

Figure 9: The representation of the broken fibers condensation for a 12-generation system, with 10-fiber bundles. The X-axis is the fiber position (from 1 to 20480) and the Y-axis is the successive position of the fibers that break under the load. The right figure is a zoom on the first bifurcation points.

4. CONCLUSION

We have compared two simple models that allow to study the rupture of an interface in a mode I sollicitation, particularly during the bifurcation process. The two models differ by the representation of the interface. In one case, we have used a continuous model as in the other one we have used a discrete model. We have first presented a system that does not take into account a redistribution. It is just made to propose an equivalent criterium for the bifurcation in a continuous model and the discrete one. Hence, the divergence of the avalanche mean size is showed to be equivalent to the loss of stability (second-order work becomes negative) for the continuous case. Then, we have presented a more realistic model that takes into account a mechanical redistribution along the interface. We have shown that for both models, the continuous and the discrete one, the initiation of the first crack is the result of a bifurcations cascade.

References

[1] Hill, R., *Some Basic Principles in the Mechanics of Solids without a Natural Time*, J. Mech. Phys. Solids, Vol. 7, pp. 209-225, 1959
 [2] Pijaudier-Cabot, G., Benallal, A., *Strain Localization and Bifurcation in a Nonlocal Continuum*, Int. J. Solids Structures, Vol. 30, N 13, pp. 1761-1775, 1993
 [3] Bazant, Z.P. and Cedolin, L., *Stability of Structures*, Oxford University Press, 1991.
 [4] Daniels, H. E., "The Statistical Theory of the Strength of Bundles of Threads", *Proceedings of the Royal Society*, London, Vol. A 183, pp. 405-435, 1945.
 [5] Hemmer, Per C., Hansen, A., "The distribution of simultaneous fiber failures in fiber bundles", *Journal of Applied Mechanics*, december 1992, Vol.59, pp. 909-914.

An estimation of overall properties of heterogeneous Cosserat materials

R. Dendievel, S. Forest* and G. Canova

GPM2-INPG/UJF, BP. 46, 38402 Saint-Martin-d'Hères cedex, France
 * Centre des Matériaux, UMR 7633 du CNRS, École des Mines de Paris, BP. 87, 91003 Evry, France

Abstract : In this work, we try to answer the following question: What is the effective characteristic length of a mixture of Cosserat continua? More generally, homogenization methods for heterogeneous Cosserat media are proposed and applied to the case of linear elasticity. They first application deals with a beam network regarded as a discrete Cosserat medium and the second with a continuous heterogeneous Cosserat continuum.

1 INTRODUCTION

Recently, there has been an increasing interest for so-called "materials with microstructures". For such materials, mechanical properties exhibit significant size effects. Such effects may arise when the wave length of the applied loading conditions are not much greater than microstructural length scales such as grain size for polycrystals or particle spacing for composites. An other possibility is when large strain or stress gradients occur at the scale of these microstructural elements.

In the last three decades, a certain number of homogenization theories have been developed in order to predict the macroscopic behaviour of heterogeneous materials from their constituents. Generally, these theories are developed in the field of the classical continuum mechanics and give results which are independent of the scale of the microstructure. In particular, it is the case of all the predictions based on an Eshelby type inclusion problem. To take into account such size effects, higher order theories have been developed. They involve additional degrees of freedom [1,2] or/and higher order gradients of the displacement fields. In the present paper, we focus on the Cosserat continuum for which independent rotational degrees of freedom are introduced at each point of the continuum in addition to the displacement field.

From an experimental point of view, there has been a long quest for the experimental evidence of Cosserat manifestation and determination of the elastic constants for real materials. The first results were not very successful [3,4]. More recently, Cosserat elastic parameters were determined for some composite materials (assembling of sliding blocks in [5], layered materials in [6]), cellular materials (polymeric foams [7], bones or similar human tissues [8]).

From a theoretical point of view, some approaches have been recently proposed in order to derive the overall behaviour of heterogeneous materials described in the field of higher order theories [9]. In this paper, after recalling the basic equations of general isotropic elasticity in Cosserat media (section 1), we propose two systematic approaches for estimating the effective parameters of two different types of Cosserat media.

The first one is a discrete Cosserat medium (section 2). It could be a cellular material (honeycomb, polymeric or metallic foam) or a composite material exhibiting a skeleton of reinforcing fibres (e.g. whiskers reinforced matrix). A Representative Volume Element (RVE) of such a medium is idealized by a percolating network of elastic beams. Some studies have been already performed for 2D simple periodic structures [10,11,12]. The present study deals with tridimensional randomly distributed networks. The effective properties of the assembly are derived using Finite Element calculations with adequate imposed boundary conditions.

