On the design of single crystal turbine blades

G. Cailletaud , J.-L. Chaboche**, S. Forest*, L. Rémy*

* Centre des Matériaux de l'École Nationale Supérieure des Mines de Paris, UMR CNRS 7633, Evry
** ONERA-DMSE, Châtillon-sous-Bâgneux

After a short historical review, this paper recalls the successive steps of the life prediction of single crystal turbine blades, paying attention to a proper modelling of the material, to the mechanical aspects in the blades and to the boundary conditions. The code built around the FE solver ZéBuLoN is now able to predict crack initiation by post-processing of the 3D elastoviscoplastic computations.

■ INTRODUCTION

The lifetime prediction for the critical components of the aeronautical engine, especially the turbine blades, comes at the final stage of the design, after aerodynamics, after the optimization of the cooling system, after solving the thermal problem. The successive steps followed to get a number of cycles to failure are the definition of the constitutive equations, the computation of stresses and strain under realistic loadings, and the lifetime prediction itself, that is the determination of the number of cycles for crack initiation. Some of the authors of the paper have been involved in this process for 30 years, specially with Snecma, with a need to consider the problem from metallurgical or mechanical point of view. This paper gives the opportunity to give a review of the various tasks to be fulfilled in during the design stage. One will successively find :

– three sections on the material element, namely a quick description of the materials used in the blades, the constitutive equations and the damage model ;

– the application of the models to the blades, with first a discussion of the mechanical model chosen to represent the blade, then a description of the software needed, and finally some results on real blades.

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■ MATERIALS

Sitting just downstream of the burner the blades experience temperatures of more than a thousand degree Celsius. Progress can be made by means of better design and cooling systems, as shown below, but impressive improvements were also made with materials in the last three decades. Refractory alloys are needed, and usually Ni-based alloys present the best performance. Starting from polycrystals, collaborative studies between researchers and engineers have lead to directionally solidified materials, then to single crystals. All of them exhibit precipitation hardening, due to the γ–γ' microstructure. The metallurgical features of such an arrangement have been studied for a long time, and studies have been recently developed in single crystals to bridge the gap between the observation of the slip systems and the mechanical models (13). On a lower scale, a well know phenomenon is the evolution of the γ' precipitates from an initially cubic shape toward "rafts", as shown in *figure 1*. Specific thermal treatments can be derived from this kind of studies, but, amazingly, the knowledge is not strong enough to have a full micro-macro model taking into account the two-phase character of the material, even if the last attempts are promising, on the experimental side (6) or concerning more physical models (11, 10). This is the reason why, as shown later, only macroscopic crystallographic models are used for the simulation of the components in the Finite Element codes.

■ CONSTITUTIVE EQUATIONS

Even if the nickel-based superalloys presented in the previous section are clearly two-phase materials, this aspect is usually not taken into account in the models used for design. The main reason is that the effect on the mechanical properties of all the microstructural changes are not fully understood yet. Opposite variations may be due to a change of the volume fraction, the shape of the precipitates, and the local dislocation densities in the channels. The current practice uses pure macroscopic models. Polycrystals are modelled with von Mises criterion, and single crystals by a crystallographic approach. In both cases, viscoplastic constitutive equations are needed to account for stress redistributions. Isotropic and kinematic isotropic hardening are requested to correctly represent cyclic behaviour. These types of models are described in textbooks (14, 2). In the following, a short description of the crystallographic model (16) is given.

Dimensionnement des aubes de turbines monocristallines

G. Cailletaud*, J.-L. Chaboche**, S. Forest*, L. Rémy*

* Centre des Matériaux de l'École Nationale Supérieure des Mines de Paris, UMR CNRS 7633, Evry
** ONERA-DMSE, Châtillon-sous-Bâgneux

L'analyse de la durée de vie des pièces critiques de turbomachines, en particulier des aubes de turbines, constitue l'étape finale du dimensionnement, après celles de l'aérodynamique, et de l'aérothermique, avec l'optimisation des circuits de refroidissement, après l'analyse thermique. Elle comporte les étapes successives de définition des lois de comportement, de calcul des contraintes et déformations sous des cycles de type « marche-arrêt » du moteur, de calcul de durée de vie à amorçage des fissures. Après un bref historique, le présent article décrit ces différentes étapes, en s'attachant notamment à la bonne prise en compte du comportement du matériau, à la mécanique de l'aube, et aux conditions de calcul sous sollicitation réaliste.

