A hierarchy of higher order and higher grade continua
Application to the plasticity and fracture of metallic foams

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Plan

1. The micromorphic approach to plasticity
   - Continuum thermomechanics
   - Full micromorphic and microstrain theories
   - Internal constraint in the micromorphic approach
2. Application to nickel foams
   - Micromorphic anisotropic compressible plasticity model
   - Simulation of crack propagation
3. Micromechanical approach

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Plan

State space

- **observable and controllable variables** (temperature, strain...)

$$\{T, \quad \underset{\sim}{\mathbf{F}}\}$$

- **internal degrees of freedom** (controllable variables that account for some aspects of the microstructre)

$$\{\alpha, \quad \nabla\alpha\}$$

they have associated stresses and α or its associated force can be prescribed at the boundary

- **internal variables** are the remembrance of internal degrees of freedom; they cannot be controlled

$$\{\alpha\}$$

gradient of internal variable (extra–entropy flux)

$$\{\alpha, \quad \nabla\alpha\}$$

[Maugin, 1990] [Maugin, 1997]

The micromorphic approach (1)

- Start from an initial classical elastoviscoplastic model with internal variables

$$DOF0 = \{\underline{\mathbf{u}}\}, \quad STATE0 = \{\underset{\sim}{\mathbf{F}}, \quad T, \quad \alpha\}$$

- Select one variable $\phi \in STATE0$ and introduce the associated micromorphic variable ${}^{\chi}\phi$ as an additional degree of freedom and, possibly, state variable:

$$DOF = \{\underline{\mathbf{u}}, \quad {}^{\chi}\phi\}, \quad STATE = \{\underset{\sim}{\mathbf{F}}, \quad T, \quad \alpha, \quad {}^{\chi}\phi, \quad \boldsymbol{\nabla}\, {}^{\chi}\phi\}$$

- Extend the power of internal forces

$$\mathcal{P}^{(i)}(\underline{\mathbf{v}}^{\star}, {}^{\chi}\dot{\phi}^{\star}) = -\int_{\mathcal{D}} p^{(i)}(\underline{\mathbf{v}}^{\star}, {}^{\chi}\dot{\phi}^{\star})\, dV$$

$$p^{(i)}(\underline{\mathbf{v}}^{\star}, {}^{\chi}\dot{\phi}^{\star}) = \underset{\sim}{\boldsymbol{\sigma}} : \boldsymbol{\nabla}\underline{\mathbf{v}}^{\star} + a\, {}^{\chi}\dot{\phi}^{\star} + \underline{\mathbf{b}}\,.\boldsymbol{\nabla}\, {}^{\chi}\dot{\phi}^{\star}$$

$a, \underline{\mathbf{b}}$ generalized stresses, *microforces* (Gurtin, 1996)

- Derive additional balance equation and boundary conditions

$$\mathrm{div}\, \underline{\mathbf{b}} - a = 0, \quad \forall \underline{\mathbf{x}} \in \Omega, \quad \underline{\mathbf{b}}\,.\underline{\mathbf{n}} = a^c, \forall \underline{\mathbf{x}} \in \partial\Omega$$

The micromorphic approach (2)

- More generally, in the presence of volume generalized forces:

$$\mathrm{div}\,(\underline{\mathbf{b}} - \underline{\mathbf{b}}^{\,e}) - a + a^e = 0, \quad \forall \underline{\mathbf{x}} \in \Omega, \quad (\underline{\mathbf{b}} - \underline{\mathbf{b}}^{\,e}).\underline{\mathbf{n}} = a^c, \forall \underline{\mathbf{x}} \in \partial\Omega$$

- Enhance the local balance of energy and the entropy inequality

$$\rho\dot{\epsilon} = p^{(i)} - \mathrm{div}\,\underline{\mathbf{q}} + \rho r, \quad -\rho(\dot{\psi} + \eta\dot{T}) + p^{(i)} - \frac{\underline{\mathbf{q}}}{T}.\boldsymbol{\nabla} T \geq 0$$

- Consider the constitutive functionals:

$$\begin{aligned}
\psi &= \hat{\psi}(\underset{\sim}{\mathbf{F}}^e, T, \alpha, {}^{\chi}\phi, \boldsymbol{\nabla}^{\chi}\phi), \; \eta = \hat{\eta}(\underset{\sim}{\mathbf{F}}^e, T, \alpha, {}^{\chi}\phi, \boldsymbol{\nabla}^{\chi}\phi) \\
\underset{\sim}{\boldsymbol{\sigma}} &= \hat{\underset{\sim}{\boldsymbol{\sigma}}}(\underset{\sim}{\mathbf{F}}^e, T, \alpha, {}^{\chi}\phi, \boldsymbol{\nabla}^{\chi}\phi) \\
a &= \hat{a}(\underset{\sim}{\mathbf{F}}^e, T, \alpha, {}^{\chi}\phi, \boldsymbol{\nabla}^{\chi}\phi), \quad \underline{\mathbf{b}} = \hat{\underline{\mathbf{b}}}(\underset{\sim}{\mathbf{F}}^e, T, \alpha, {}^{\chi}\phi, \boldsymbol{\nabla}^{\chi}\phi)
\end{aligned}$$

- Derive the state laws (Coleman and Noll, 1963)

$$\underset{\sim}{\boldsymbol{\sigma}} = \rho\frac{\partial\hat{\psi}}{\partial\underset{\sim}{\mathbf{F}}^e}.\underset{\sim}{\mathbf{F}}^{eT}, \; \eta = -\frac{\partial\hat{\psi}}{\partial T}, \; X = \rho\frac{\partial\hat{\psi}}{\partial\alpha}, \quad a = \frac{\partial\hat{\psi}}{\partial\,{}^{\chi}\phi}, \quad \underline{\mathbf{b}} = \frac{\partial\hat{\psi}}{\partial\boldsymbol{\nabla}\,{}^{\chi}\phi}$$

- Residual dissipation $\qquad D^{res} = W^p - X\dot{\alpha} - \frac{\underline{\mathbf{q}}}{T}.\boldsymbol{\nabla} T \geq 0$

The micromorphic approach (3)

- Take a simple quadratic potential

$$\psi(\underset{\sim}{F}, T, \alpha, {}^{\chi}\phi, \nabla {}^{\chi}\phi) = \psi_1(\underset{\sim}{F}, \alpha, T) + \psi_2(e = \phi - {}^{\chi}\phi, \nabla {}^{\chi}\phi, T)$$

$$\rho\psi_2 = \frac{1}{2} H_\chi (\phi - {}^{\chi}\phi)^2 + \frac{1}{2} A \nabla {}^{\chi}\phi . \nabla {}^{\chi}\phi$$

$$a = \rho \frac{\partial \psi}{\partial {}^{\chi}\phi} = -H_\chi(\phi - {}^{\chi}\phi), \quad \underline{\mathbf{b}} = \rho \frac{\partial \psi}{\partial \nabla {}^{\chi}\phi} = A \nabla {}^{\chi}\phi$$

- Simple form of the partial differential equation (homogeneous, isothermal...)

$$a = \operatorname{div} \underline{\mathbf{b}} \implies {}^{\chi}\phi - \frac{A}{H_\chi} \Delta {}^{\chi}\phi = \phi$$

  Helmholtz equation with a minus sign and a source term

- Coupling modulus H_χ and characteristic length of the medium

$$l_c^2 = \frac{A}{H_\chi}$$

  Stability $H_\chi > 0, \quad A > 0$

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Micromorphic continuum

Micromorphic continuum according to (Eringen and Suhubi, 1964; Mindlin, 1964)

- Select variable:

$$\phi \equiv \underset{\sim}{\mathbf{F}}, \quad {}^{\chi}\phi \equiv \underset{\sim}{\chi}$$

$$p^{(i)} = \underset{\sim}{\sigma} : \nabla \underline{\dot{\mathbf{u}}} + \underset{\sim}{\mathbf{a}} : \dot{\underset{\sim}{\chi}} + \underset{\approx}{\mathbf{B}} \vdots \nabla \dot{\underset{\sim}{\chi}}$$