The second one is a continuous heterogeneous Cosserat medium (section 4 and 5). It is for example a polycrystal where each grain can be regarded as a Cosserat constituent if the density of geometrically

3 IDENTIFICATION OF OVERALL MODULI FOR DISCRETE COSSERAT MEDIA

Let us consider now an idealized medium made of a tridimensional lattice of randomly distributed beams. The term "beam" employed here signifies that the elements are capable of supporting axial, shear, flexural and torsion loads. The intersection points between the beams define nodes which are characterised by three displacements and three rotations, corresponding to the kinematical freedom of the points in a Cosserat continuum representation. At these nodes, the links are assumed perfect (i.e. infinitely rigid), so that both displacements and rotations are fully transmitted. The different parameters are the geometrical and mechanical characteristics of the beams (length, section and inertia moduli, Young's modulus, Poisson's ratio) as well as the topological definition of the lattice (volume fraction of the beams, type of spatial arrangement). The aim is then to verify if such a discrete medium can be seen as a homogeneous Cosserat medium. Contrary to the previous studies performed on periodic lattices (Lakés, Adachi), it is impossible here to derive analytically the overall Cosserat moduli. We have chosen to perform Finite Element calculations over a cubic representative volume containing several hundreds of beams. The intersections of the beams with the faces of the cube determine the boundary nodes. It is then possible to identify the overall moduli by applying appropriate homogeneous conditions at the boundaries, i.e. setting the displacements and the rotations of the boundary nodes so that they fulfill prescribed homogeneous displacement gradient, rotation or rotation gradient. The identification results of the equality between the double of the deformation energy in the cubic volume and the external work, which is calculated from the forces and the moments obtained at the boundary nodes by the FEM simulation. Six simple tests allow us to obtain the six Cosserat coefficients. In the following we give for each test the homogeneous imposed boundary value in regard with the value of the external work where the unknown coefficients (all the other kinematic values are zero) appear:

- II) homogeneous displacement gradient : $u_{i,1} \rightarrow W_I = (2\mu + \lambda) u_{i,1}^2$
- III) homogeneous displacement gradient : $u_{i,2} \rightarrow W_{II} = (\mu + \mu_c) u_{i,2}^2$
- IV) homogeneous rotation : $\phi_1 \rightarrow W_{III} = 4\mu_c \phi_1^2$
- V) homogeneous rotation gradient : $\phi_{i,1} \rightarrow W_{IV} = (2\beta + \alpha) \phi_{i,1}^2$
- VI) homogeneous symmetric rotation gradient with $\chi_{12} = \chi_{21} = \phi_{1,2} \rightarrow W_V = 4\beta \phi_{1,2}^2$

The external work is calculated from the forces and the moments obtained at the boundary nodes by the FEM simulation. Values of the elastic parameters follow immediately. It is to be noted that, in a first step, it is possible to verify the isotropy hypothesis by performing the different combinations of values for indices 1,2 or 3.

Different networks with different characteristics have been generated. In the present article, we will give, as an example, the results for networks of beams which were taken of length unity (in μm), with a diameter set to 0.01 (i.e. an aspect ratio of 100). The Young's modulus and Poisson's ratio were taken respectively to be 150 GPa and 0.3. These particular values correspond to a "real" material studied in the lab (network of natural cellulosic whiskers acting as reinforcements for a polymeric matrix [13]. Figure 1 gives an example of VER and the table displays the results for different volume fractions of beams (the size of the cube remaining constant). In this particular case, it can be seen that the coefficients β and γ are approximately equal as well as the length scales l and l' . Moreover, the values of the classical Lamé parameter are much greater than the value for the μ_c one. In this specific case, it seems that the material is not a strong Cosserat material.

Some other tests can lead to the following remarks. Decreasing the value of the beam section S leads to the decrease of the Cosserat parameters, but l and l' are unchanged. However, these two characteristic lengths are linked to the geometry of the structure : for example, an increasing beam density tends to decrease it. A measure of the inherent structure length scale could be the average beam length. When this quantity is plotted against the beam volume fraction, a decreasing trend is observed as for the Cosserat lengths. However, close to the percolation threshold, the lengths l and l' seem to go to infinity, which is not the case for the average beam length. Moreover, at a given

necessary dislocations becomes important. Assuming that a RVE made of a heterogeneous Cosserat material can be defined, we try to solve a micro-mechanical boundary value problem. The problem of determining effective properties of heterogeneous continuous Cosserat media must be distinguished from that of constructing a Cosserat homogeneous equivalent medium starting from a heterogeneous classical material. This last problem has been investigated in [17]. In this last reference, it is shown how the pertinence of the found effective properties can be assessed : Computations involving the actual so-called "coarse grain structure" with every heterogeneity can be compared in some situations to the response of the homogeneous equivalent medium.

2 THE COSSERAT FORMALISM

Throughout the paper, both tensorial and indicial notation will appear. In the body of the text, the first one will be often used. In this case, $\mathbf{x}, \mathbf{x}, \underline{\mathbf{x}}$ respectively denote a vector, a second-rank tensor and a fourth-rank tensor. The Levi-Civita third-order permutation tensor will be denoted by $\underline{\underline{\epsilon}}$ if applied to a second-rank tensor or $\underline{\underline{\epsilon}}$ if applied to a vector.