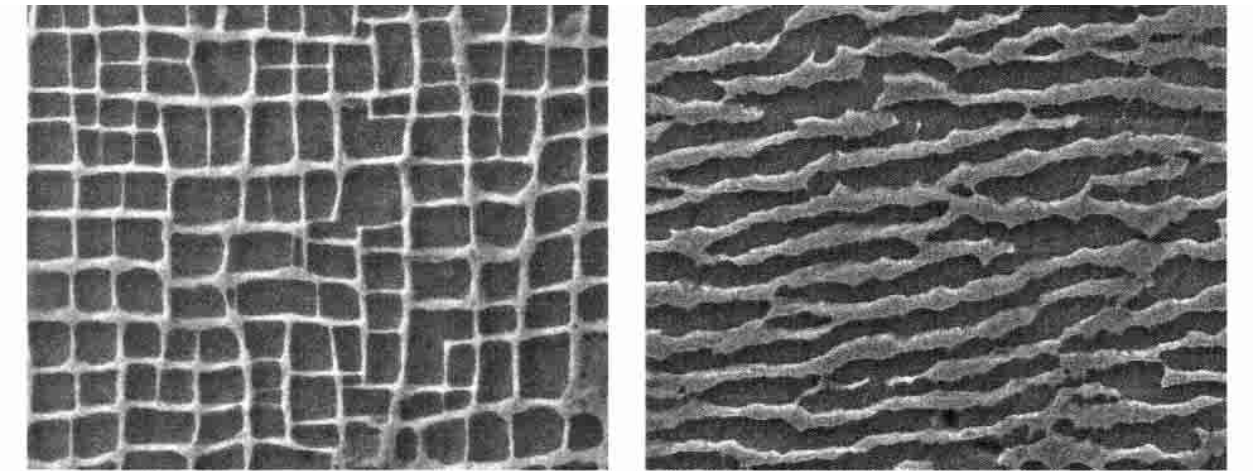
Les modèles de comportement utilisent le critère de von Mises dans le cas des polycristaux, et une approche cristallographique dans le cas des monocristaux. Les approches utilisées ont été identifiées et validées en chargement cyclique et sous conditions anisothermes. Les algorithmes de calcul utilisés en éléments finis font appel aux méthodes maintenant classiques d'intégration implicite, et fournissent la matrice tangente cohérente pour la résolution de l'étape globale. Ils sont implémentés dans le code ZéBuLoN, et peuvent être également utilisés comme sous-programme utilisateur de la plupart des grands codes commerciaux. Des développements particuliers, réalisés sur la plate-forme Z-set/ZéBuLoN, sont brièvement décrits dans l'article : le plus important pour les calculs à réaliser dans les aubes est la possibilité de traitement parallèle. Le calcul parallèle utilise la méthode FETI de décomposition en sous-domaines. Le parallélisme est complet ; il porte sur les jeux de données, le calcul de l'équilibre global, et sur l'intégration des lois de comportement. Les gains obtenus sur cluster de PC se sont avérés pratiquement linéaires jusqu'à quelques dizaines de machines. Mis à part le solveur, la boîte à outil du code comporte des « opérateurs », Z-sim (simulateur d'un chargement multiaxial, ou uniaxial, sur un élément de volume isolé) et Z-optim (optimiseur général contenant de nombreux algorithmes de minimisation), et le module Z-life qui permet, en post-traitement (de grands codes commerciaux ou de ZéBuLoN) de calculer la durée de vie à amorçage de fissure, avec des lois qui prennent en compte les interactions fatigue-fluage-oxydation.

Le « fonctionnement » de l'aube est également discuté. On énumère les différentes sollicitations extérieures, force centrifuge, force aérodynamique, contraintes thermomécaniques, et on discute les modèles mécaniques possibles, avec par ordre de complexité croissante :

- *l'application de la méthode des tranches sur une section d'aube, en ne conservant que les contraintes normales (méthode CALIFAT) ;*
- *l'assimilation de l'aube à une « poutre viscoplastique » qui réunit entre elles N sections, le long d'une ligne neutre dont la position n'est pas connue a priori, chacune étant considérée comme une tranche de la section précédente ;*
- *le calcul par éléments finis 3D, sur une géométrie réduite ou, de plus en plus, sur l'aube complète, avec des modèles allant de 100 000 à 500 000 degrés de liberté, ce qui correspond aux calculs utiles en ce moment pour la conception. Sur de tels problèmes, il est maintenant possible de calculer une cinquantaine de cycles en quelques jours. Au rythme de progression des machines, cette durée devrait tomber à quelques heures dans 2 ou 3 ans.*

Le dernier point traité concerne les difficultés liées à la géométrie de plus en plus complexe des aubes, en particulier à la présence des trous de refroidissement situés en bord d'attaque. Une méthode d'homogénéisation a été mise en oeuvre pour obtenir le matériau équivalent « monocristal troué », ce qui permet d'utiliser un maillage relativement lâche dans un premier temps, dans lequel les trous ne sont pas présents, mais où ils sont représentés par le matériau équivalent. En réutilisant les données du premier calcul, on relocalise ensuite la solution pour trouver le champ réel local au voisinage des trous, et appliquer un critère local d'amorçage.