- application of the principle of (infinitesimal) material frame indifference, (infinitesimal) change of observer of rate $\underset{\sim}{\mathbf{w}}$:

$$\nabla \underline{\dot{\mathbf{u}}} \Longrightarrow \nabla \underline{\dot{\mathbf{u}}} + \underset{\sim}{\mathbf{w}}, \quad \underset{\sim}{\chi} \Longrightarrow \underset{\sim}{\chi} + \underset{\sim}{\mathbf{w}}$$

$\Longrightarrow \underset{\sim}{\sigma} + \underset{\sim}{\mathbf{a}}$ must be symmetric. Rewrite the virtual power:

$$p^{(i)} = \underset{\sim}{\sigma} : \dot{\underset{\sim}{\varepsilon}} + \underset{\sim}{\mathbf{s}} : (\nabla \underline{\dot{\mathbf{u}}} - \dot{\underset{\sim}{\chi}}) + \underset{\approx}{\mathbf{S}} \vdots \nabla \dot{\underset{\sim}{\chi}}$$

- two balance equations:

$$\mathrm{div}\,(\underset{\sim}{\sigma} + \underset{\sim}{\mathbf{s}}) + \rho \underline{\mathbf{f}} = 0, \quad \mathrm{div}\,\underset{\approx}{\mathbf{S}} + s = 0$$

Microstrain continuum

Microstrain continuum after (Forest and Sievert, 2006)

- Select

$$\phi \equiv \underset{\sim}{\mathbf{C}} = \underset{\sim}{\mathbf{F}}^T . \underset{\sim}{\mathbf{F}}, \quad {}^\chi\phi \equiv {}^\chi\underset{\sim}{\mathbf{C}}, \quad \text{or} \quad \phi \equiv \underset{\sim}{\varepsilon}, \quad {}^\chi\phi \equiv {}^\chi\underset{\sim}{\varepsilon}$$

$$p^{(i)} = \underset{\sim}{\sigma} : \dot{\underset{\sim}{\varepsilon}} + \underset{\sim}{\mathbf{a}} : {}^\chi\dot{\underset{\sim}{\varepsilon}} + \underset{\approx}{\mathbf{b}} \,\vdots\, \nabla^\chi\underset{\sim}{\varepsilon}$$

- Constitutive coupling between macro and microstrain via the relative strain

$$\underset{\sim}{\mathbf{e}} := \underset{\sim}{\varepsilon} - {}^\chi\underset{\sim}{\varepsilon}$$

$$\psi(\underset{\sim}{\varepsilon}^e, \quad T, \quad \alpha, \quad \underset{\sim}{\mathbf{e}} := \underset{\sim}{\varepsilon} - {}^\chi\underset{\sim}{\varepsilon}, \quad \underset{\approx}{\mathbf{K}} := \nabla^\chi\underset{\sim}{\varepsilon})$$

- Take a quadratic potential

$$\underset{\sim}{\mathbf{a}} = H_\chi \underset{\sim}{\mathbf{e}}, \quad \underset{\approx}{\mathbf{b}} = A \nabla^\chi\underset{\sim}{\varepsilon}$$

- Extra–balance equation

$${}^\chi\underset{\sim}{\varepsilon} - l_c^2 \Delta^\chi\underset{\sim}{\varepsilon} = \underset{\sim}{\varepsilon}, \quad \text{with} \quad l_c^2 = \frac{A}{H_\chi}$$

example: microfoams (Dillard et al., 2006)

Cosserat continuum

Cosserat continuum

$$\phi = \underset{\sim}{\mathbf{R}}, \quad {}^{\chi}\phi \equiv {}^{\chi}\underset{\sim}{\mathbf{R}}$$

$$p^{(i)} = \underset{\sim}{\boldsymbol{\sigma}}^{s} : \dot{\underset{\sim}{\boldsymbol{\varepsilon}}} - \underset{\sim}{\boldsymbol{\sigma}}^{a} : ((\boldsymbol{\nabla}\underline{\dot{\mathbf{u}}})^{a} - \dot{\underset{\sim}{\mathbf{R}}}.\underset{\sim}{\mathbf{R}}^{T}) + \underset{\sim}{\mathbf{M}} : \dot{\underset{\sim}{\boldsymbol{\kappa}}}$$

A hierarchy of higher order continua

name	number of DOF	DOF (finite case)	DOF (infinitesimal case)	references
Cauchy	3	$\underline{\mathbf{u}}$	$\underline{\mathbf{u}}$	[Cauchy, 1823]
micromorphic	12	$\underline{\mathbf{u}}\,,\quad \underset{\sim}{\chi}$	$\underline{\mathbf{u}}\,,\quad \underset{\sim}{\chi}^{s} + \underset{\sim}{\chi}^{a}$	[Eringen, 1964] [Mindlin, 1964]

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microdilatation	4	$\underline{\mathbf{u}}, \quad \chi$	$\underline{\mathbf{u}}, \quad \chi$	[Goodman, Cowin, 1972] [Steeb, Diebels, 2003]
micromorphic	12	$\underline{\mathbf{u}}, \quad \underset{\sim}{\boldsymbol{\chi}}$	$\underline{\mathbf{u}}, \quad \underset{\sim}{\boldsymbol{\chi}}^s + \underset{\sim}{\boldsymbol{\chi}}^a$	[Eringen, 1964] [Mindlin, 1964]

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Cosserat	6	$\underline{\mathbf{u}}, \quad \underset{\sim}{\mathbf{R}}$	$\underline{\mathbf{u}}, \quad \underline{\mathbf{\Phi}}$	[Kafadar, Eringen, 1976]
micromorphic	12	$\underline{\mathbf{u}}, \quad \underset{\sim}{\boldsymbol{\chi}}$	$\underline{\mathbf{u}}, \quad \underset{\sim}{\boldsymbol{\chi}}^s + \underset{\sim}{\boldsymbol{\chi}}^a$	[Eringen, 1964] [Mindlin, 1964]

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Cosserat	6	$\underline{\mathbf{u}}, \quad \underset{\sim}{\mathbf{R}}$	$\underline{\mathbf{u}}, \quad \underline{\mathbf{\Phi}}$	[Kafadar, Eringen, 1976]
microstretch	7	$\underline{\mathbf{u}}, \quad \chi, \quad \underset{\sim}{\mathbf{R}}$	$\underline{\mathbf{u}}, \quad \chi, \quad \underline{\mathbf{\Phi}}$	[Eringen, 1990]
micromorphic	12	$\underline{\mathbf{u}}, \quad \underset{\sim}{\boldsymbol{\chi}}$	$\underline{\mathbf{u}}, \quad \underset{\sim}{\boldsymbol{\chi}}^s + \underset{\sim}{\boldsymbol{\chi}}^a$	[Eringen, 1964] [Mindlin, 1964]

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Cosserat	6	$\underline{\mathbf{u}}\,, \quad \underset{\sim}{\mathbf{R}}$	$\underline{\mathbf{u}}\,, \quad \underline{\boldsymbol{\Phi}}$	[Kafadar, Eringen, 1976]
microstretch	7	$\underline{\mathbf{u}}\,, \quad \chi, \quad \underset{\sim}{\mathbf{R}}$	$\underline{\mathbf{u}}\,, \quad \chi, \quad \underline{\boldsymbol{\Phi}}$	[Eringen, 1990]
microstrain	9	$\underline{\mathbf{u}}\,, \quad {}^{\chi}\underset{\sim}{\mathbf{C}}$	$\underline{\mathbf{u}}\,, \quad {}^{\chi}\underset{\sim}{\boldsymbol{\varepsilon}}$	[Forest, Sievert, 2006]
micromorphic	12	$\underline{\mathbf{u}}\,, \quad \underset{\sim}{\boldsymbol{\chi}}$	$\underline{\mathbf{u}}\,, \quad \underset{\sim}{\boldsymbol{\chi}}^{s} + \underset{\sim}{\boldsymbol{\chi}}^{a}$	[Eringen, 1964] [Mindlin, 1964]

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Internal constraint and gradient of internal variable approach

- Impose the internal constraint that

$$^{\chi}\phi \simeq \phi \quad \Longrightarrow \quad \underline{\mathbf{K}} \simeq \boldsymbol{\nabla}\phi$$

Then, the generalized stress a becomes a Lagrange multiplier.

