

# Mines ParisTech

## A short personal view

Samuel Forest  
*Centre des Matériaux, Mines ParisTech CNRS,  
PSL Research University, France*



# Mines ParisTech



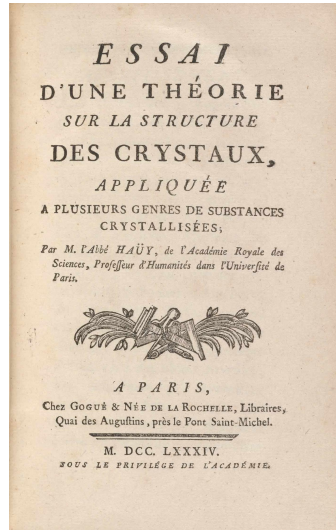
- The last royal engineering school created in 1783
- 180 “general” engineers and 100 PhD students graduate each year
- 250 permanent researchers and professors
- Research towards industry in: Materials sciences, Geosciences, Applied mathematics, Energy, Social sciences (Economics, sociology...)
- Several locations: Luxembourg garden, Evry, Fontainebleau, Sophia-Antipolis
- Member of ParisTech: A network of 10 graduate schools
- Member of PSL Research University, a community of 10 universities/schools  
Paris Sciences Lettres, 17000 students, 4500 researchers

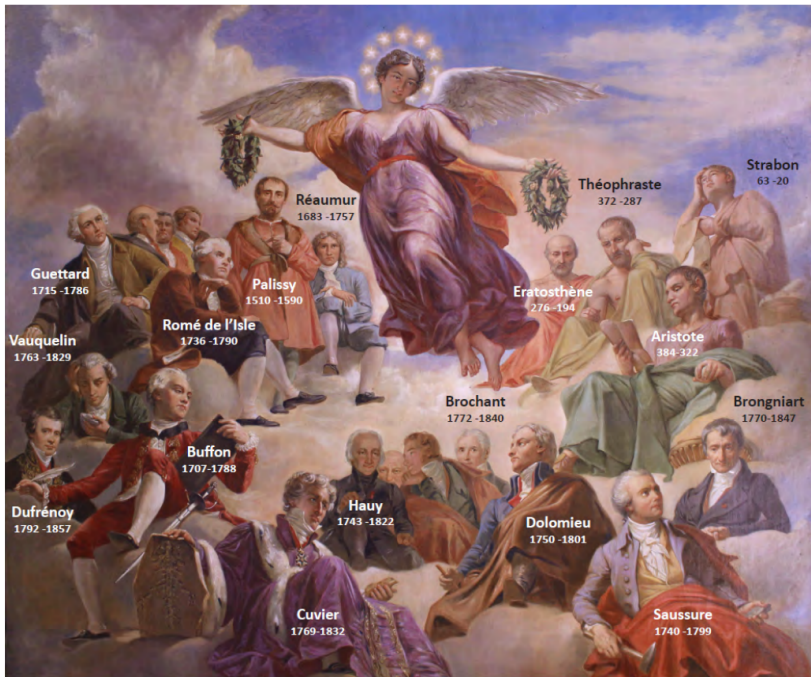
# Theory and practice



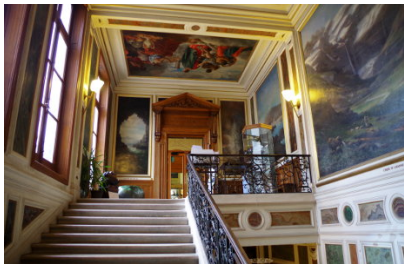
**Déodat de Dolomieu** gave its name to the Dolomites, a mountain range of northeastern Italy, to its building sedimentary rock, *dolomite*, and to a crater of the volcano *Piton de la Fournaise*.

# René-Just Haüy: Founder of crystallography





# Mineralogy museum



One of the largest collection in the world: 100000 samples, 4000 exposed  
(British museum, Freiberg museum)  
in connection with *Museum d'histoire naturelle*

## A president in the mineralogy museum



## A president in the mineralogy museum





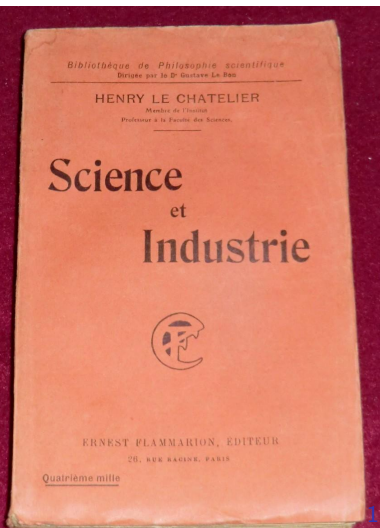
## A president in the mineralogy museum



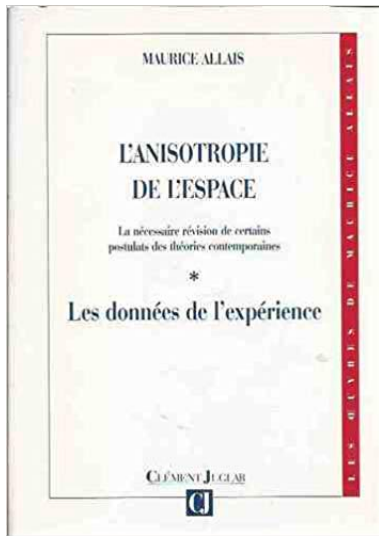
# A president among the engineering students



# Henry Le Chatelier



# Maurice Allais: Nobel prize in economic sciences (1988)



M. Allais wanted a Nobel prize in Physics!

# Local approach to fracture

Fracture mechanics in nuclear engineering and aeronautics: Role of material microstructure

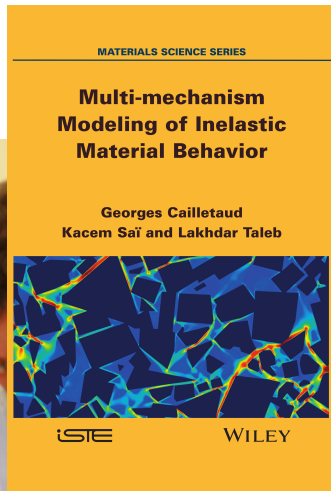


Dominique François, André Pineau and André Zaoui

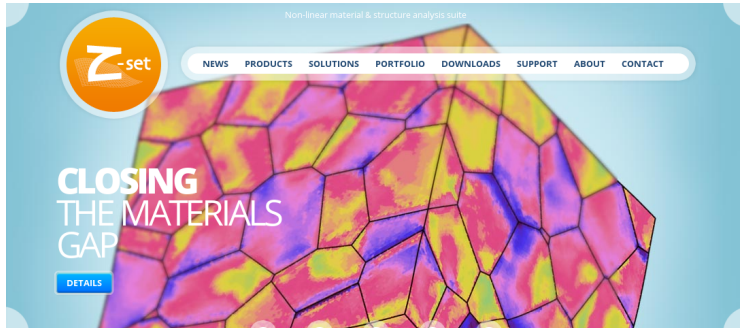
# Material modelling and computational mechanics



Georges Cailletaud and Jean-Louis Chaboche



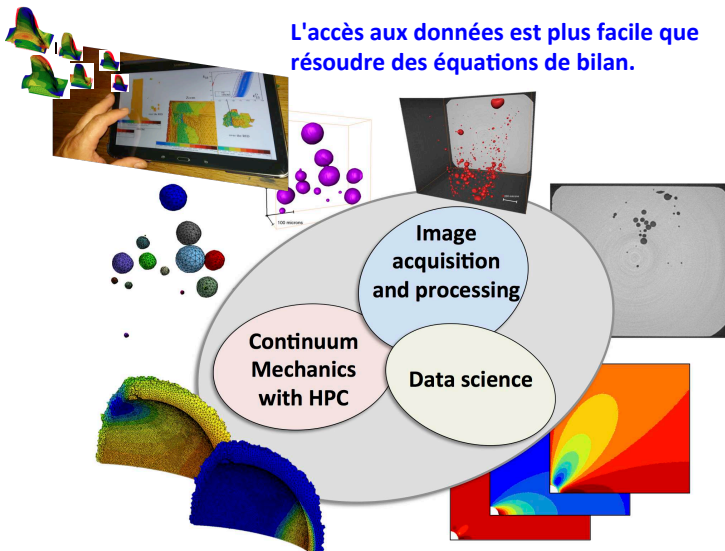
# Z-set Non-linear material & structure analysis suite



[www.zset-software.com](http://www.zset-software.com)

# Mechanics and materials at the age of Big Data

L'accès aux données est plus facile que résoudre des équations de bilan.