It is assumed that slip is the predominant deformation mechanism, and that Schmid's law is valid. The resolved shear stress can then be used as a critical variable to evaluate the inelastic flow. A threshold is introduced both in positive and negative direction on each slip system : twelve octahedral slip systems and six cubic slip systems will be used. Two variables are defined for each slip system s, r^s and x^s, corresponding respectively to isotropic hardening (expansion of the elastic domain), and kinematic hardening (translation of the elastic domain). A system will be active provided its resolved shear stress τ^s is greater than $x^s + r^s$ or less than $x^s - r^s$ and the slip rate will be known as long as stress and the hardening variables are known. The state variables used to define the evolution of r^s and x^s are the accumulated slip v^s for isotropic hardening and the variable α^s for kinematic hardening. Knowing the stress tensor applied to the single crystal, $\underline{\sigma}$ the resolved shear stress for system τ^s can be classically written according to equation [1], $\vec{n}^s$ and $\vec{m}^s$ being res-



a) b)

Figure 1 – a) Initial cuboidal microstructure ; b) raft shaped precipitates.

Figure 1 – a) Microstructure cuboïdale initiale ; b) mise en radeau des précipités.

$$\tau^s = \underset{\sim}{\sigma} : \underset{\sim}{\mathbf{m}}^s = \frac{1}{2}\,\underset{\sim}{\sigma} : (\vec{n}^s \otimes \vec{m}^s + \vec{m}^s \otimes \vec{n}^s) \quad [1]$$

$$x^s = c\,\alpha^s \quad ; \quad r^s = R_0 + Q_1 \sum_r h_{rs}\left\{1 - e^{-b_1 v^r}\right\} + Q_2\left\{1 - e^{-b_2 v^s}\right\} \quad [2]$$

$$\dot{\gamma}^s = \dot{v}^s\, sign\,(\tau^s - x^s) \quad ; \quad \underset{\sim}{\dot{\varepsilon}}^p = \sum_s \underset{\sim}{\mathbf{m}}^s\,\dot{\gamma}^s \quad [3]$$

$$\dot{v}^s = \left\langle \frac{|\tau^s - x^s| - r^s}{K} \right\rangle^n \quad \text{with } \langle x \rangle = Max(x, 0) \quad ; \quad \dot{\alpha}^s = \dot{\gamma}^s - d\,\alpha^s\,\dot{v}^s \quad [4]$$

pectively, for the system *s*, the normal to the slip plane and the slip direction in this plane. The hardening variables x^s and r^s can then be expressed as a function of α^s and v^s following equation [2], their current values allowing then to compute the viscoplastic slip rate $\dot{\gamma}^s$, the viscoplastic strain rate tensor $\underset{\sim}{\dot{\varepsilon}}^p$ (equ. [3]), and the hardening rules (equ. [4]). The present formulation gives a saturation of the hardening in both monotonic and cyclic loading, and takes the interactions between the slip systems into account. Seven material and temperature dependent coefficients are defined for each slip system family (*K, n, c, d, R^o, Q, b*).

■ DAMAGE MODELS AND VALIDATION

The operating conditions of the engines involve very high temperatures and long holding times. This is why many types of damage can be found, namely creep, fatigue, and oxidation. The basic model was developed at Onera in the seventies, combining a creep damage law describing cavitation and intergranular damage (Rabotnov) and a fatigue law describing the nucleation and growth of transgranular fatigue microcracks. A nonlinear accumulation rule is obtained by simply adding each damage contribution, so that creep (resp. fatigue) damage increases fatigue (resp. creep) damage rate. The model is able to correctly reproduce the frequency effects, hold time effect, and the effect of complex thermomechanical loadings.