- Examples
  - ⋆ $\phi \equiv \underset{\sim}{\mathbf{F}}$ second gradient model (Mindlin, 1965)
  - ⋆ $\phi \equiv p$ Aifantis model (Aifantis, 1987; Fleck and Hutchinson, 2001)
  - ⋆ $\phi \equiv \underset{\sim}{\varepsilon}^p$ strain gradient plasticity (Forest and Sievert, 2003; Gurtin, 2003)

$$p^{(i)} = \underset{\sim}{\boldsymbol{\sigma}} : \dot{\underset{\sim}{\varepsilon}}^e + \underset{\sim}{\mathbf{s}} : \dot{\underset{\sim}{\varepsilon}}^p + \underset{\approx}{\mathbf{S}} \,\vdots\, \dot{\underset{\approx}{\varepsilon}}^p, \quad \mathrm{div}\, \underset{\approx}{\mathbf{S}} = \underset{\sim}{\mathbf{s}} - \underset{\sim}{\boldsymbol{\sigma}}^{dev}$$

The yield condition becomes a PDE (Aifantis, Fleck–Hutchinson, Gurtin):

$$\sigma_{eq} = \sigma_Y + Hp - A\Delta p$$

- What do the boundary conditions become?
  - ⋆ For the second gradient theory, intricate b.c. involving surface curvature
  - ⋆ For gradient of plastic strain, $\underset{\approx}{\mathbf{S}}.\underline{\mathbf{n}} = \underset{\sim}{\mathbf{m}}$

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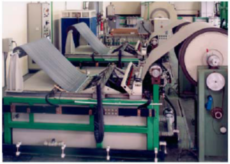
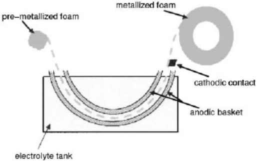
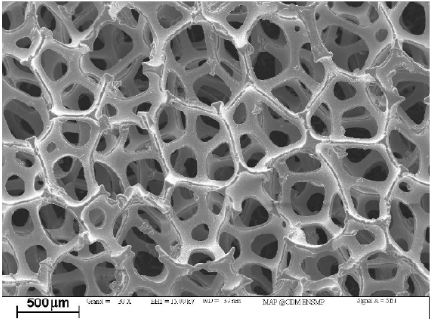
Processing of nickel foams

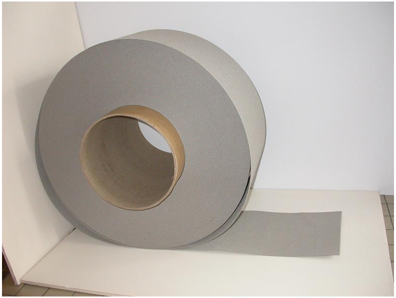
from PU template

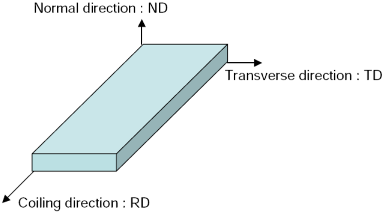


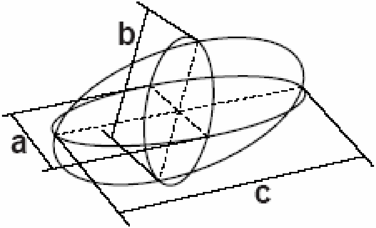




Foam sheet axes and cell shape







$$a \leq b \leq c$$

Anisotropic Compressible Continuum Plasticity Model

Yield criterion

$$f(\underset{\sim}{\sigma}) = \sigma_{eq} - R, \quad \sigma_{eq} = \left(\frac{3}{2} C \underset{\sim}{\sigma}^{\text{dev}} : \underset{\approx}{\mathbf{H}} : \underset{\sim}{\sigma}^{\text{dev}} + F(\underset{\sim}{\mathbf{P}} : \underset{\sim}{\sigma})^2\right)^{\frac{1}{2}}$$

$$[H] = \begin{bmatrix} h_a & 0 & 0 & 0 & 0 & 0 \\ 0 & h_b & 0 & 0 & 0 & 0 \\ 0 & 0 & h_c & 0 & 0 & 0 \\ 0 & 0 & 0 & h_d & 0 & 0 \\ 0 & 0 & 0 & 0 & h_e & 0 \\ 0 & 0 & 0 & 0 & 0 & h_f \end{bmatrix}, \quad [P] = \begin{bmatrix} p & 0 & 0 \\ 0 & q & 0 \\ 0 & 0 & r \end{bmatrix}$$

Normality rule $\quad \dot{\underset{\sim}{\varepsilon}}^p = \dot{p} \frac{\partial f}{\partial \underset{\sim}{\sigma}} = \dot{p} \left(\frac{3}{2} C \underset{\approx}{\mathbf{H}} : \underset{\sim}{\sigma}^{\text{dev}} + F(\underset{\sim}{\mathbf{P}} : \underset{\sim}{\sigma}) \underset{\sim}{\mathbf{P}}\right)$

Consistency condition (cumulative plastic strain p)

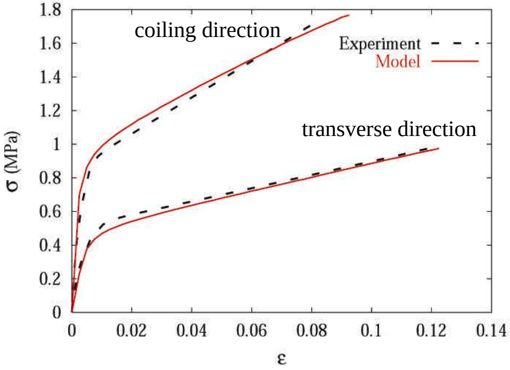
$$\dot{p} = \frac{\dfrac{\partial f}{\partial \underset{\sim}{\sigma}} : \underset{\approx}{\mathbf{E}} : \dot{\underset{\sim}{\varepsilon}}}{\dfrac{\partial f}{\partial \underset{\sim}{\sigma}} : \underset{\approx}{\mathbf{E}} : \dfrac{\partial f}{\partial \underset{\sim}{\sigma}} + \dfrac{\partial R}{\partial p}}$$

Nonlinear isotropic hardening

$$R = R_0 + Hp + Q(1 - \exp(-bp))$$

Parameter identification

C	F	R_0 (MPa)	Q (MPa)	b	H (MPa)	h_a	h_b	h_c	h_d	h_e	h_f
1	$6{,}8.10^{-4}$	0,30	0,37	3,3	8,17	0,41	1,8	1,75	1,45	1	1



p	q	r
1	37	23

Metal foam as a micromorphic continuum : The *microfoam* model

Degrees of freedom at material point

$$\text{displacement vector } \underline{\mathbf{u}}\,, \quad \text{microdeformation tensor } \underset{\sim}{\chi}$$

after [Mindlin, 1964] [Eringen, 1964]

Deformation measures

$$\text{strain tensor} \quad \underset{\sim}{\varepsilon} = (\underline{\mathbf{u}} \otimes \boldsymbol{\nabla} + \boldsymbol{\nabla} \otimes \underline{\mathbf{u}})/2$$

$$\text{relative deformation tensor} \quad \underset{\sim}{\mathbf{e}} = \underline{\mathbf{u}} \otimes \boldsymbol{\nabla} - \underset{\sim}{\chi}$$

$$\text{microdeformation gradient} \quad \underset{\approx}{\mathbf{K}} = \underset{\sim}{\chi} \otimes \boldsymbol{\nabla}$$

Power density of internal forces / generalized stress tensors

$$p^{(i)} = \underset{\sim}{\sigma} : \dot{\underset{\sim}{\varepsilon}} + \underset{\sim}{\mathbf{s}} : \dot{\underset{\sim}{\mathbf{e}}} + \underset{\approx}{\mathbf{M}} \vdots \dot{\underset{\approx}{\mathbf{K}}}$$

