ij} = \alpha\kappa_{kk}\delta_{ij} + 2\beta\kappa_{(i,j)} + 2\gamma\kappa_{\{i,j\}} \tag{8}$$

where $e_{(i,j)}$ and $e_{\{i,j\}}$ respectively denote the symmetric and skew-symmetric part of e_{ij}. Four additional elasticity moduli appear in addition to the classical Lamé constants. Several characteristic lengths can then be defined; among them and after Nowacki (1986):

$$l_1^2 = \frac{(\mu+\mu_c)(\beta+\gamma)}{4\mu\mu_c}, \quad l_2^2 = \frac{\alpha+2\beta}{4\mu_c} \tag{9}$$

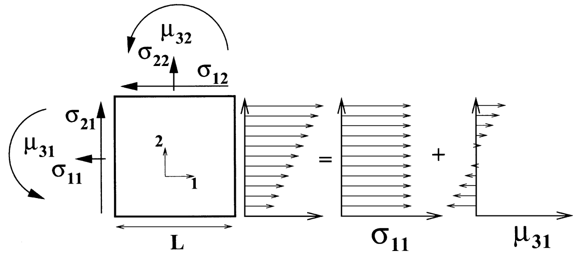

Figure 1
Volume element of Cosserat mechanics.

Cosserat elastoplasticity also is a well-established theory. Von Mises classical plasticity can be extended to micropolar continua in a straightforward manner (see references in Mühlhaus 1995). The yield criterion depends on both force- and couple-stresses:

$$f(\sigma, \mu) = \sqrt{\tfrac{3}{2}(a_1 s_{ij} s_{ij} + a_2 s_{ij} s_{ji} + b_1 \mu_{ij} \mu_{ij} + b_2 \mu_{ij} \mu_{ji})} - R \tag{10}$$

where s denotes the stress deviator and a_i, b_i are material parameters. It can be seen that the ratio a_i/b_i has the dimension of the square of a length. The normality rule can be used to compute the amount and direction of plastic flow and curvature. The isotropic hardening variable R can be a function of cumulative plastic deformation and curvature.

2. *Cosserat Materials*

The Cosserat continuum provides a relevant description of the mechanical behavior of several classes of materials with microstructure; some examples are briefly reported here.

2.1 Liquid Crystals

Liquid crystals naturally possess directors that make them good candidates for Cosserat modeling. Linear Cosserat viscoelasticity has been used to describe the shear flow of nematic liquid crystals, their change of orientation due to magnetic fields and the bending and torsion of smectic crystals (Lee and Eringen 1973).

2.2 Rocks and Granular Media

The inelastic response and the stability of block-systems in masonry are adequately predicted using the Cosserat continuum (Besdo 1985). The material degrees of freedom are related to the rigid body motion of individual blocks. The approach has also been successfully applied to faulted rocks in geomechanics. The flow of mineral charges in blast furnaces or in silos is often simulated experimentally by the motion of stacked cylinders that can both translate and rotate. The elastic and inelastic behavior of granular materials like sands and soils can also be described properly by generalized continua.

2.3 Cellular Solids and Composites

Beam networks, frames and reinforced grids display strong Cosserat effects when their bending stiffness is relatively high compared to their shear stiffness. Accordingly, the Cosserat properties of polymeric foams, bones and other open or closed-cell periodic or random cellular solids have been extensively studied regarding elastic and failure properties. More generally, heterogeneous materials and composites undergoing strong overall deformation gradients can be modeled as homogeneous Cosserat media. The dependence of the effective properties of metal matrix composites on the size of the particles or fibers can be accounted for by treating the matrix as a Cosserat or generalized medium.

2.4 Dislocated Crystals

The idea that a crystal element containing dislocations can carry both surface forces and couples goes back to Günther and MacClintock in the late 1950s. The influence of so-called geometrically necessary dislocations on the hardening behavior of metals can be accounted for using generalized continua (Fig. 2). This provides an efficient way to model size effects in crystals (Forest 1998).

3. *Applications*

3.1 Dispersion of Elastic Waves in Heterogeneous Elastic Solids

Heterogeneous materials can become dispersive under dynamical conditions in the elastic regime when the incident wavelength and the size of the heterogenities have the same order of magnitude. Longitudinal waves in a Cosserat medium remain nondispersive as in the classical case but dispersion relations exist for transverse and rotational waves. They can be used to interpret the results of nondestructive testing methods.

3.2 Regularization Methods in the Computation of Localization Phenomena

Finite element simulations of localization deformation modes, like shear banding in elastoplastic materials exhibiting a strain-softening behavior, lead to spurious mesh-dependence as a result of the loss of ellipticity of the field equations (de Borst 1991). Generalized continua have been used as regularization methods to

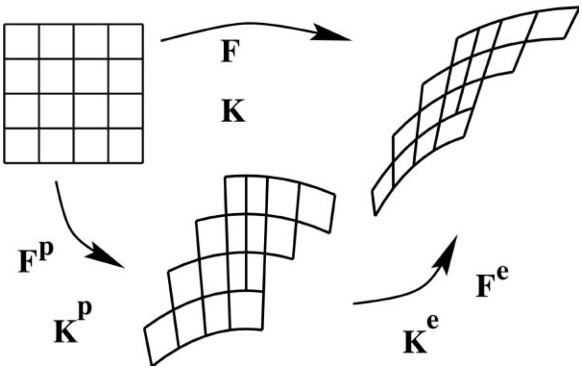

Figure 2
Cosserat crystal plasticity: initial, intermediate, and final configurations (F and K denote the deformation gradient and curvature).

predict mesh-independent load-displacement curves. The finite width of shear bands can be predicted using the Cosserat approach. Applications deal with sands and soils as well as fracture and damage in metals and concrete.

4. Conclusions

The mechanics of generalized continua can be useful in the modeling of heterogeneous materials when the size of the heterogenities (internal length) and the wavelength of the loading conditions (external length) have the same order of magnitude. It may apply when discrete simulations remain untractable and conventional continuum modeling breaks down. The corresponding validity range may be sometimes rather narrow, which contributes to a long-standing debate on the physical interest of generalized continua.

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