Full order models (FOM) – Reduced order models (ROM) – Data science



## Thanks to Carlo Sansour!



with Gérard Maugin (CISM course in Udine, Italy, 2007)

# Modelling the mechanical behaviour of nickel base foams for DPF applications

A. Burteau, T. Dillard, J.D. Bartout, F. N'Guyen, S. Forest, Y.  
Bienvenu

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Samuel.Forest@ensmp.fr

S. Saberi

INCO SP, Mississauga, Canada

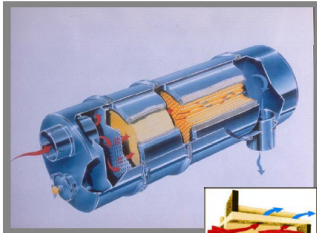


# Plan

- 1 Introduction
- 2 Microstructure evolution during material processing and deformation
  - Nickel based open cell foams
  - Microtomography analysis
- 3 Phenomenological modelling of plasticity and fracture
  - Micromorphic anisotropic compressible plasticity model
  - Simulation of crack propagation
- 4 Micromechanical approach
  - Strut behaviour
  - Representative Volume Element size
- 5 Conclusions

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« Source : Eminox »

- Environment : protocol of Kyoto (1997)
- Petrol engine vs. Diesel engine
- Reduction of greenhouse gas  
=> increase in the rejection of particles
- EURO V standard in 2008

- Necessity of the PF: technology « ceramics » vs. « metals »
- Advantage of metallic PF:
  - better catalytic efficiency
  - easier designed
  - easier recycled



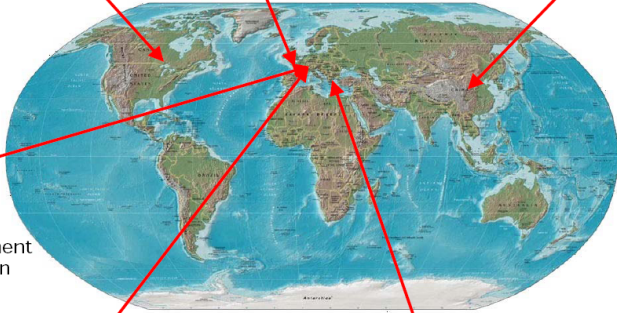
# INCO special products

- Management
- Optimisation of the filters



- Mechanical properties
- Characterization of the foams

- Production of PU and nickel foams



**IFAM**  
Institut für  
Fließungstechnik  
Materialforschung

- Alloy development
- Industrialisation



- Catalytic coating



- Thermodynamics
- Test in working conditions

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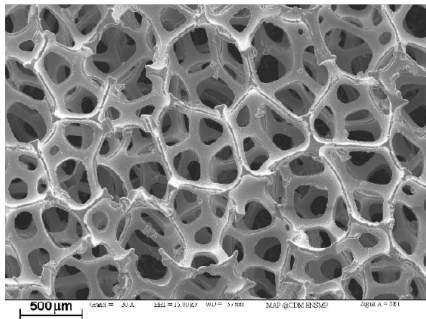
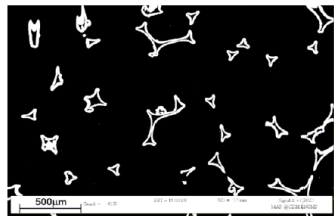
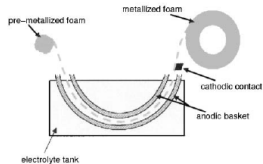
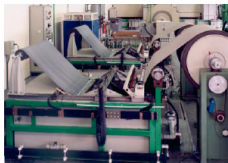
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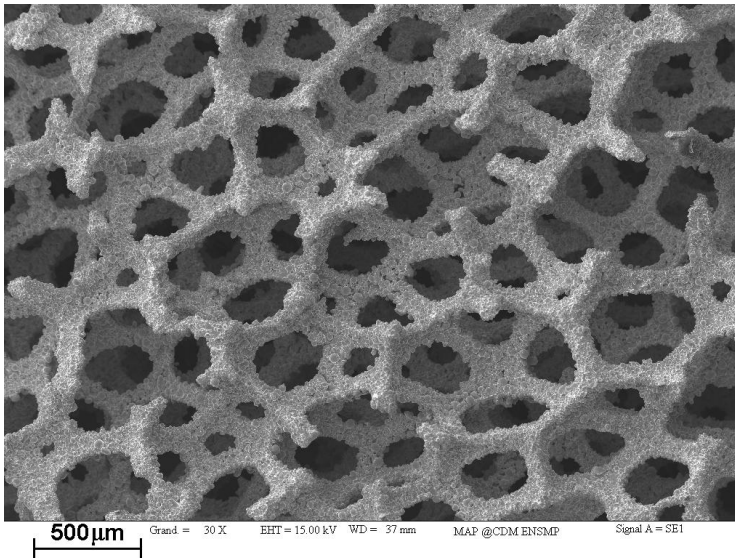
# Processing of nickel foams

from PU template

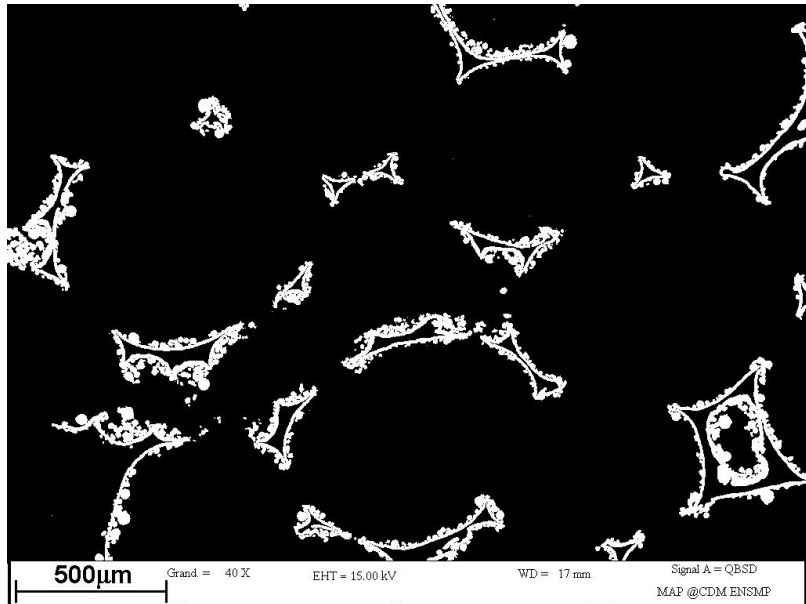


# Microstructure of superalloy foam

composition close to IN625



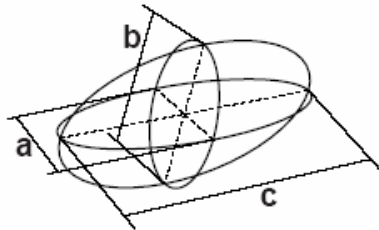
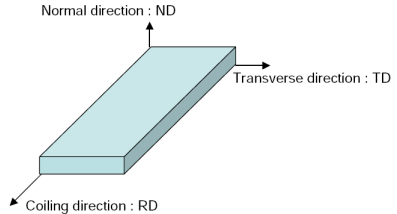
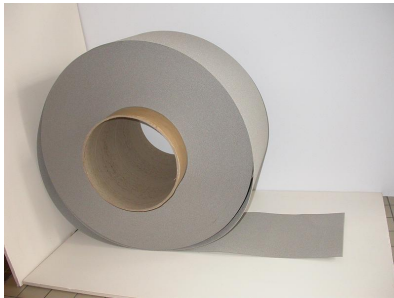
# Microstructure of superalloy foam



# Plan

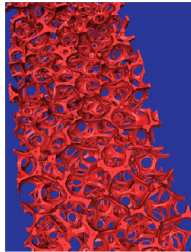
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# Foam sheet axes and cell shape

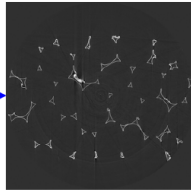


$$a \leq b \leq c$$

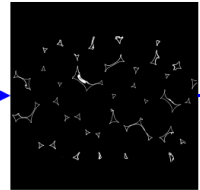
# 3D image analysis from X-ray microtomography experiments at ESRF



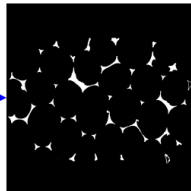
*3D picture built from  
X-ray tomography data*



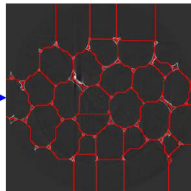
*Slice from X-ray tomography  
data (grey level)*



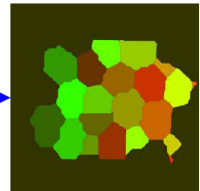
*Binary slice*



*Filled binary slice*

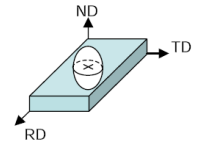
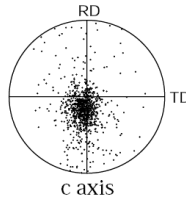
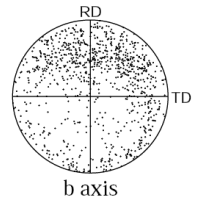
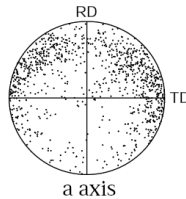
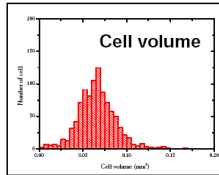
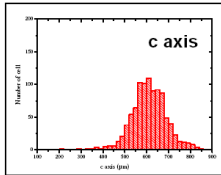
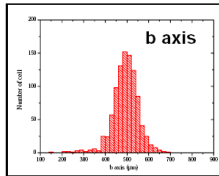
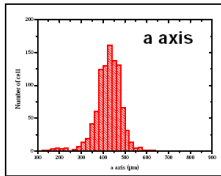


*Segmentation of cells*



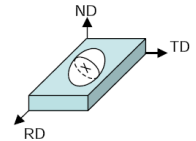
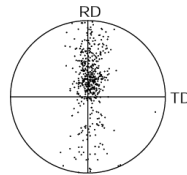
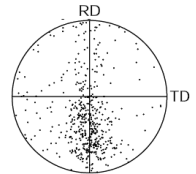
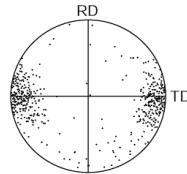
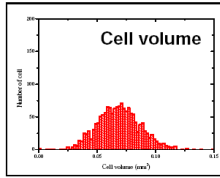
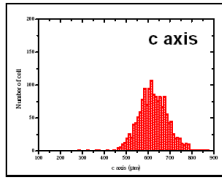
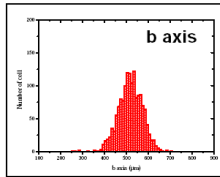
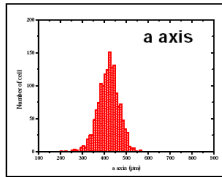
*Cleaned out slice and*