Balance of momentum $\quad (\underset{\sim}{\sigma} + \underset{\sim}{\mathbf{s}}).\boldsymbol{\nabla} = 0$

Balance of micromorphic momentum $\quad \underset{\approx}{\mathbf{M}}.\boldsymbol{\nabla} + \underset{\sim}{\mathbf{s}} = 0$

Boundary conditions (traction and double traction vectors)

$$\underline{\mathbf{t}} = (\underset{\sim}{\sigma} + \underset{\sim}{\mathbf{s}}).\underline{\mathbf{n}}\,, \quad \underset{\sim}{\mathbf{T}} = \underset{\approx}{\mathbf{M}}.\underline{\mathbf{n}}$$

Variational formulation for finite element implementation

$$\int_V p^{(i)}\, dV = \int_{\partial V} \left(\underline{\mathbf{t}}\,.\dot{\underline{\mathbf{n}}} + \underset{\sim}{\mathbf{T}} : \dot{\underset{\sim}{\chi}} \right) dS$$

The *microfoam* model : Constitutive equations

Strain partition

$$\underset{\sim}{\varepsilon} = \underset{\sim}{\varepsilon}^e + \underset{\sim}{\varepsilon}^p, \quad \underset{\sim}{\mathbf{e}} = \underset{\sim}{\mathbf{e}}^e + \underset{\sim}{\mathbf{e}}^p, \quad \underset{\approx}{\mathbf{K}} = \underset{\approx}{\mathbf{K}}^e + \underset{\approx}{\mathbf{K}}^p$$

Linear elasticity

$$\underset{\sim}{\boldsymbol{\sigma}} = \underset{\approx}{\mathbf{c}} : \underset{\sim}{\varepsilon}^e, \quad \underset{\sim}{\mathbf{s}} = \underset{\approx}{\mathbf{a}} : \underset{\sim}{\mathbf{e}}^e, \quad \underset{\approx}{\mathbf{M}} = \underset{\approx\approx}{\mathbf{A}} \vdots \underset{\approx}{\mathbf{K}}^e$$

simplified moduli as proposed by [Shu et al., 1999] $\underset{\approx}{\mathbf{M}} = l_c^2 \, \underset{\approx}{\mathbf{c}} : \underset{\approx}{\mathbf{K}}^e$
characteristic length l_c
when $||\underset{\approx}{\mathbf{a}}||$ is very large, and if $\underset{\sim}{\mathbf{e}}^p = 0$, then $\underline{\mathbf{u}} \otimes \boldsymbol{\nabla} \simeq \underset{\sim}{\boldsymbol{\chi}}$ (second gradient theory)

The *microfoam* model : Constitutive equations

Yield criterion

$$f(\underset{\sim}{\boldsymbol{\sigma}}) = \sigma_{eq} - R$$

$$\sigma_{eq} = \left(\frac{3}{2} C \underset{\sim}{\boldsymbol{\sigma}}^{\mathrm{dev}} : \underset{\approx}{\mathbf{H}} : \underset{\sim}{\boldsymbol{\sigma}}^{\mathrm{dev}} + F(\underset{\sim}{\mathbf{P}} : \underset{\sim}{\boldsymbol{\sigma}})^2 + a_1 \underset{\sim}{\mathbf{s}}^{\mathrm{dev}} : \underset{\sim}{\mathbf{s}}^{\mathrm{dev}} + a_2 \underset{\sim}{\mathbf{s}}^{\mathrm{dev}} : \underset{\sim}{\mathbf{s}}^{\mathrm{devT}} + \right.$$

$$\left. + b_1 \underset{\approx}{\mathbf{M}} \vdots \underset{\approx}{\mathbf{M}} \right)^{\frac{1}{2}}$$

Normality rule

$$\dot{\underset{\sim}{\boldsymbol{\varepsilon}}}^p = \dot{p} \frac{\partial f}{\partial \underset{\sim}{\boldsymbol{\sigma}}}, \quad \dot{\underset{\sim}{\mathbf{e}}}^p = \dot{p} \frac{\partial f}{\partial \underset{\sim}{\mathbf{s}}}, \quad \dot{\underset{\approx}{\mathbf{K}}}^p = \dot{p} \frac{\partial f}{\partial \underset{\approx}{\mathbf{M}}}$$

Microfoam: $a_1 = a_2 = a_3 = b_1 = 0, ||\underset{\sim}{\mathbf{a}}||\text{very large}$
constrained microdeformation (second gradient), linear relationship between gradient of microdeformation and hyperstress tensor

Microstrain **continuum**

degrees of freedom $(\underline{\mathbf{u}}\,, \underset{\sim}{\boldsymbol{\chi}}^s)$
strain measures :
$(\underset{\sim}{\varepsilon}, \underset{\sim}{\varepsilon} - \underset{\sim}{\boldsymbol{\chi}}^s, \underset{\sim}{\boldsymbol{\chi}}^s \otimes \boldsymbol{\nabla})$

$$S_{ijk,k} + s_{ij} = 0$$

$$A\chi^s_{ij,kk} + b(\varepsilon_{ij} - \chi^s_{ij}) = 0$$

$$\underset{\sim}{\boldsymbol{\sigma}} = \underset{\approx}{\mathbf{C}} : (\underset{\sim}{\varepsilon} - \underset{\sim}{\varepsilon}^p)$$

$$\underset{\sim}{\mathbf{s}} = b(\underset{\sim}{\varepsilon} - \underset{\sim}{\boldsymbol{\chi}}^s)$$

$$\underset{\approx}{\underline{\mathbf{S}}} = A\underset{\sim}{\boldsymbol{\chi}}^s \otimes \boldsymbol{\nabla}$$

$$\varepsilon_{ij} = \chi^s_{ij} - l^2 \Delta \chi^s_{ij}$$

Link with "implicit gradient–enhanced elastoplasticity models" [Engelen *et al.*, 2003]

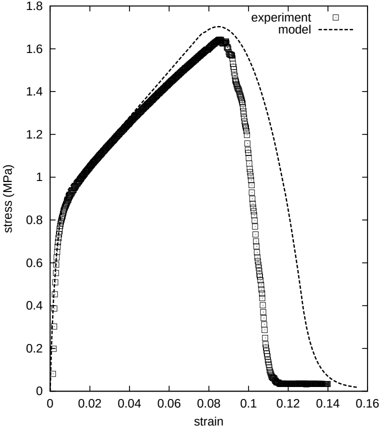
$$\operatorname{div}(\underset{\sim}{\boldsymbol{\sigma}} + \underset{\sim}{\mathbf{s}}) = 0$$

$$\operatorname{div} \underset{\approx}{\underline{\mathbf{S}}} + \underset{\sim}{\mathbf{s}} = 0$$

Boundary conditions :
(u_i, χ^s_{ij}) or
$(\sigma_{ij} + s_{ij})n_j, S_{ijk} n_k$