Figure 2 shows the lifetime prediction on a large data base on polycrystalline IN 100 alloy, protected by a vapour phase aluminization. Isothermal tests used for the identification are reported, but also the

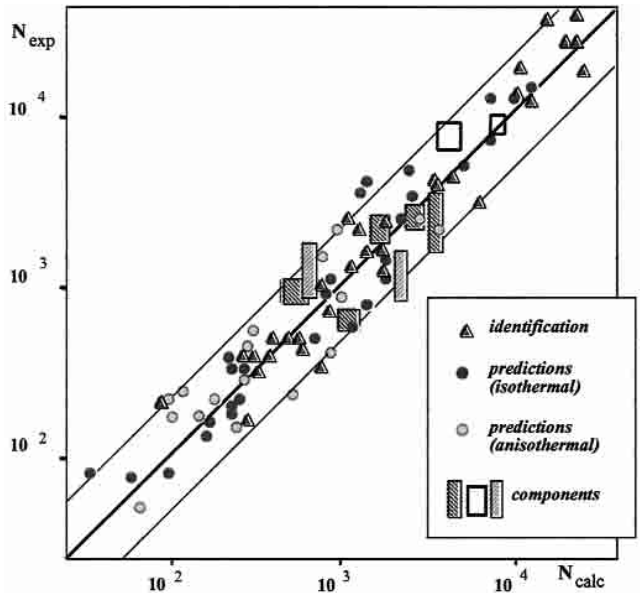


Figure 2 – Lifetime prediction in fatigue and thermomechanical fatigue on specimens and on components for IN 100 superalloy.

Figure 2 – Prévision de durée de vie en fatigue et en fatigue thermomécanique sur des éprouvettes et des composants en alliage IN 100.

prediction for non-isothermal tests (various levels), for thermomechanical tests, and some prediction on actual turbine blades (5), and other components (15).

Single crystal materials like AM1 demand more advanced models. The purpose of this type of approach is to account for fatigue-oxidation interaction (12) (time effect enhanced by environment during the initiation phase, then regular creep-fatigue interaction when microcracks propagate. The first stage corresponds to the cracking of the protection layer (type C1 A) and to the diffusion effect.

■ GEOMETRICAL MODELS

A large number of cycles are needed to have a good estimation of the stress-strain states, to account for the stress redistribution, and perform a good prediction of the crack initiation. This is the reason why very crude approximations have been developed in the past, replacing the full thermo-viscoplastic analysis by a one dimensional approach. This tool is still used at the first step of the design procedure.

• The simplest method consists in isolating a *plane slice* of the blade, and applying then kinematical conditions following Bernoulli assumption : an initially plane section remains plane during the deformation process (*fig. 3 a,b*). Normal stresses in the plane and shear stresses are neglected, and the problem can be reduced to the treatment of three degrees of freedom, one extension, and two deflections, with known resulting forces, produced by the centrifugal force and aerodynamical forces. Moreover, the non-uniform temperature field induces thermomechanical stresses, which can be very high on cooled blades. The simplified problem is then one dimensional, each small volume element being locally subjected to tension-compression loading. A code (CALIFAT) was developed at Onera following these assumptions. It is in operation at Snecma since 1980. A macroscopic viscoplastic law is used to describe the material, and creep-fatigue interaction models to predict crack initiation.

• When it is difficult to decide a priori the location of the critical section of the blade, the next step consists in considering a series of sections, combined in a *viscoplastic beam* (*fig. 3c*). Such codes have been tested in the seventies (4) : a first industrial application was made on a Snecma turbine blade. The industrial implementation has been made in the nineties, in a new environment (1), to account for the deformation of the neutral line, which is now the result of the competition between aerodynamical and centrifugal forces. The momentum in each section is then an unknown, the value of which derives from the computation of the solution of the beam bending problem, introducing a series of cross sections.

• *Full 3D analyses* were also performed in the beginning of the nineties, starting with a 3D section (generalized plane strain assumption), allowing the computation to take into account transverse stresses and local stress concentration due to geometry. Since the shape of the blade is more and more complex, the critical areas are very sensitive to external loading, and to the temperature field. The top of the blade experience larger temperatures, but large shear stresses may be active near the disk, cooling holes on the leading edge introduce stress concentration... Large 3D computations have then to be performed to cover all these aspects. The constitutive equations do not change, nevertheless, as dicussed later, parallel computations have to be introduced.