# Cell shape in PU580 template foam



<b>(964)</b>	a axis	b axis	c axis	Cell volume
unit	µm	µm	µm	mm <sup>3</sup>
mean	424	492	612	0.066
Std dev	57	60	81	0.021

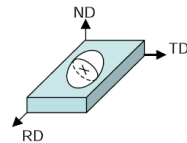
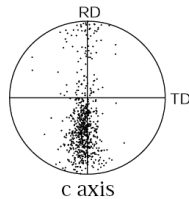
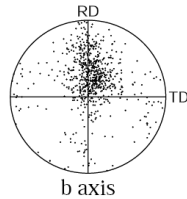
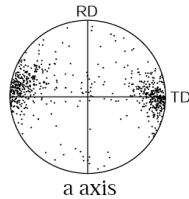
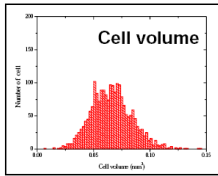
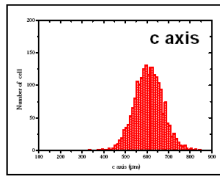
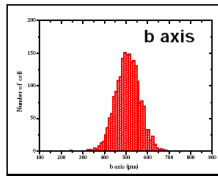
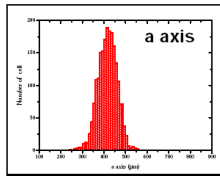
# Cell shape in Ni580 foam



(1502)	a axis	b axis	c axis	Cell volume
unit	μm	μm	μm	mm <sup>3</sup>
mean	418	517	622	0.0687
Std dev	48	56	70	0.019



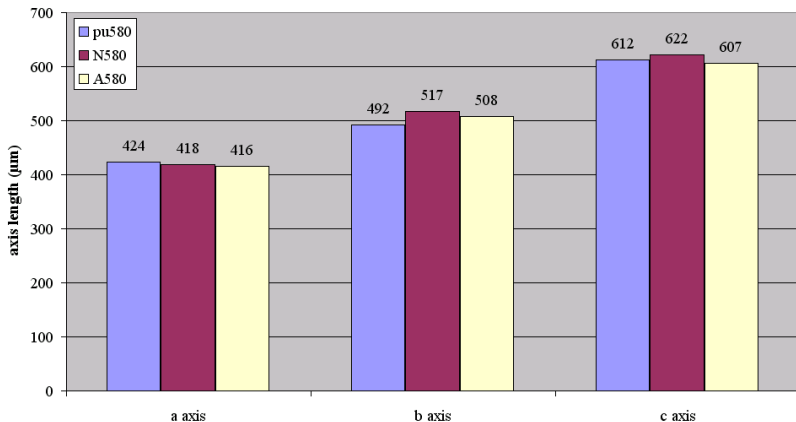
# Cell shape in A580 foam



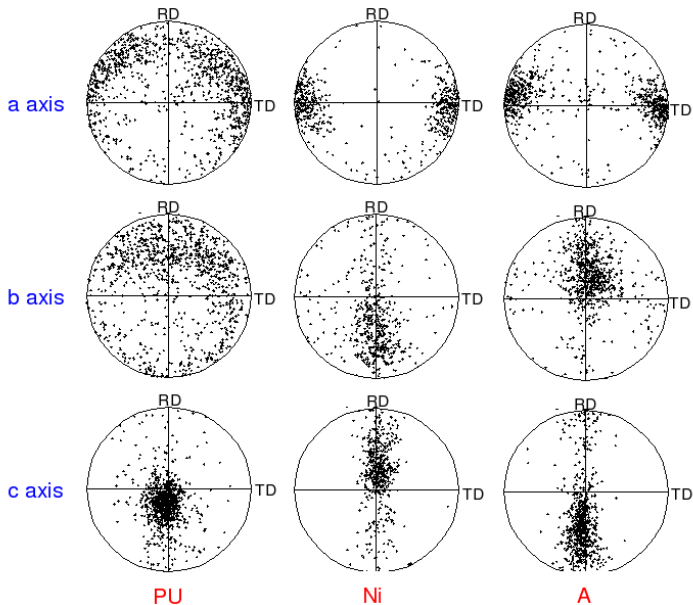
(2071)	a axis	b axis	c axis	Cell volume
unit	$\mu\text{m}$	$\mu\text{m}$	$\mu\text{m}$	$\text{mm}^3$
mean	416	508	607	0.066
Std dev	44	55	65	0.018

# Evolution of cell shape during the process

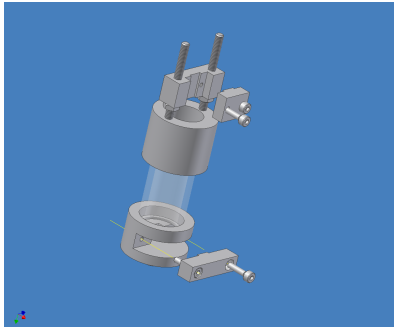
mean cell dimensions of 580  $\mu\text{m}$  foams



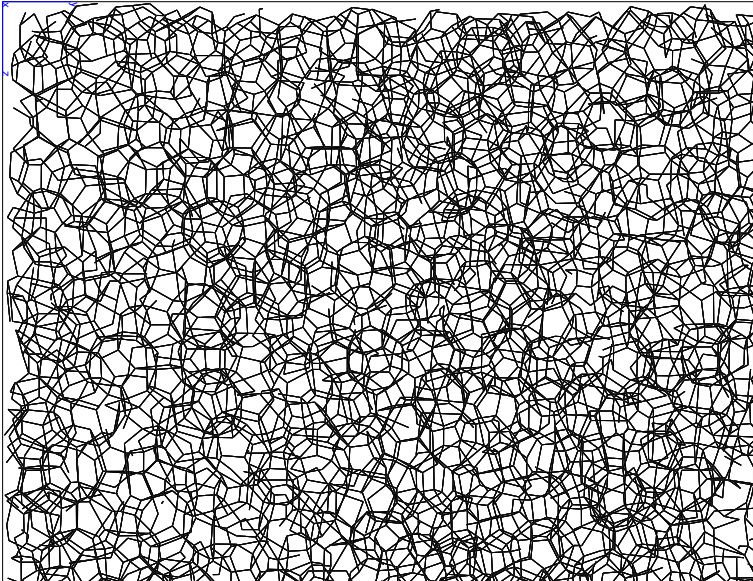
# Evolution of cell orientation during the process



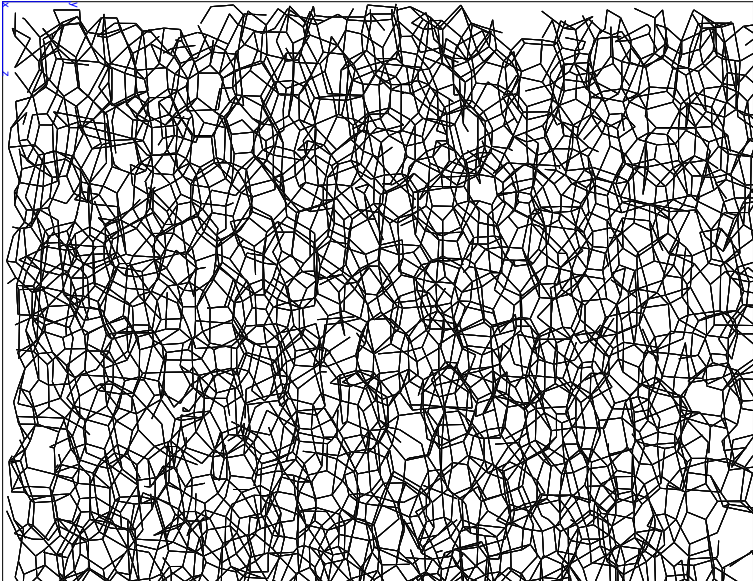
# Deformation of PU foam



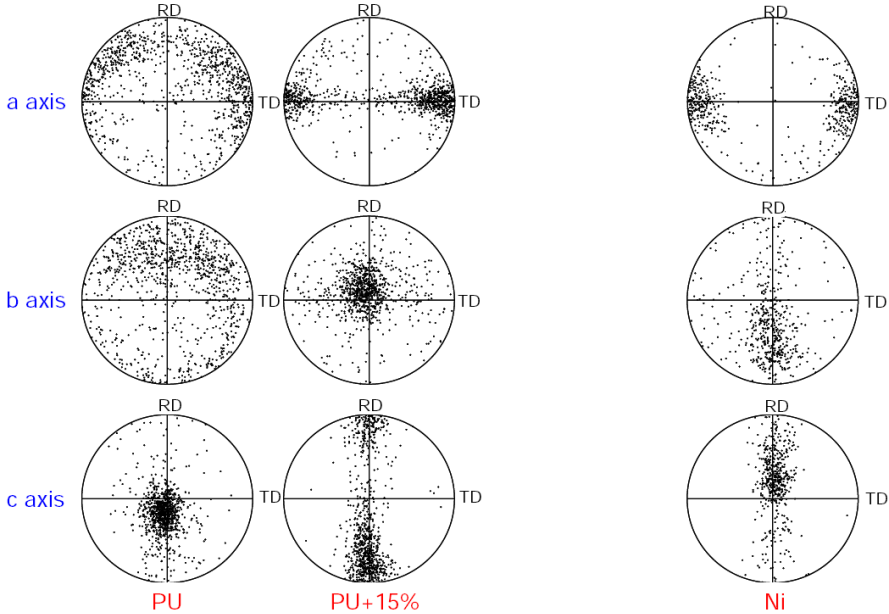
# Evolution of cell orientation after deformation