Plan

Fracture of nickel foams



simple fracture criterion

$$p = p_{crit} = 0.08$$

limited scatter in p_{crit}

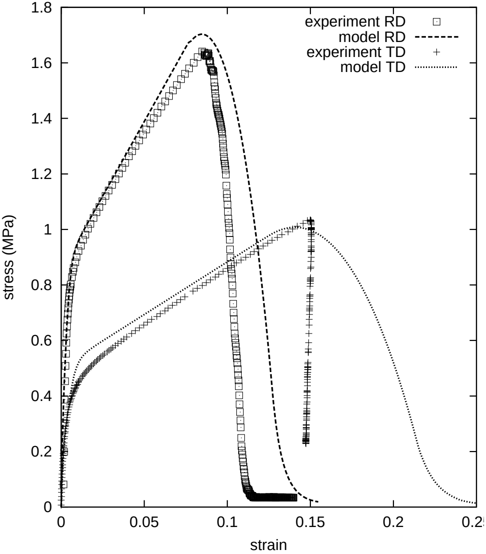
softening law for $p > p_{crit}$

$$R = R(p > p_{crit})$$

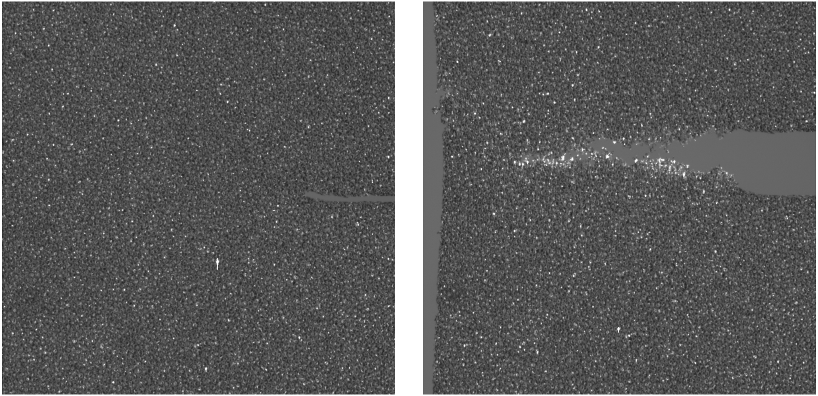
tensile test in direction RD

Fracture anisotropy of nickel foams

prediction of fracture stress for tension along TD

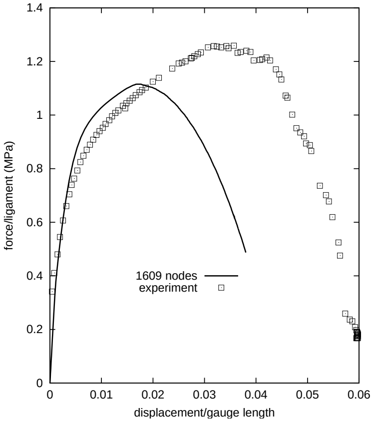


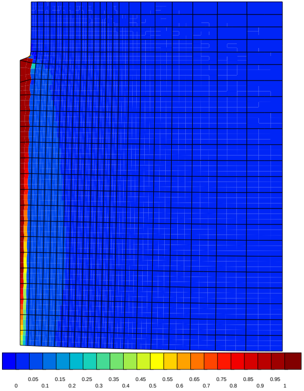
Tension of central crack nickel foam plate



crack length : 10 mm

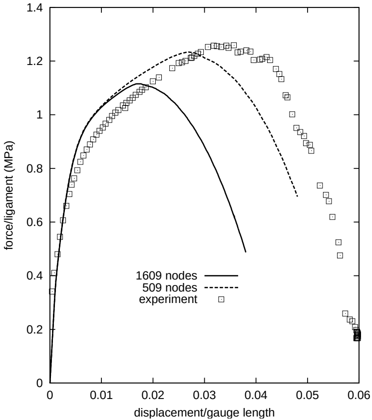
Simulation with a classical compressible plasticity model

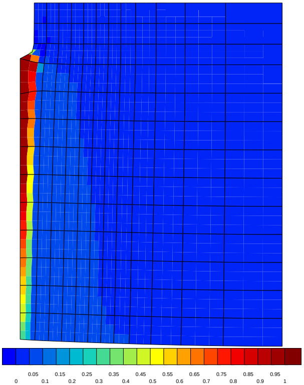




strain component ε_{11}

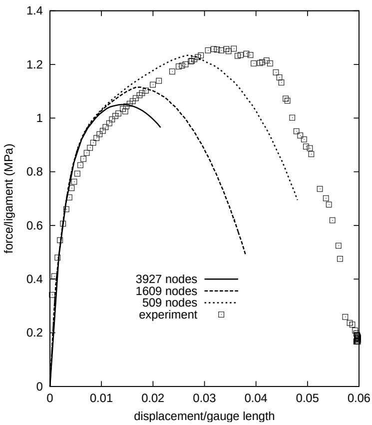
Simulation with a classical compressible plasticity model

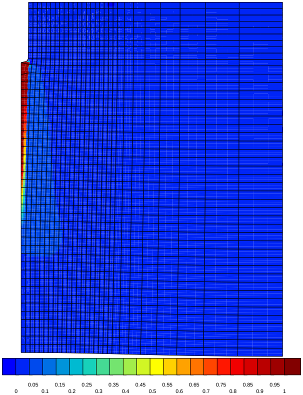




strain component ε_{11}

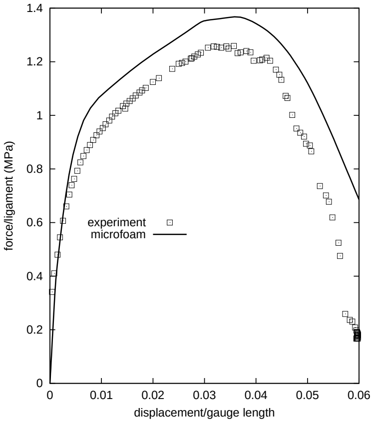
Simulation with a classical compressible plasticity model

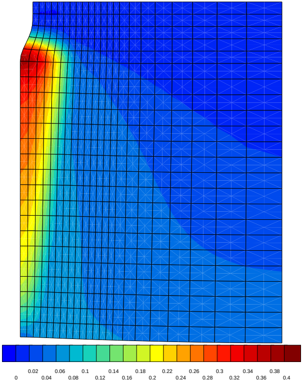




strain component ε_{11}

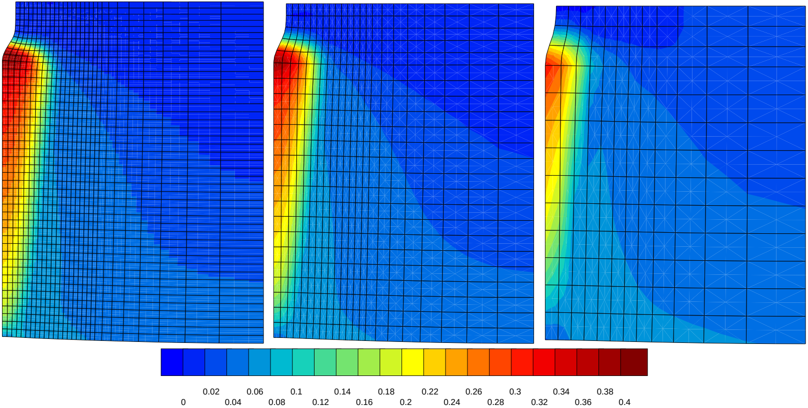
Simulation with the micromorphic foam model