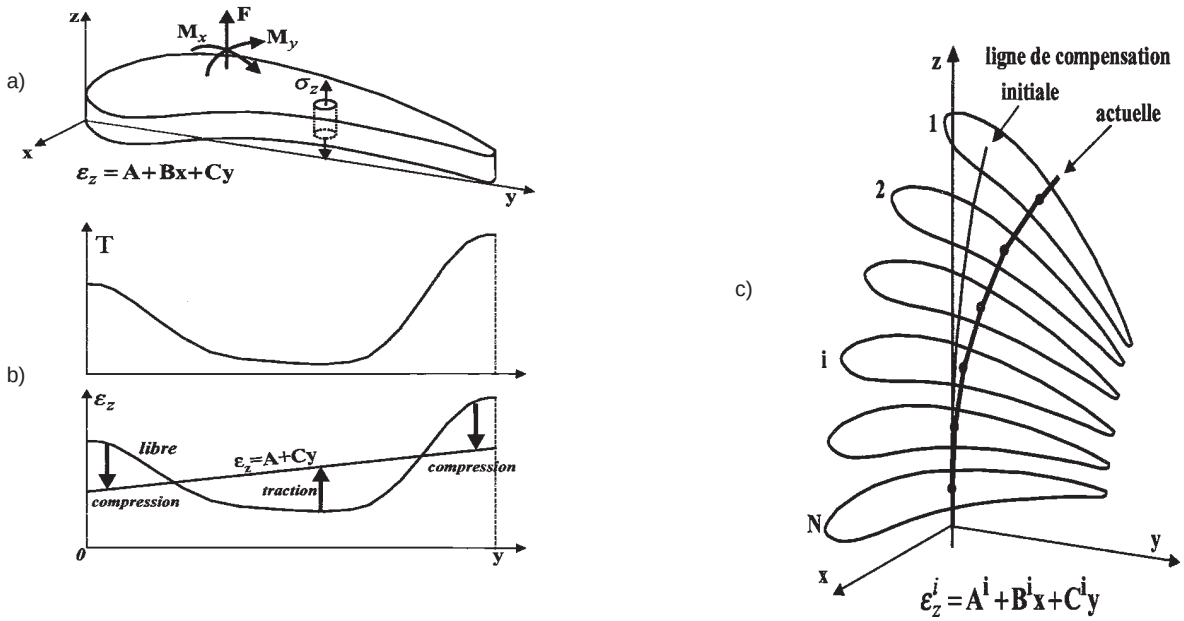


Figure 3 – The plane section method. a) Discretization of the section ; b) typical temperature and strain fields ; c) viscoplastic beam method.

Figure 3 – La méthode de la section plane. a) Discrétisation de la section ; b) champs de température et déformation typiques ; c) méthode de la « poutre viscoplastique ».

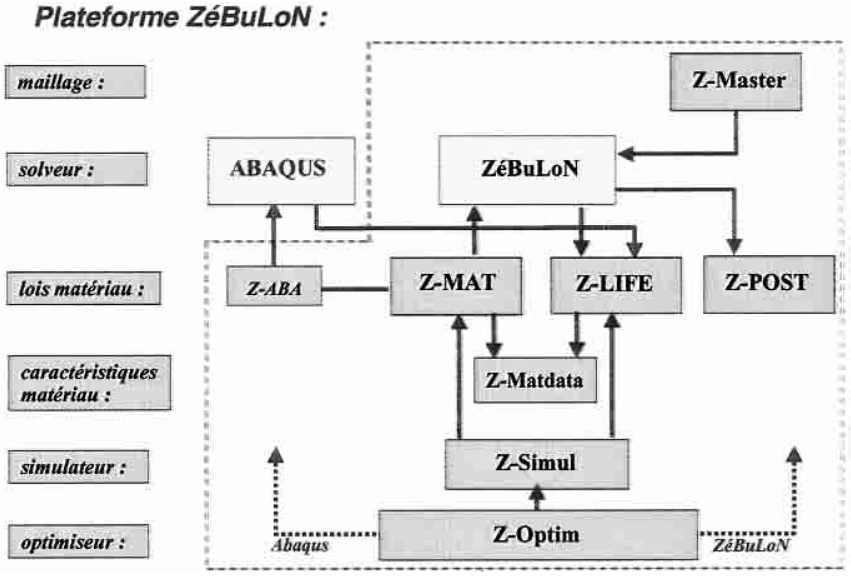


Figure 4 – The code Z-Set / ZéBuLoN.

Figure 4 – Le code Z-Set / ZéBuLoN.