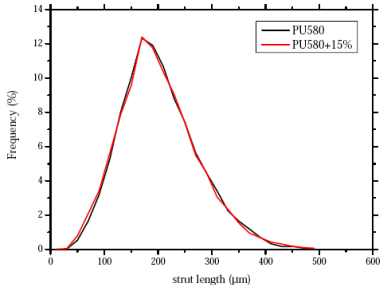
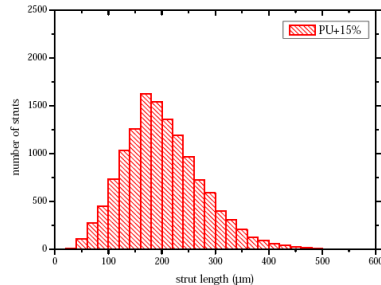
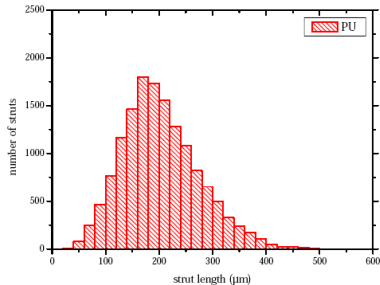
# Evolution of cell orientation after deformation



# Evolution of cell orientation after deformation



# Evolution of strut length after deformation



	PU	PU+15%
Strut length (μm)	203	202



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# Anisotropic Compressible Continuum Plasticity Model

Yield criterion

$$f(\underline{\underline{\sigma}}) = \sigma_{eq} - R, \quad \sigma_{eq} = \left( \frac{3}{2} C \underline{\underline{\sigma}}^{\text{dev}} : \underline{\underline{H}} : \underline{\underline{\sigma}}^{\text{dev}} + F(\underline{\underline{P}} : \underline{\underline{\sigma}})^2 \right)^{\frac{1}{2}}$$

$$[H] = \begin{bmatrix} h_a & 0 & 0 & 0 & 0 & 0 \\ 0 & h_b & 0 & 0 & 0 & 0 \\ 0 & 0 & h_c & 0 & 0 & 0 \\ 0 & 0 & 0 & h_d & 0 & 0 \\ 0 & 0 & 0 & 0 & h_e & 0 \\ 0 & 0 & 0 & 0 & 0 & h_f \end{bmatrix}, \quad [P] = \begin{bmatrix} p & 0 & 0 \\ 0 & q & 0 \\ 0 & 0 & r \end{bmatrix}$$

Normality rule  $\dot{\underline{\underline{\epsilon}}}^p = \dot{p} \frac{\partial f}{\partial \underline{\underline{\sigma}}} = \dot{p} \left( \frac{3}{2} C \underline{\underline{H}} : \underline{\underline{\sigma}}^{\text{dev}} + F(\underline{\underline{P}} : \underline{\underline{\sigma}}) \underline{\underline{P}} \right)$

Consistency condition (cumulative plastic strain  $p$ )

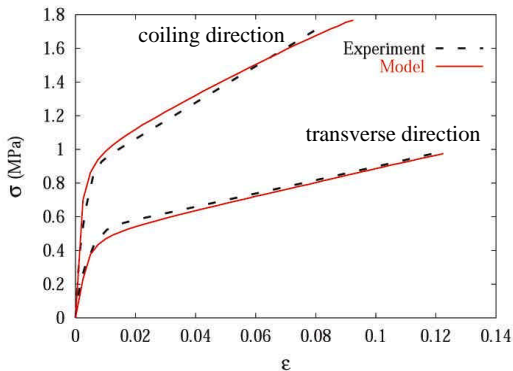
$$\dot{p} = \frac{\frac{\partial f}{\partial \underline{\underline{\sigma}}} : \underline{\underline{E}} : \dot{\underline{\underline{\epsilon}}}}{\frac{\partial f}{\partial \underline{\underline{\sigma}}} : \underline{\underline{E}} : \frac{\partial f}{\partial \underline{\underline{\sigma}}} + \frac{\partial R}{\partial p}}$$

Nonlinear isotropic hardening

$$R = R_0 + Hp + Q(1 - \exp(-bp))$$

## Parameter identification

C	F	$R_0$ (MPa)	Q (MPa)	b	H (MPa)	$h_a$	$h_b$	$h_c$	$h_d$	$h_e$	$h_f$
1	$6,8 \cdot 10^{-4}$	0,30	0,37	3,3	8,17	0,41	1,8	1,75	1,45	1	1



p	q	r
1	37	23

# Metal foam as a micromorphic continuum : The *microfoam* model

*Degrees of freedom at material point*

displacement vector  $\underline{\mathbf{u}}$ , microdeformation tensor  $\underline{\underline{\chi}}$

after [Mindlin, 1964] [Eringen, 1964]

*Deformation measures*

$$\text{strain tensor } \underline{\underline{\varepsilon}} = (\underline{\mathbf{u}} \otimes \nabla + \nabla \otimes \underline{\mathbf{u}})/2$$

$$\text{relative deformation tensor } \underline{\underline{\mathbf{e}}} = \underline{\mathbf{u}} \otimes \nabla - \underline{\underline{\chi}}$$

$$\text{microdeformation gradient } \underline{\underline{\mathbf{K}}} = \underline{\underline{\chi}} \otimes \nabla$$

*Power density of internal forces / generalized stress tensors*

$$\rho^{(i)} = \underline{\underline{\boldsymbol{\sigma}}} : \underline{\underline{\dot{\varepsilon}}} + \underline{\underline{\mathbf{s}}} : \underline{\underline{\dot{\mathbf{e}}}} + \underline{\underline{\mathbf{M}}} : \underline{\underline{\dot{\mathbf{K}}}}$$

*Balance of momentum*

$$(\underline{\underline{\boldsymbol{\sigma}}} + \underline{\underline{\mathbf{s}}}) \cdot \nabla = 0$$

*Balance of micromorphic momentum*

$$\underline{\underline{\mathbf{M}}} \cdot \nabla + \underline{\underline{\mathbf{s}}} = 0$$

*Boundary conditions (traction and double traction vectors)*

$$\underline{\underline{\mathbf{t}}} = (\underline{\underline{\boldsymbol{\sigma}}} + \underline{\underline{\mathbf{s}}}) \cdot \underline{\underline{\mathbf{n}}}, \quad \underline{\underline{\mathbf{T}}} = \underline{\underline{\mathbf{M}}} \cdot \underline{\underline{\mathbf{n}}}$$

*Variational formulation for finite element implementation*

$$\int_V \rho^{(i)} dV = \int_{\partial V} (\underline{\underline{\mathbf{t}}} \cdot \underline{\underline{\mathbf{n}}} + \underline{\underline{\mathbf{T}}} : \underline{\underline{\dot{\chi}}}) dS$$

# The *microfoam* model : Constitutive equations

*Strain partition*

$$\underline{\underline{\varepsilon}} = \underline{\underline{\varepsilon}}^e + \underline{\underline{\varepsilon}}^p, \quad \underline{\underline{\mathbf{e}}} = \underline{\underline{\mathbf{e}}}^e + \underline{\underline{\mathbf{e}}}^p, \quad \underline{\underline{\mathbf{K}}} = \underline{\underline{\mathbf{K}}}^e + \underline{\underline{\mathbf{K}}}^p$$

*Linear elasticity*

$$\underline{\underline{\boldsymbol{\sigma}}} = \underline{\underline{\mathbf{c}}} : \underline{\underline{\varepsilon}}^e, \quad \underline{\underline{\mathbf{s}}} = \underline{\underline{\mathbf{a}}} : \underline{\underline{\mathbf{e}}}^e, \quad \underline{\underline{\mathbf{M}}} = \underline{\underline{\mathbf{A}}} : \underline{\underline{\mathbf{K}}}^e$$

simplified moduli as proposed by [Shu et al., 1999]  $\underline{\underline{\mathbf{M}}} = l_c^2 \underline{\underline{\mathbf{c}}} : \underline{\underline{\mathbf{K}}}^e$

characteristic length  $l_c$

when  $\|\underline{\underline{\mathbf{a}}}\|$  is very large, and if  $\underline{\underline{\mathbf{e}}}^p = 0$ , then  $\underline{\underline{\mathbf{u}}} \otimes \nabla \simeq \underline{\underline{\chi}}$  (second gradient theory)

# The *microfoam* model : Constitutive equations

*Yield criterion*

$$f(\underline{\sigma}) = \sigma_{eq} - R$$

$$\sigma_{eq} = \left( \frac{3}{2} C \underline{\underline{\sigma}}^{\text{dev}} : \underline{\underline{\mathbf{H}}} : \underline{\underline{\sigma}}^{\text{dev}} + F(\underline{\underline{\mathbf{P}}} : \underline{\underline{\sigma}})^2 + a_1 \underline{\underline{\mathbf{s}}}^{\text{dev}} : \underline{\underline{\mathbf{s}}}^{\text{dev}} + a_2 \underline{\underline{\mathbf{s}}}^{\text{dev}} : \underline{\underline{\mathbf{s}}}^{\text{devT}} + b_1 \underline{\underline{\mathbf{M}}} : \underline{\underline{\mathbf{M}}} \right)^{\frac{1}{2}}$$

*Normality rule*

$$\underline{\underline{\dot{\epsilon}}}^P = \dot{p} \frac{\partial f}{\partial \underline{\underline{\sigma}}}, \quad \underline{\underline{\dot{e}}}^P = \dot{p} \frac{\partial f}{\partial \underline{\underline{\mathbf{s}}}}, \quad \underline{\underline{\dot{\mathbf{K}}}}^P = \dot{p} \frac{\partial f}{\partial \underline{\underline{\mathbf{M}}}}$$

*Microfoam:*  $a_1 = a_2 = a_3 = b_1 = 0$ ,  $\|\underline{\underline{\mathbf{a}}}\|$  very large

constrained microdeformation (second gradient), linear relationship between gradient of microdeformation and hyperstress tensor