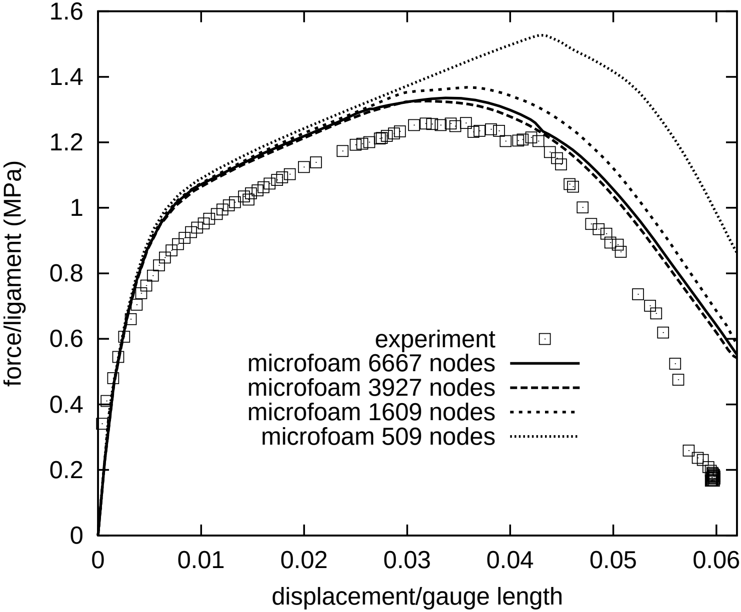


strain component ε_{11}

Damage zone size

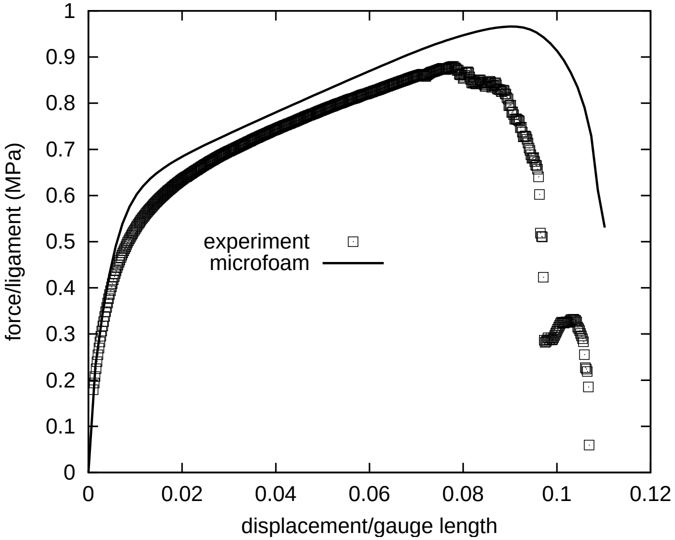


Convergence of the load–displacement curve



Fracture anisotropy of nickel foams

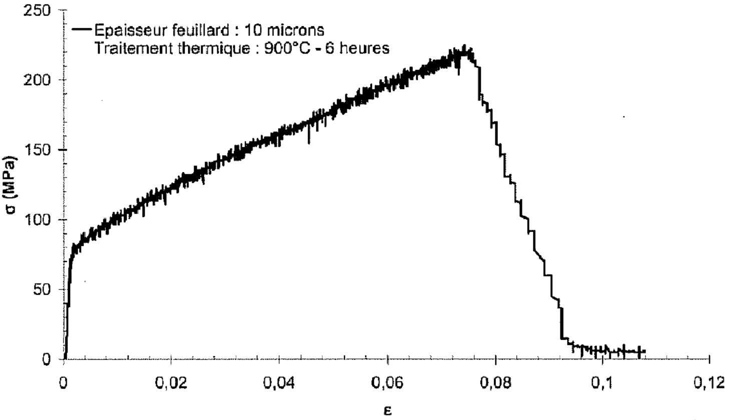
prediction of fracture stress for tension along TD



Plan

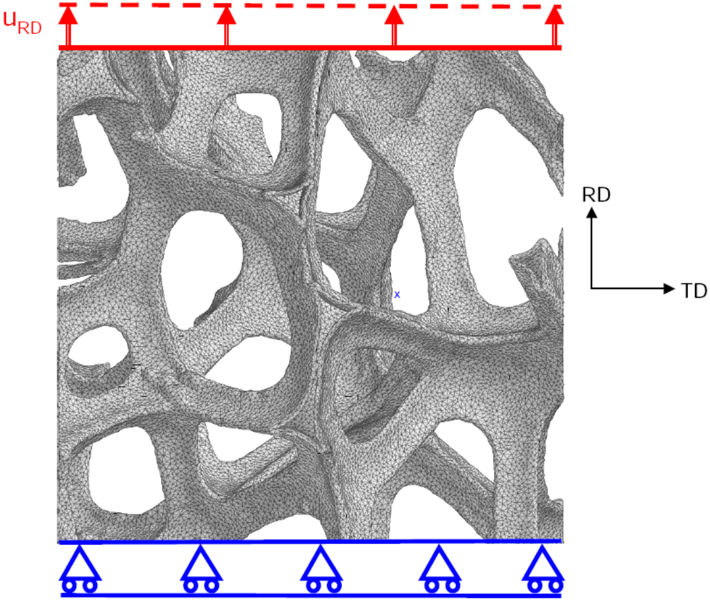
1. The micromorphic approach to plasticity
   - Continuum thermomechanics
   - Full micromorphic and microstrain theories
   - Internal constraint in the micromorphic approach
2. Application to nickel foams
   - Micromorphic anisotropic compressible plasticity model
   - Simulation of crack propagation
3. Micromechanical approach

Behaviour of Nickel foils

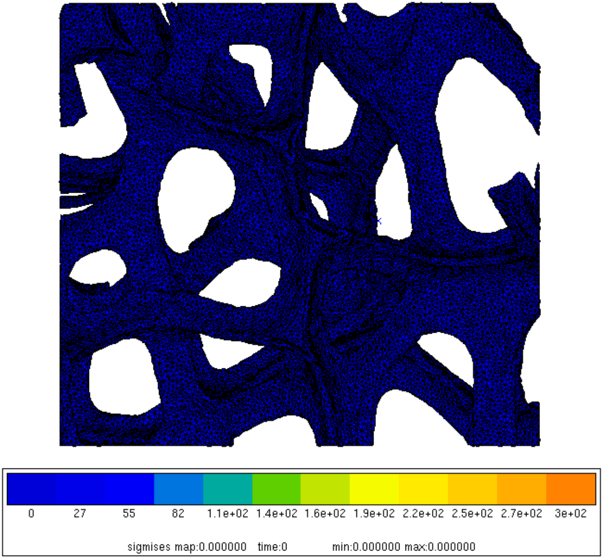


elastoplastic model with linear isotropic hardening

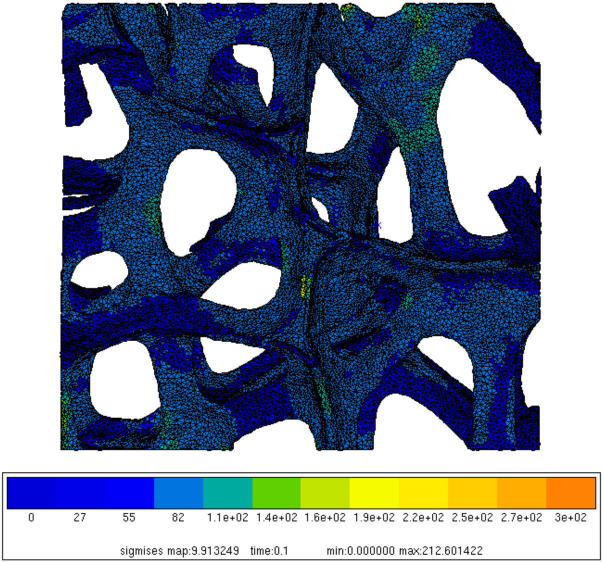
Computing one real single cell



Computing one real single cell



Computing one real single cell



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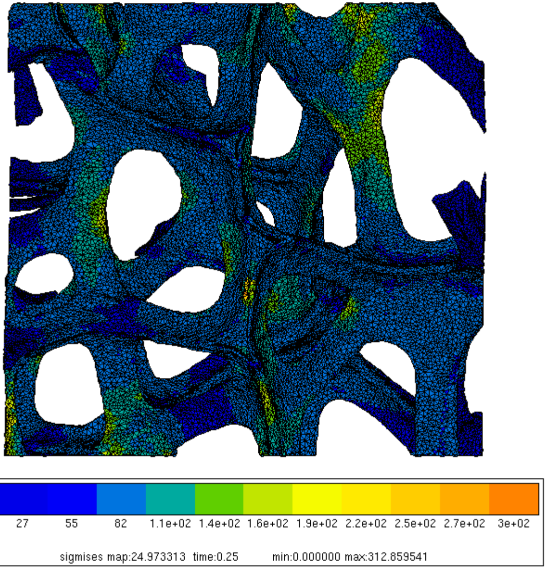
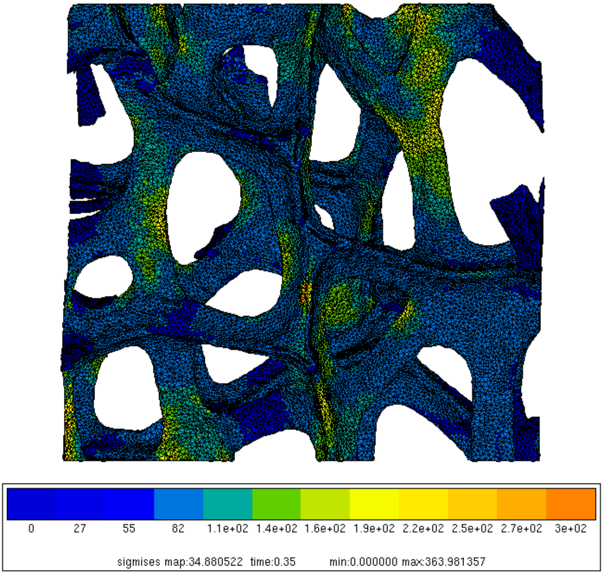
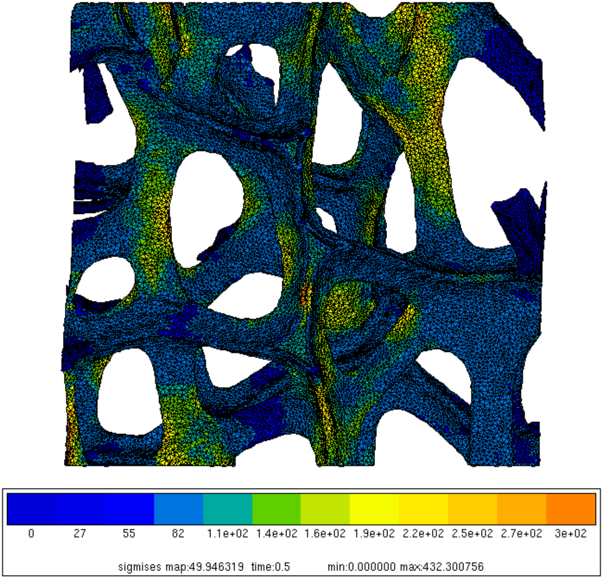