■ THE PREDICTION CHAIN

The analysis of the turbine blades is performed at Snecma by using several codes, specially the code Z-Set, including the FE solver ZéBuLoN. Originally elaborated at École des Mines, this platform is now co-developed by École des Mines, Onera and North West Numerics (nwnumerics.com). *Figure 4* shows the main modules in the code. One can note that all the constitutive equations (cyclic elasto-viscoplasticity, coupled damage cumulation...) are available as a huge user subroutine (Z-mat) in commercial codes (Z-aba for Abaqus, but also versions for Marc, Ansys and Cosmos). Other interfaces are in progress, for instance with the code Mecano. The material parameters are identified by means of a constitutive equation driver (Z-sim) allowing the user to load a representative volume element in onedimensional or multiaxial loadings, which can be implemented in an optimizing loop using Z-opt, which is a general purpose optimizer with several strategies available (using the gradient, heuristic approaches, evolutionary procedures). Since Z-sim has a direct access to Z-mat, the same code is used for the identification stage and the FE computation, with exactly the same source and the same material file, so that the risk of introducing an error between the two is virtually suppressed. This remains true if the computation is made with the ZéBuLoN solver, but also with any other commercial code like Abaqus. The final module used for life prediction is Z-life, in which the models previously described are implemented (creep-fatigue interaction, creep-fatigue-oxidation interaction).

The parallel version of the code was developed at Onera (8). The solution of the system is obtained by subdomains, using the FETI method (7). This approach implements a full parallelism (data sets, solution of the global equilibrium, integration of the local constitutive equations). It is very efficient on MIMD clusters. PC clusters will probably offer impressive capabilities for this type of code, so that the size of the acceptable meshes will dramatically increase. The scalability of the method is illustrated in *figure 5*, with a comparison with results obtained by other codes (Boulder University, “collecteur échappement”). The diagram demonstrates that domain decomposition produces large CPU reduction and that the method is well adapted to large problems, for which the computation time is big if compared with time spent for communications).

In addition to parallel computation, an acceleration of the convergence toward the mechanical steady state is obtained through a special “cycle skip” method. Some prototypes were made in the past (17), and the method has been recently reimplemented in ZéBuLoN and Abaqus (9). The number of cycles skipped during the computations is small during the first stage of operation, but it increases

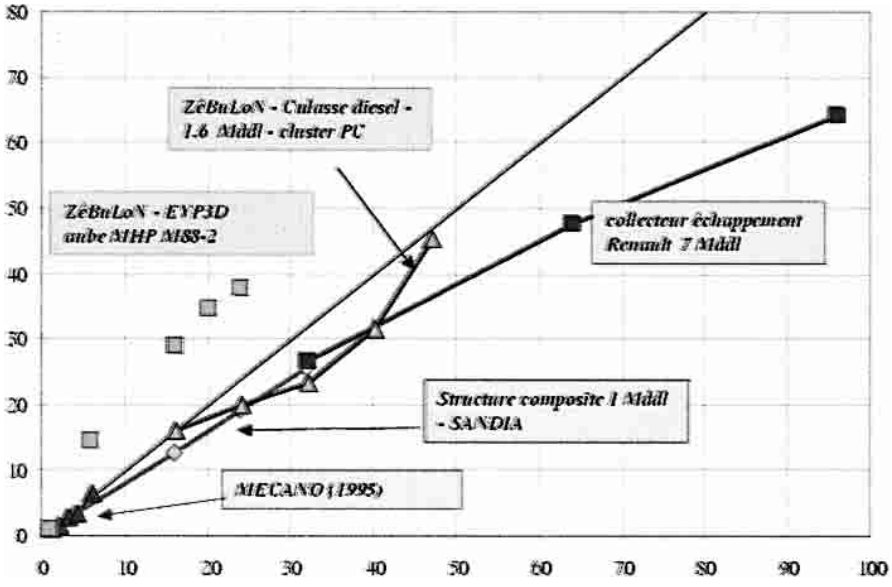


Figure 5 – Illustration of the scalability of the FETI method in the code ZéBuLoN.

Figure 5 – Illustration des performances de la méthode FETI dans le code ZéBuLoN.

gradually, and the method becomes specially efficient if a large number of cycles have to be computed. *Figure 6* demonstrates the good agreement between a reference calculation with all the cycles and the results obtained with the “cycle skip” method.