## Microstrain continuum

degrees of freedom ( $\underline{\mathbf{u}}, \underline{\chi}^s$ )

strain measures :

$$(\underline{\varepsilon}, \underline{\varepsilon} - \underline{\chi}^s, \underline{\chi}^s \otimes \nabla)$$

$$\underline{\sigma} = \underline{\underline{\mathbf{C}}} : (\underline{\varepsilon} - \underline{\varepsilon}^p)$$

$$\underline{\mathbf{s}} = b(\underline{\varepsilon} - \underline{\chi}^s)$$

$$\underline{\underline{\mathbf{S}}} = A\underline{\chi}^s \otimes \nabla$$

$$\operatorname{div}(\underline{\sigma} + \underline{\mathbf{s}}) = 0$$

$$\operatorname{div} \underline{\underline{\mathbf{S}}} + \underline{\mathbf{s}} = 0$$

$$S_{ijk,k} + s_{ij} = 0$$

$$A\chi_{ij,kk}^s + b(\varepsilon_{ij} - \chi_{ij}^s) = 0$$

$$\varepsilon_{ij} = \chi_{ij}^s - l^2 \Delta \chi_{ij}^s$$

Link with “implicit  
gradient-enhanced  
elastoplasticity models”  
[Engelen *et al.*, 2003]

Boundary conditions :

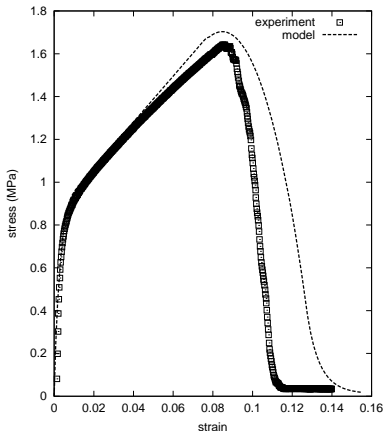
$$(u_i, \chi_{ij}^s) \text{ or} \\ (\sigma_{ij} + s_{ij})n_j, S_{ijk}n_k$$



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# Fracture of nickel foams



simple fracture criterion

$$p = p_{crit} = 0.08$$

limited scatter in  $p_{crit}$

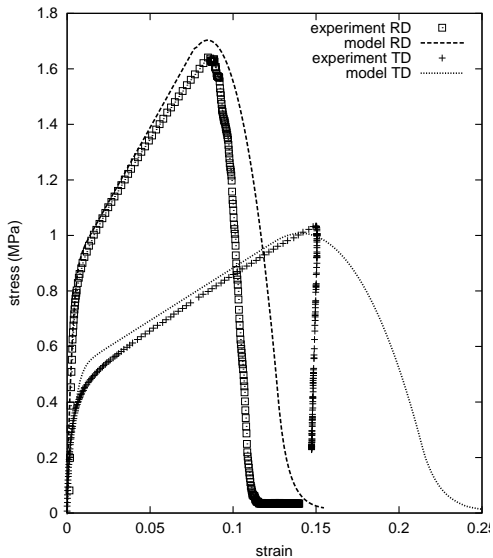
softening law for  $p > p_{crit}$

$$R = R(p > p_{crit})$$

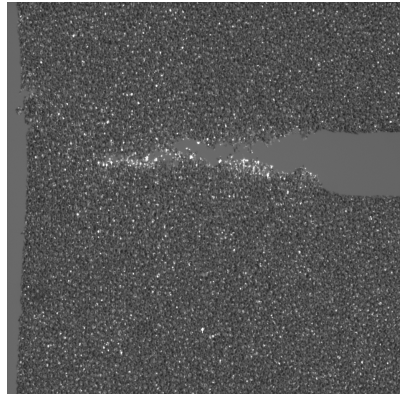
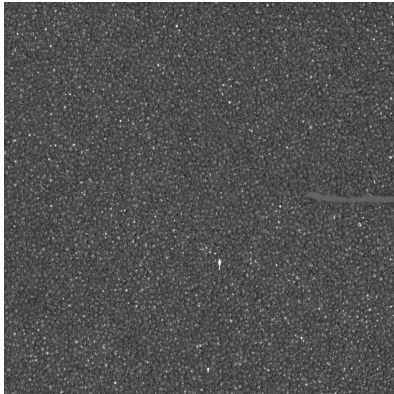
tensile test in direction RD

# Fracture anisotropy of nickel foams

prediction of fracture stress for tension along TD

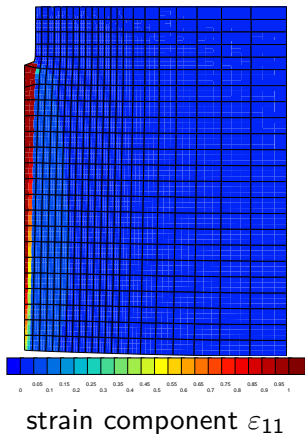
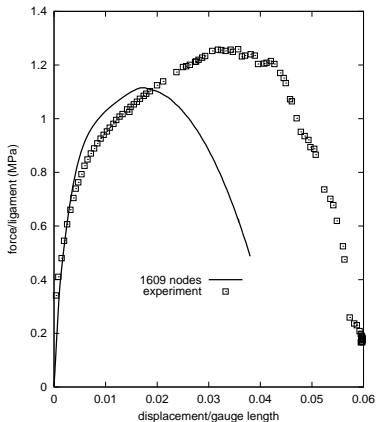


# Tension of central crack nickel foam plate

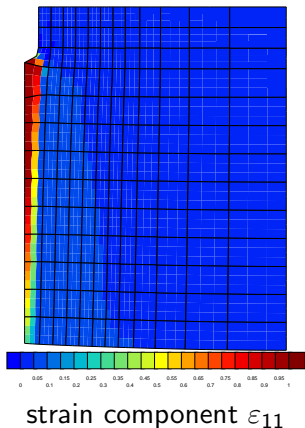
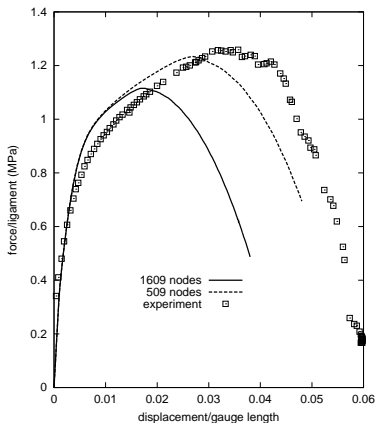


crack length : 10 mm

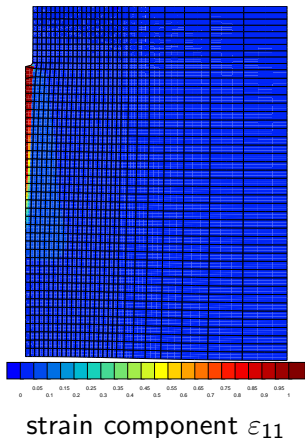
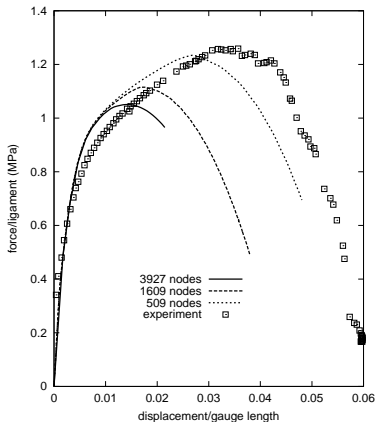
# Simulation with a classical compressible plasticity model



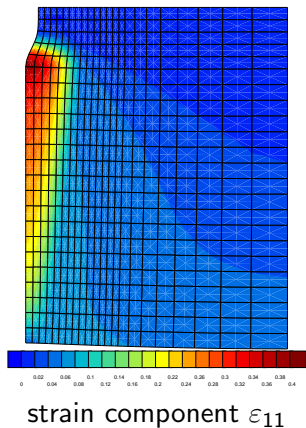
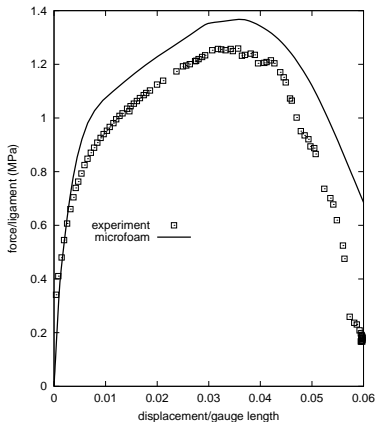
# Simulation with a classical compressible plasticity model