Computing one real single cell



Computing one real single cell



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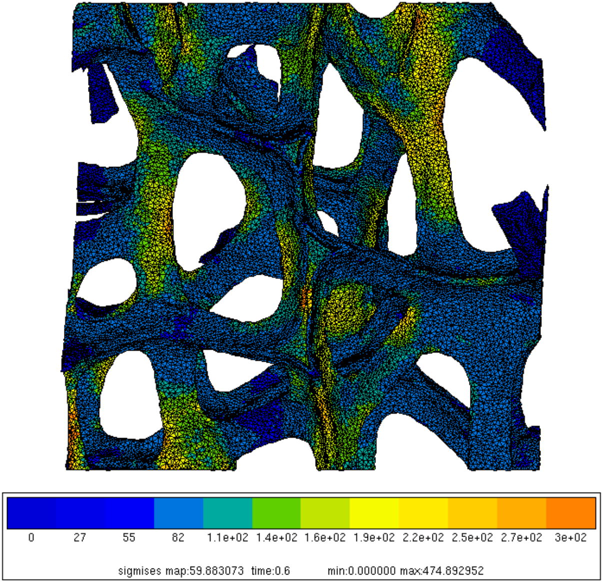
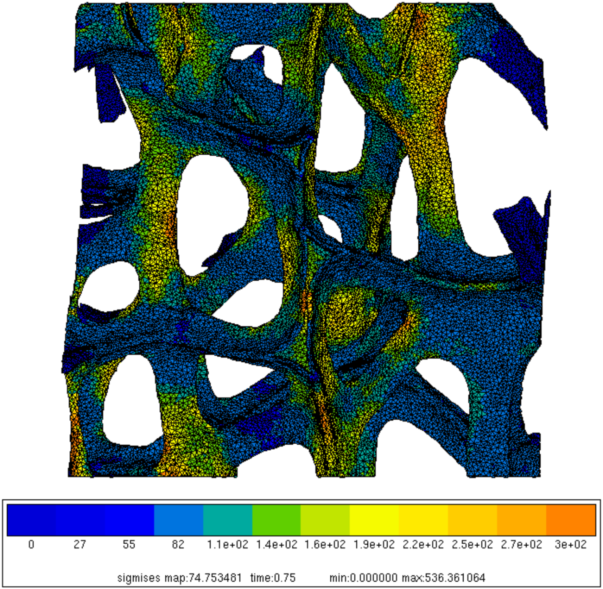
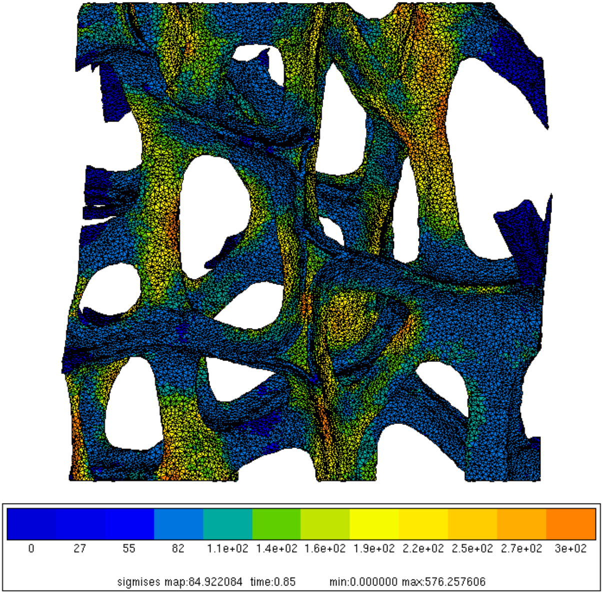