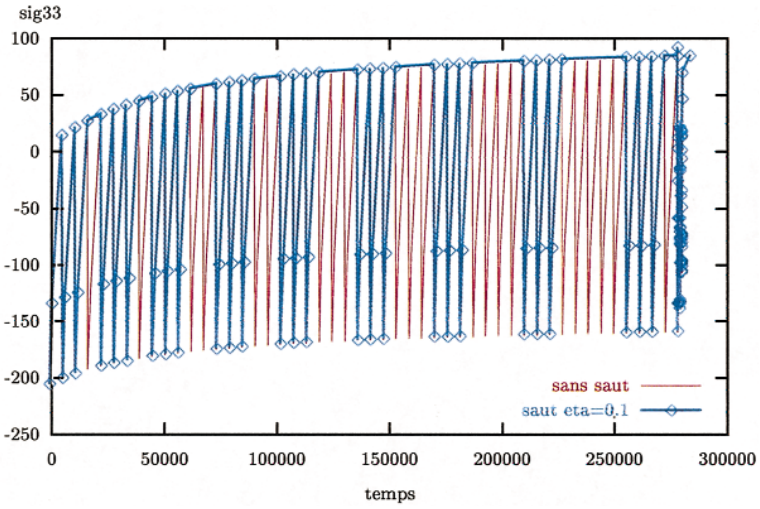


Figure 6 – Comparison of the results given with or without the cycle skip technique.

Figure 6 – Comparaison des résultats obtenus avec ou sans la méthode de saut de cycles.

■ RESULTS

Figure 7 shows a typical calculation, made some years ago. A relatively coarse mesh is used, for representing the whole blade. This type of calculation allows to have an idea of the critical areas. By the way, their location is no longer intuitive, due to the large gradients in the temperature field, and the 3D character of the stress field. The meshes presently used have a larger number of degrees of freedom, and a more precise geometrical description of the critical zones. The calculation must also take into account centrifugal forces, and aerodynamical forces. Their history is quite complex : in addition to take-off and landing, a series of regime changes have generally to be considered, so that a realistic “cycle” is in fact made of a series of 50-100 steps with 20-30 sub-cycles. The resulting stress histories have to be treated for the application of the crack initiation models. Rain flow is generally used to extract the most damaging events.

Modern blades are actively cooled. After introducing hollow blades in the seventies, engineers have built more and more complex systems. Nowadays, cool air, which is led off the compressor, is pumped through the blade and out through small holes on the leading edge and slits on the trailing edge. Such a blade can now be computed (3), accounting for the presence of holes and slits in 3D elasto-viscoplastic finite element computations. This calculation was made possible only thanks to the parallel computation, since the number of dofs is larger than 500,000. Even in that case, several days are necessary to achieve the computation. In order to get better CPU times, the presence of holes can also be accounted for by introducing at the leading edge a zone of weakened material as shown on *figure 8a*. This amounts to replacing the perforated part of the blade by a homogeneous equivalent medium. Its mechanical properties are obtained using homogenization methods. The unit cell consisting of a single crystal small volume containing one hole can be subjected to complex loading conditions including tension, shear and hydrostatic pressure. An explicit effective constitutive model is identified from the responses of the cell. Such a model of compressible single crystal plasticity has been proposed in (3). It involves the plastic slip systems and additional dilatant systems :

$$\underset{\sim}{\dot{\varepsilon}}^{p} = \sum_{s=1}^{18} \dot{\gamma}^{s}\, \underline{\mathbf{m}}^{s} \otimes \underline{\mathbf{n}}^{s} + \sum_{s=1}^{3} \dot{\delta}^{s}\, \underline{\mathbf{n}}^{s}_{dil} \otimes \underline{\mathbf{n}}^{s}_{dil} \qquad [5]$$

where the chosen expansion directions are $\underline{n}^{s}_{dil}$ = {100}.

The local stress-strain states around an underlying hole can then be deduced from the overall stress and strain at a point of the weakened zone, by means of standard concentration methods in homogenization theory (2). An example of such a reconstructed stress field around a single hole is given in *figure 8b*. It can be shown that the use of an effective material law significantly improves this local prediction. Post-processing lifetime assessment models can the be used to predict the effect of the presence of holes and slit on crack