# Simulation with a classical compressible plasticity model

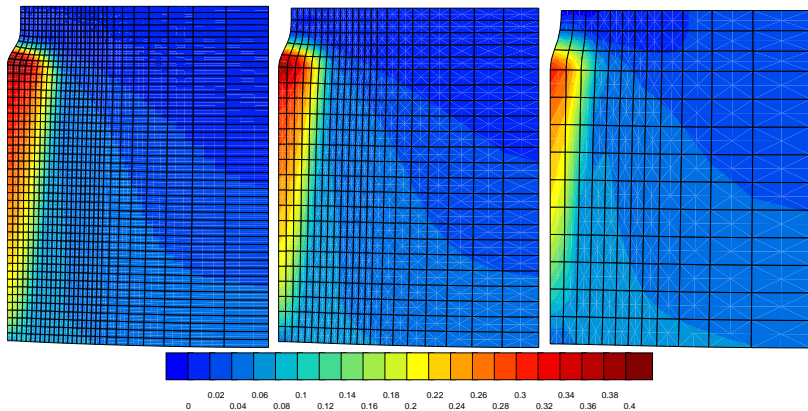


# Simulation with the micromorphic foam model

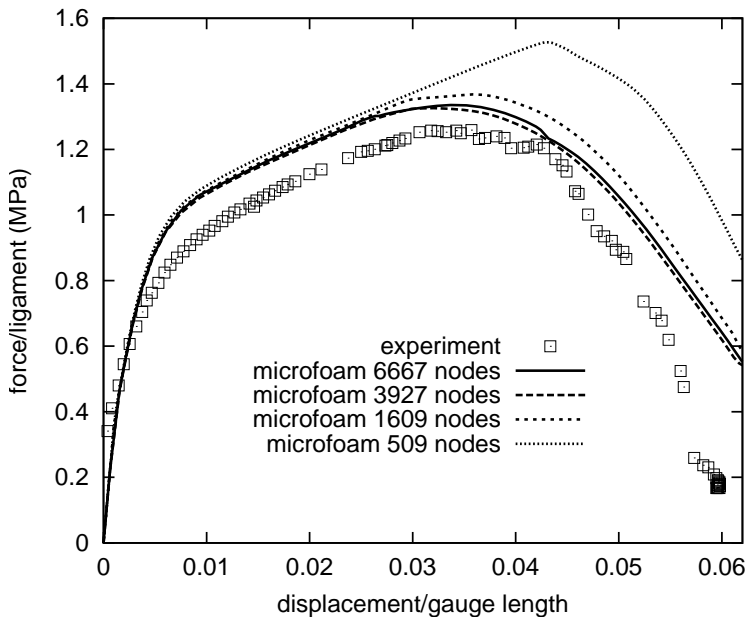




# Damage zone size

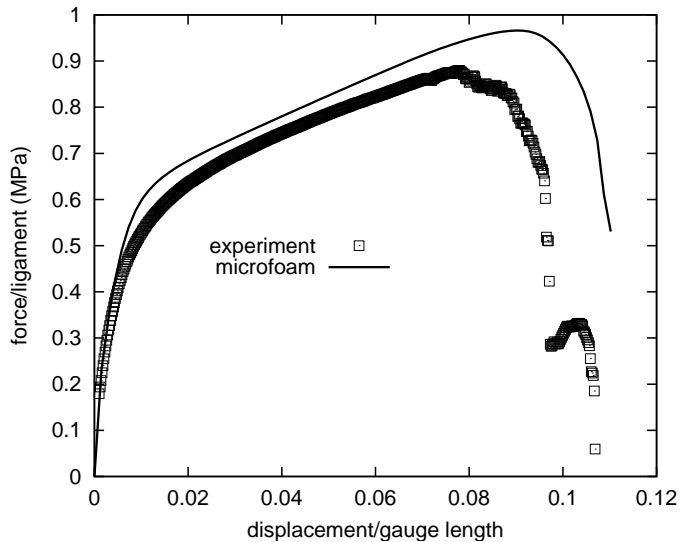


# Convergence of the load–displacement curve



# Fracture anisotropy of nickel foams

prediction of fracture stress for tension along TD



# Plan

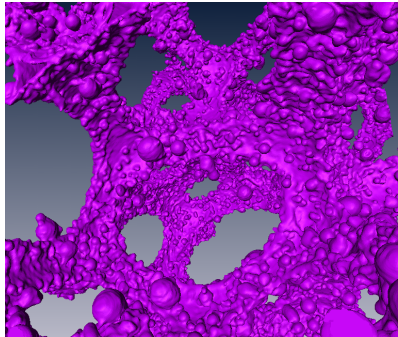
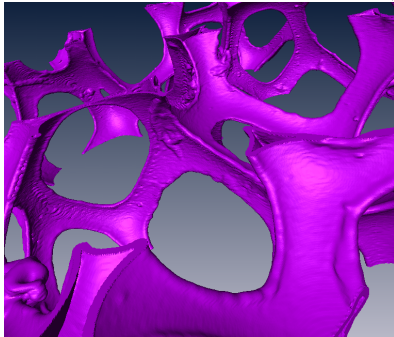
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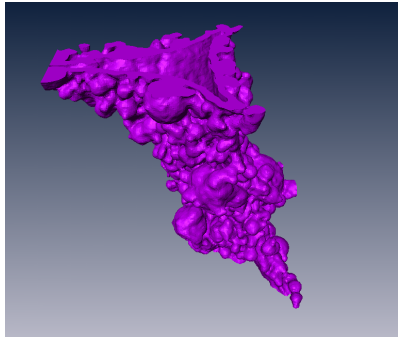
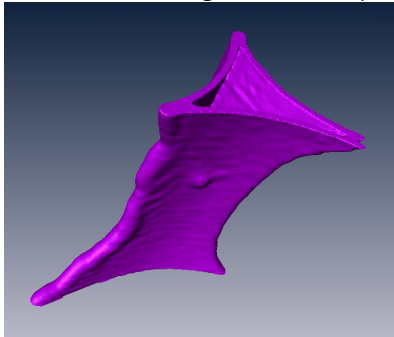
# Ductility of nickel and alloyed foams

virtual machining a strut tensile/bending specimen out of the foam



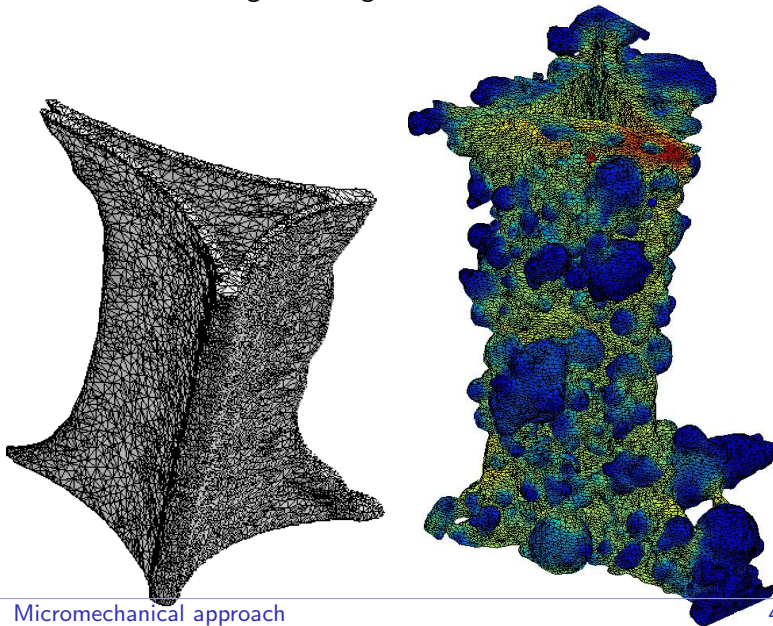
# Ductility of nickel and alloyed foams

virtual machining of a strut specimen out of the foam



# Ductility of nickel and alloyed foams

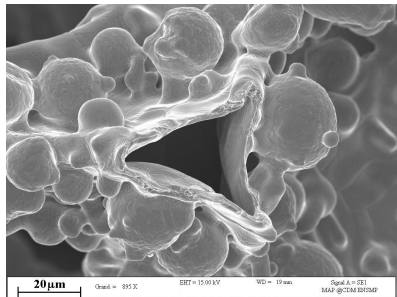
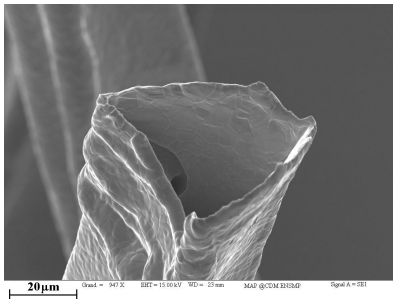
virtual tensile testing of a single strut





# Ductility of nickel and alloyed foams

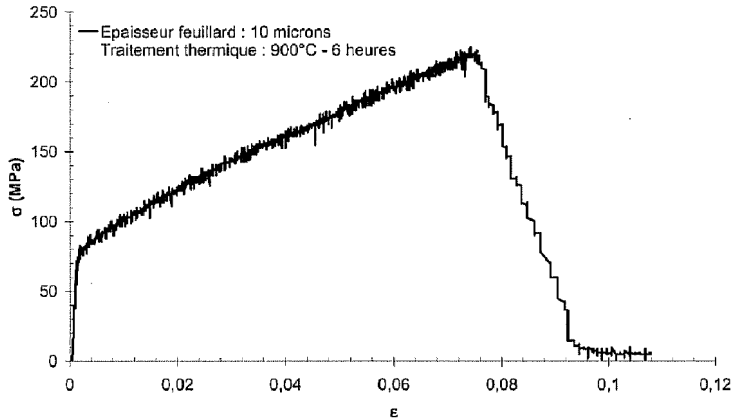
actual fracture surface of a strut



# Plan

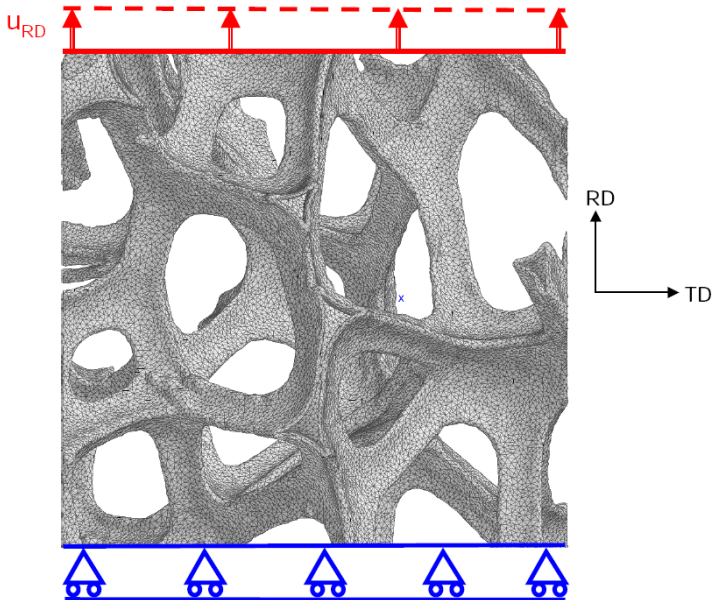
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# Behaviour of Nickel foils