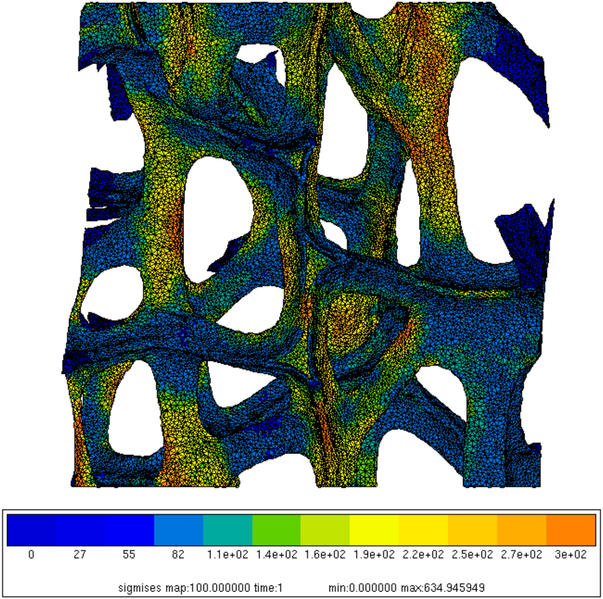
Computing one real single cell



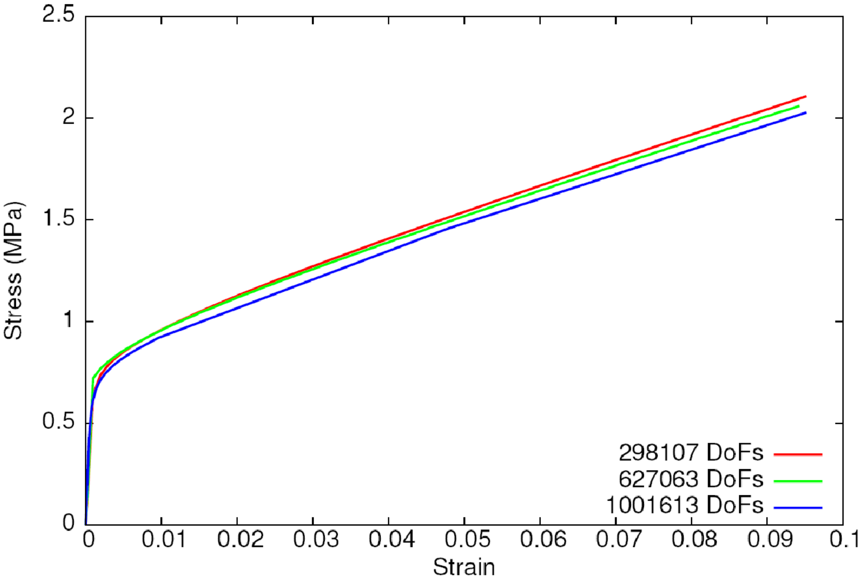
Computing one real single cell



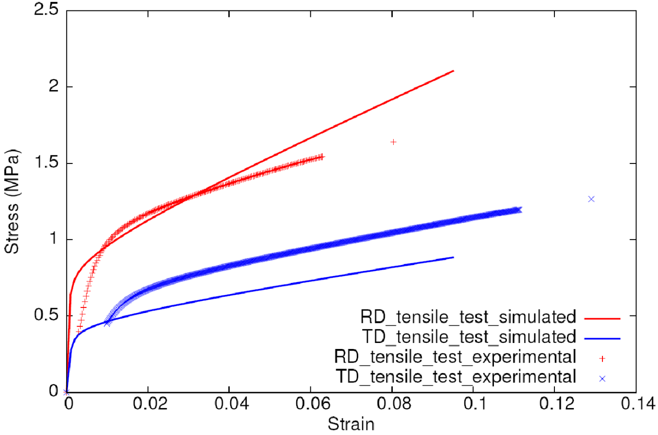
Computing one real single cell



Mesh sensitivity

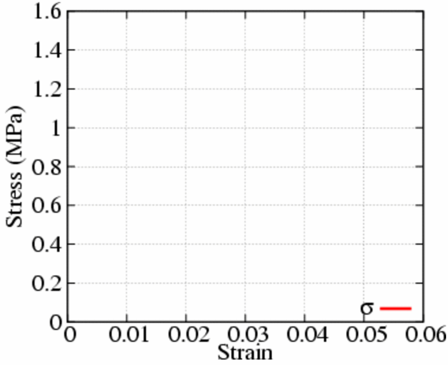


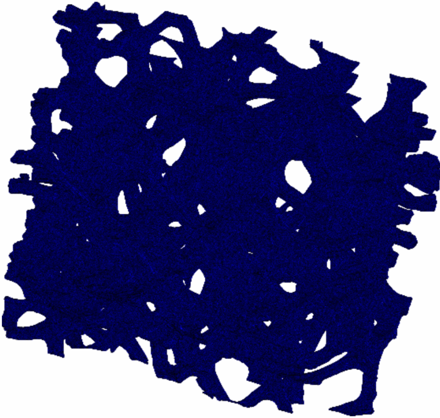
Prediction of elastoplastic properties



Young's modulus (GPa)	RD	TD	ND	E_{RD}/E_{TD}
simulation	1091	319	432	3.42
experiment	177	88		2

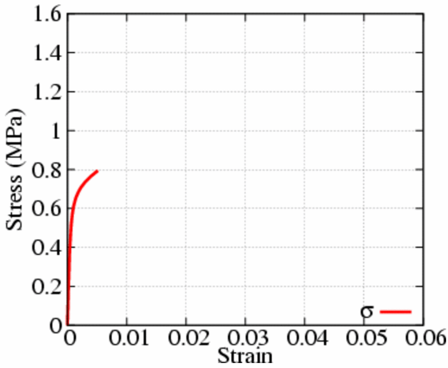
Eight cell volume element

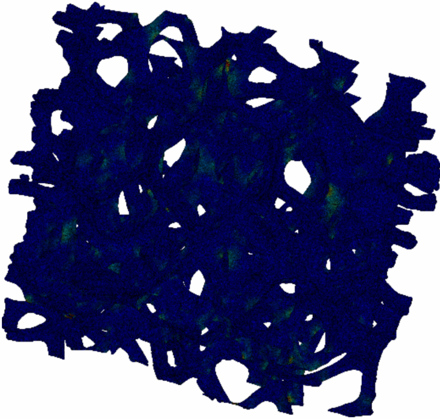




3 600 000 DOF – 36 GB – 300 hours

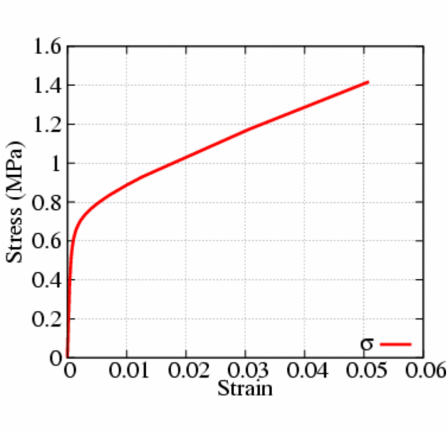
Eight cell volume element

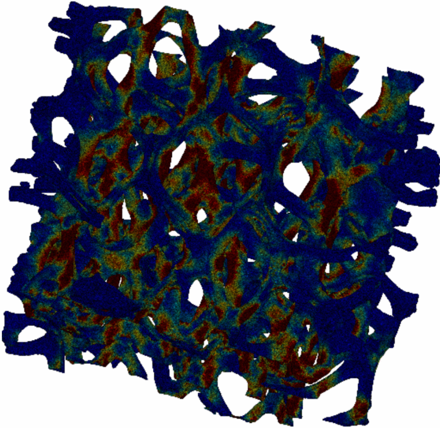




3 600 000 DOF – 36 GB – 300 hours – red: $p > 0.03$

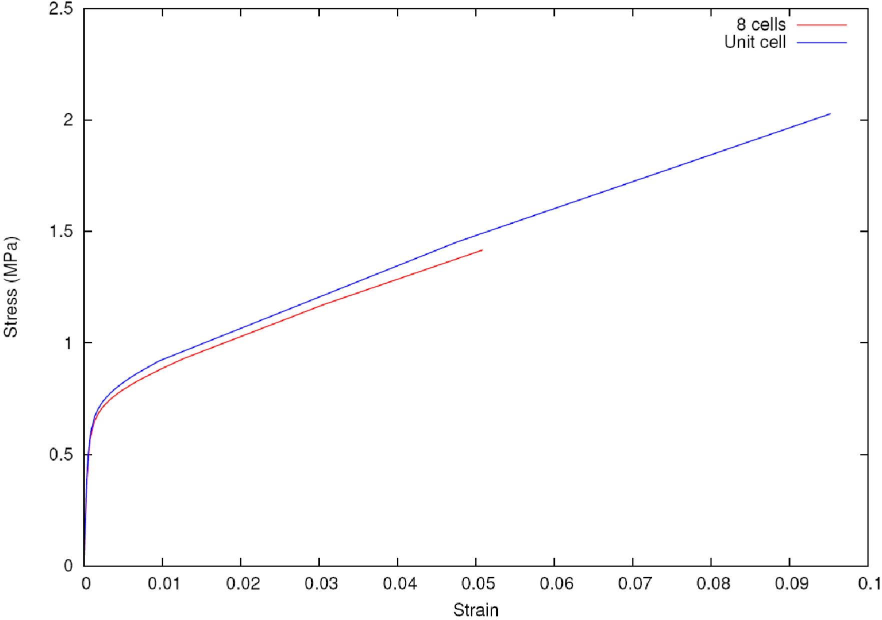
Eight cell volume element





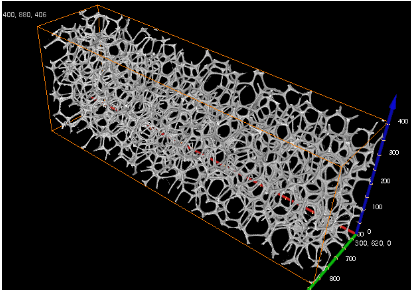
3 600 000 DOF – 36 GB – 300 hours – red: $p > 0.03$

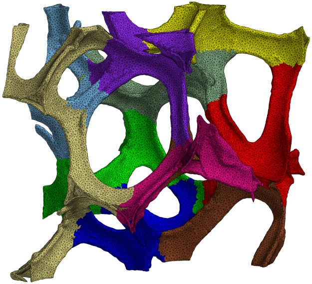
Eight cell vs. one cell volume element



Perspectives

- Micromorphic approach in the mechanics of generalized continua
- Plasticity and fracture with the *microfoam* model
- Micromechanical definition of the additional degrees of freedom :
  link between the microdeformation tensor and the deformation of individual cells
  $\Longrightarrow$ see the presentation of Anthony Burtheau





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