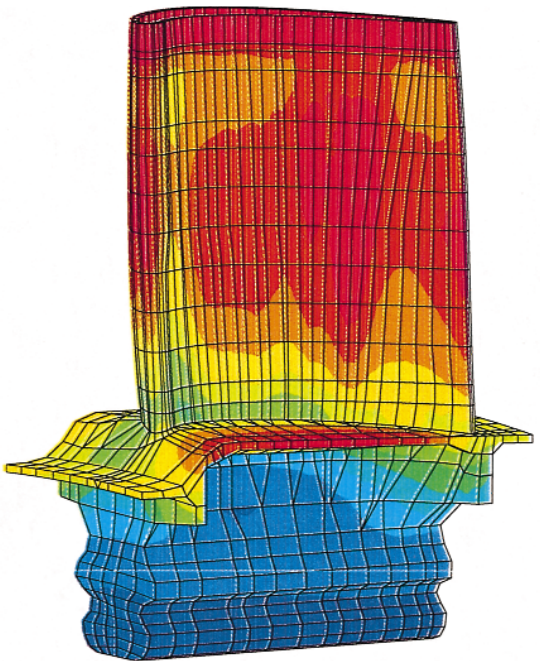
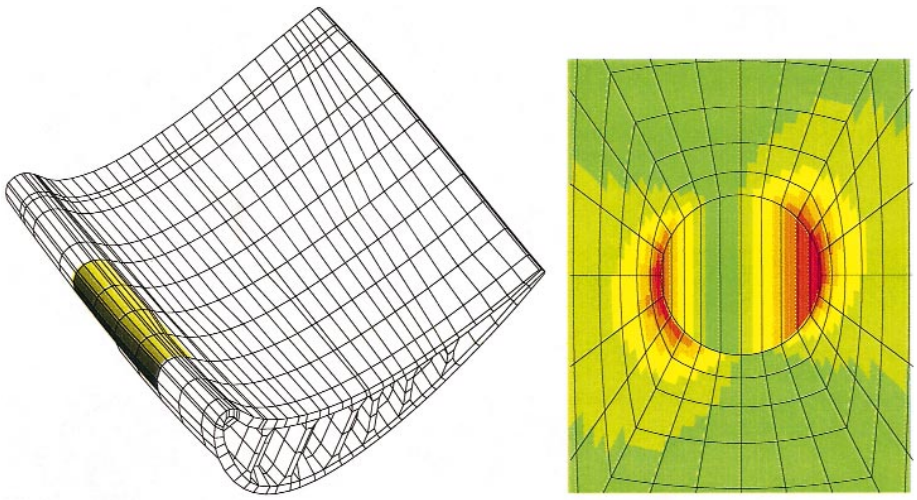


Figure 7 – Example of a temperature field on a typical turbine blade.

Figure 7 – Exemple d’un champ de température sur une aube de turbine typique.



a) b)

Figure 8 – a) Part of a turbine blade with a zone in grey weakened by the presence of holes and modelled by a homogeneous substitution material ; b) equivalent stress field around a hole reconstructed from the overall fields using homogenization techniques.

Figure 8 – Maillage d'une aube de turbine mettant en œuvre une zone de matériau homogène équivalent simulant la présence des trous de refroidissement ; b) reconstruction du champ local à partir du calcul global uitlisant les techniques d'homogénéisation.

initiation. The methodology is general enough to be extended to other multiperforated jet engine components like combustors and burners.

■ CONCLUDING REMARKS

The purpose of the paper was to make a brief review of the evolution of the design of turbine blades in the last thirty years. Major changes have to be mentioned. Single crystals are now used for the hottest part of the engine, after classical polycrystals then directionally solidified materials. The shape of the blades is also more and more complex, with incurved leading edges and the presence of holes in order to generate gaseous films to protect the material against the combustion gas. New models have been developed accordingly and, since the power of the computers increases continuously, the strategy of the component lifeprediction is also updated. In the industry, the design process needs a good cooperation between the various actors who have in charge combustion, fluid mechanics, thermal problems, and mechanics. Material engineering is also concerned, and exchanges with mechanical engineers are crucial. This type of discussion is now traditional in France, and allows the engineers to have in hand mechanical models taking into account the real microstructure of the materials.

A lifetime prediction is performed with the following steps :

– acquisition of the thermal fields and of the aeronautical forces ;

– mesh of the whole blade ;

– FE computation of a series of typical missions, which may use parallel computers for large meshes ;

– evaluation of the time to crack initiation with a post-processing which reads the stress and temperature history.

For critical cases, a structural zoom can be performed, in order to get a better representation of critical areas. This has to be made for taking into account the rôle of multiple perforation.

Improvements in the methodology are still to come. As a first direction for future research authors try to be able to predict short crack propagation (and maybe crack arrest) in single crystals, during multiaxial non-isothermal loadings. Thermal barriers are also an active research field. They involve a lot of coupled mechanisms. Understanding how they operate, characterizing the rate of damage in connection with diffusion process, being ready to give reliable predictions for spallation are still a challenge for researchers and engineers.

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