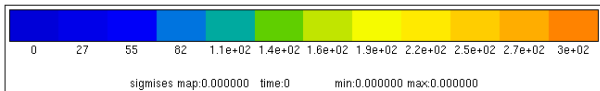
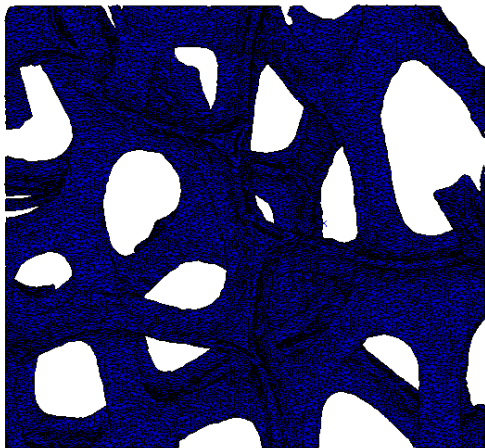


elastoplastic model with linear isotropic hardening

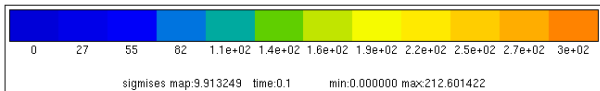
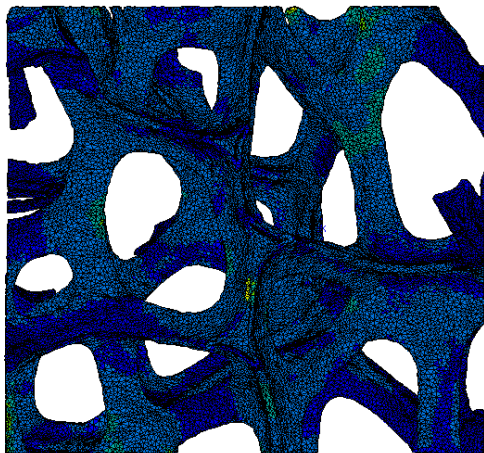
# Computing one real single cell



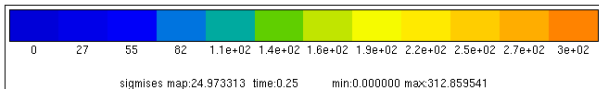
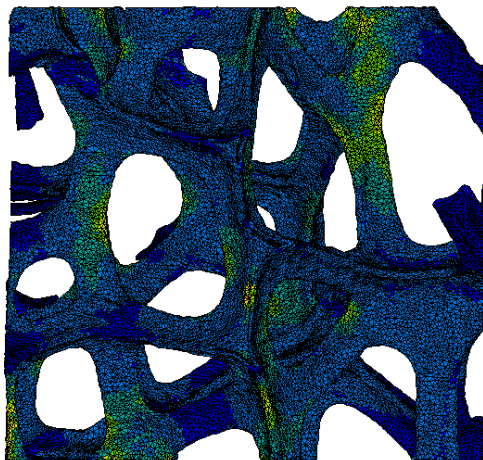
# Computing one real single cell



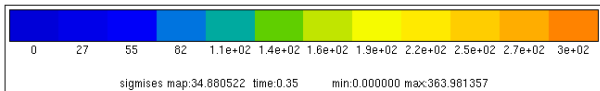
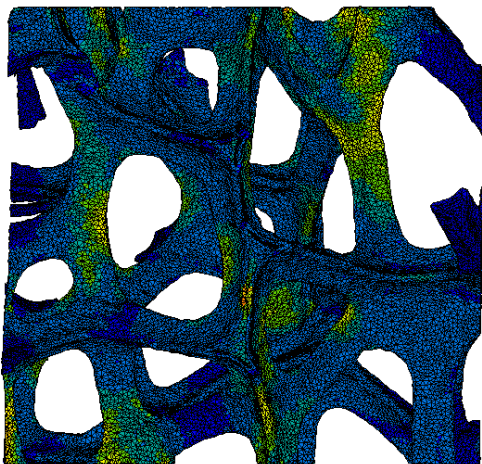
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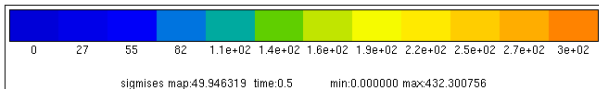
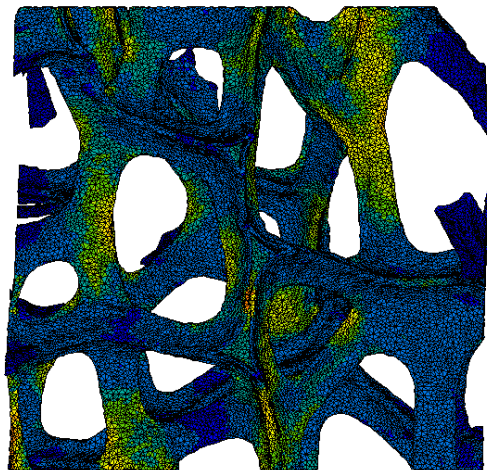


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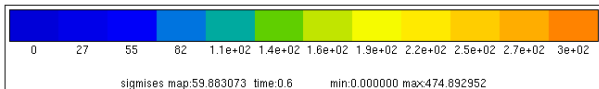
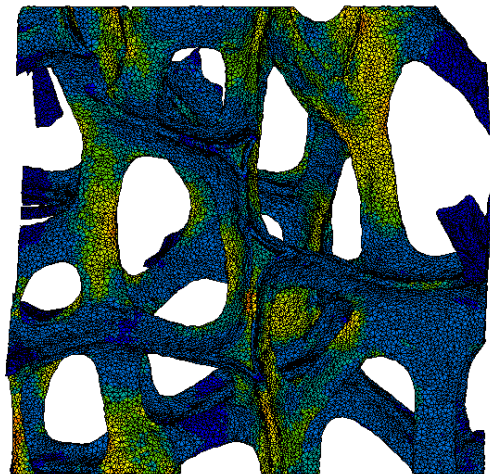




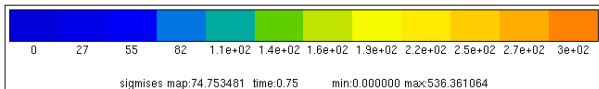
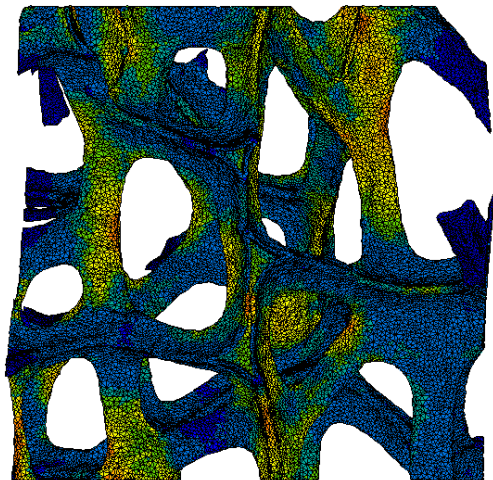
# Computing one real single cell



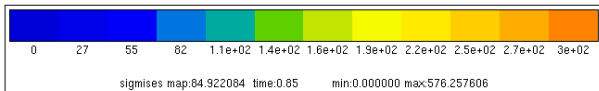
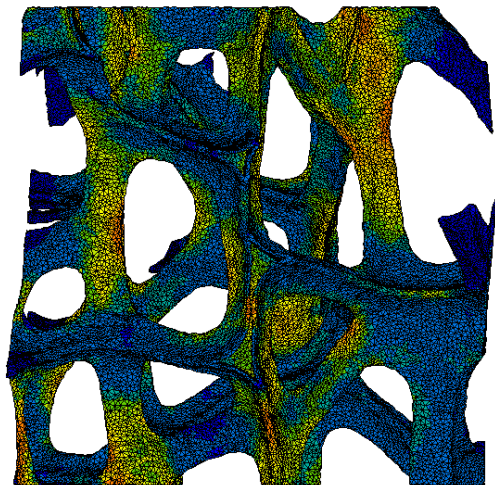
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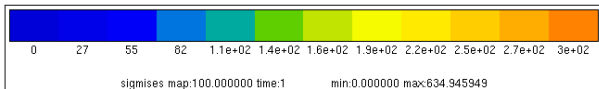
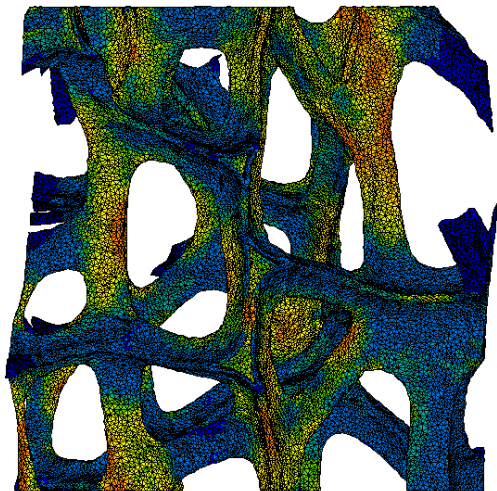
# Computing one real single cell



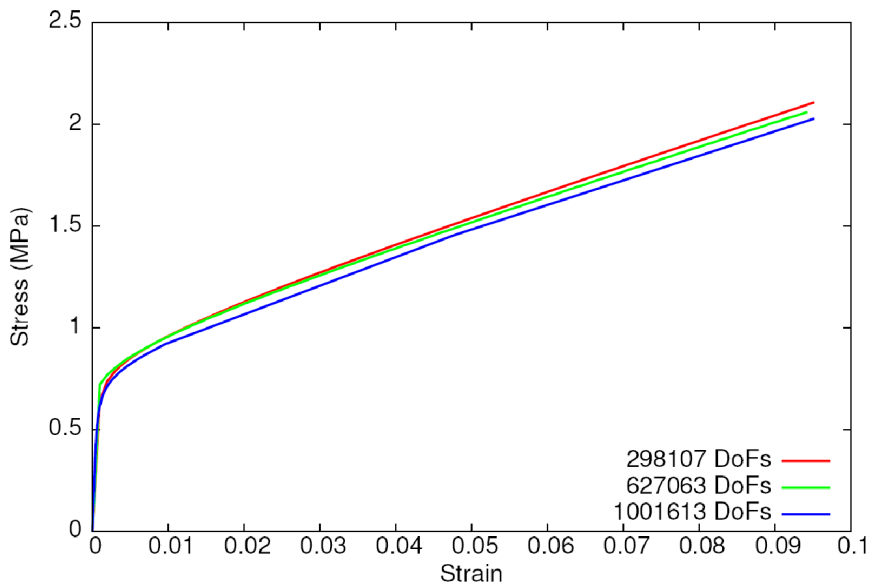
# Computing one real single cell



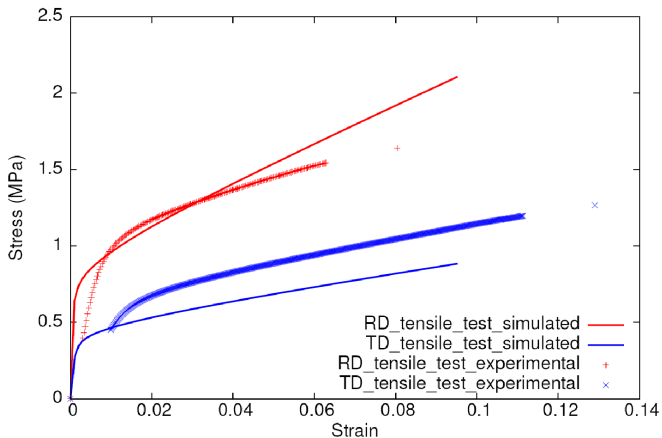
# Computing one real single cell



# Mesh sensitivity

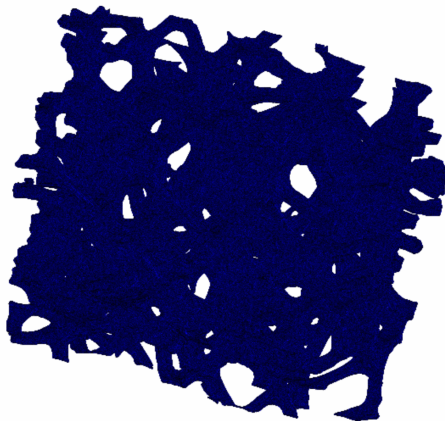
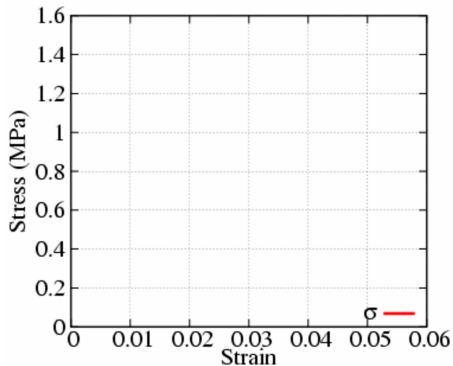


# Prediction of elastoplastic properties



Young's modulus (GPa)	RD	TD	ND	$E_{RD}/E_{TD}$
simulation	1091	319	432	3.42
experiment	177	88		2

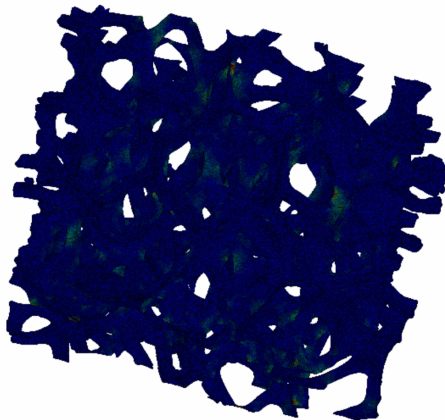
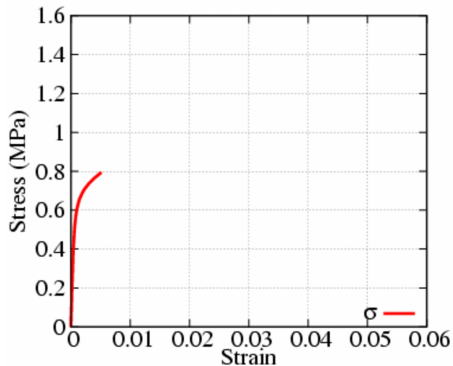
# Eight cell volume element



3 600 000 DOF – 36 GB – 300 hours

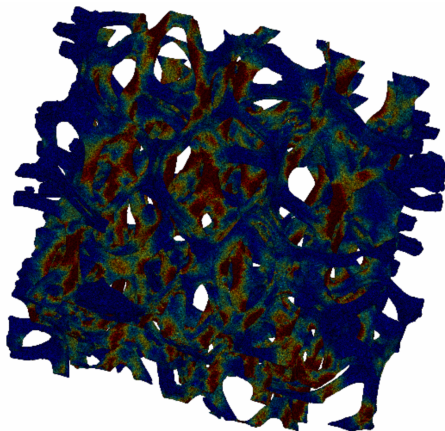
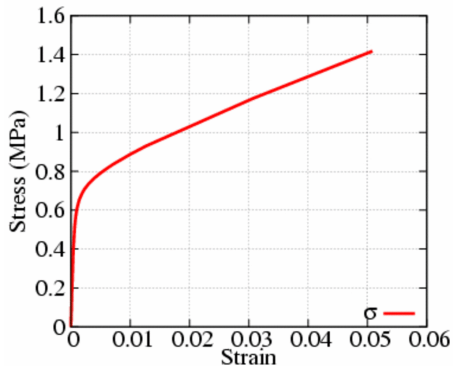


# Eight cell volume element



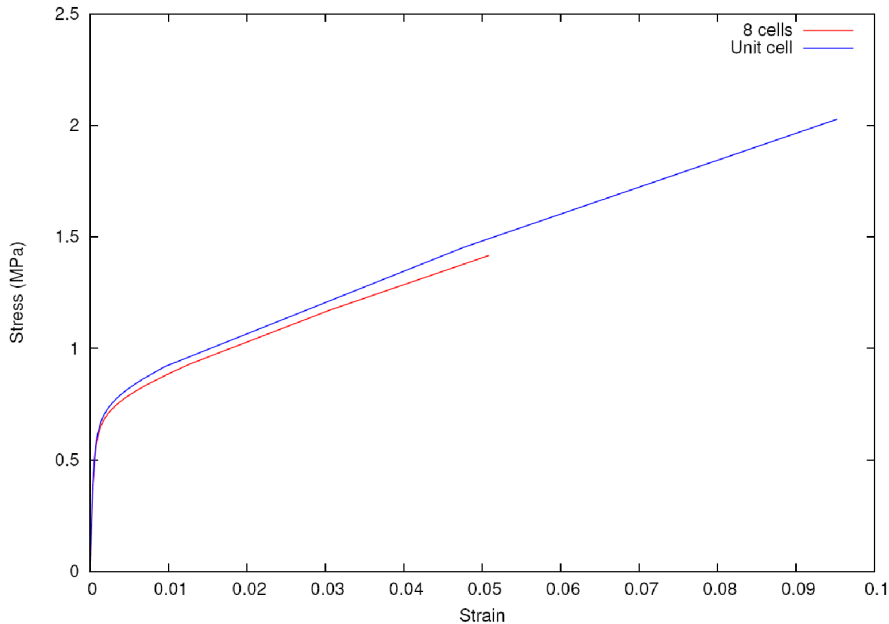
3 600 000 DOF – 36 GB – 300 hours – red:  $p > 0.03$

# Eight cell volume element



3 600 000 DOF – 36 GB – 300 hours – red:  $p > 0.03$

## Eight cell vs. one cell volume element

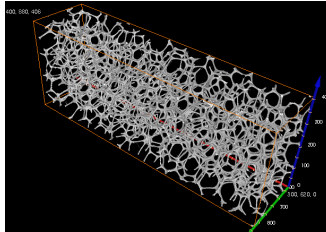


# Plan

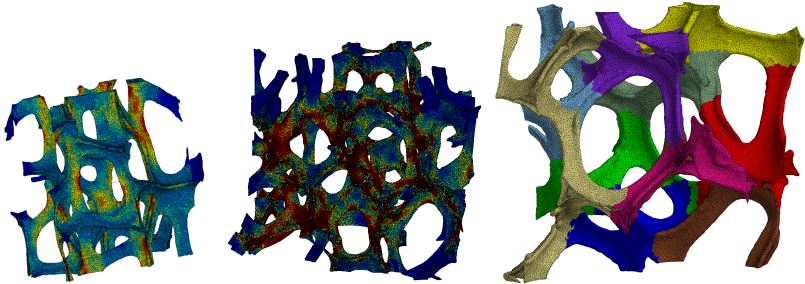
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# Perspectives

Deformation of cells by 3D microtomography (3D correlation techniques)



Systematic determination of RVE size for elastoviscoplastic properties of nickel base foams: cyclic loading, creep...



effect of BCs, statistical approach, parallel computing...