The first sections deal with isotropic elasticity in a general Cosserat continuum. Such a continuum is defined by the fact that the transfer of mechanical interactions is carried out not only by forces \mathbf{F} but also by moments \mathbf{M} acting on any elementary unit surface oriented by vector $\underline{\mathbf{n}}$. The corresponding force and couple stress tensors $\underline{\underline{\sigma}}$ and $\underline{\underline{\mu}}$ are defined by

$$F_i = \sigma_{ij} n_j \quad \text{and} \quad M_{ij} = \mu_{ij} n_j \tag{1}$$

In general, these two tensors are not symmetric. They have to fulfill the equilibrium equations

$$\sigma_{ij,j} + F_i^{ext} = 0 \quad \text{and} \quad \mu_{ij,j} - \epsilon_{ijk} \sigma_{jk} + M_i^{ext} = 0 \tag{2}$$

where \mathbf{F}^{ext} and \mathbf{M}^{ext} are volumic external forces and couples.

The kinematic variables are the classical displacement field $\underline{\mathbf{u}}$ and a new micro-rotation field which accounts for the rotation of the underlying microstructure and which will be represented by the vector $\underline{\underline{\phi}}$. This rotation is here an independent variable, whereas, in a constrained theory, it appears as the skew-symmetric part of the displacement gradient tensor. The associated deformation $\underline{\underline{\epsilon}}$ and torsion-curvature $\underline{\underline{\chi}}$ tensors are defined in the following manner :

$$\underline{\underline{\epsilon}}_{ij} = u_{i,j} + \epsilon_{ijk} \phi_k \tag{3}$$

$$\chi_{ij} = \phi_{i,j} \tag{4}$$

The constitutive equations relating the stress tensors and deformation tensors are :

$$\underline{\underline{\sigma}} = 2\mu \{ \underline{\underline{\epsilon}} \} + 2\mu_c \{ \underline{\underline{\epsilon}} \} + \lambda \text{tr}(\underline{\underline{\epsilon}}) \underline{\underline{1}} \tag{5}$$

$$\underline{\underline{\mu}} = 2\beta \{ \underline{\underline{\chi}} \} + 2\gamma \{ \underline{\underline{\chi}} \} + \alpha \text{tr}(\underline{\underline{\chi}}) \underline{\underline{1}} \tag{6}$$

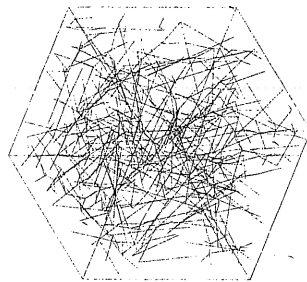
with the conditions $\mu, \beta, \mu_c, \gamma > 0$, $3\lambda + 2\mu > 0$ and $3\alpha + 2\beta > 0$. $\underline{\underline{1}}$ is the second-rank unit tensor. The symbols $\{ \cdot \}$ and $\} \cdot \}$ denote respectively the symmetric and skew-symmetric parts of a second-rank tensor. The description of a general isotropic elastic Cosserat continuum involves then six coefficients. λ and μ are the well-known Lamé coefficients of the classical elasticity. μ_c is related to the possible non symmetric nature of the stress tensor $\underline{\underline{\sigma}}$. The three other coefficients are related to the couple effects. Moreover, two intrinsic length scales can be introduced, defined by

$$l^2 = \frac{(\beta + \gamma)(\mu + \mu_c)}{4\mu\mu_c} \quad \text{and} \quad l'^2 = \frac{\alpha + 2\beta}{4\mu_c} \tag{7}$$

These characteristics lengths govern the size scale at which deviations from classical elasticity are observed. The elastic deformation energy density can be written as :

$$U_{el} = \mu \{ \underline{\underline{\epsilon}} \} : \{ \underline{\underline{\epsilon}} \} + \beta \{ \underline{\underline{\chi}} \} : \{ \underline{\underline{\chi}} \} + \gamma \{ \underline{\underline{\chi}} \} : \{ \underline{\underline{\chi}} \} + \frac{\lambda}{2} \text{tr}(\underline{\underline{\epsilon}})^2 + \frac{\alpha}{2} \text{tr}(\underline{\underline{\chi}})^2 \tag{8}$$

volume fraction of whiskers and a given average beam length, increasing the size of the representative cube leads to a decrease of the parameter α whereas the Lamé coefficients λ and μ remain constants. The Cosserat overall elastic properties turn out to depend strongly on the size of the representative volume. The effective continuum tends to a classical Cauchy continuum when this size increases. A Cosserat approach appears then relevant when the wave length of the applied loading conditions is no greater than the size of small number of beams.



	volume fraction of beams					
	1%	1.5%	3%	5%	10%	15%
λ (Pa)	1.5e07	2.5e07	1.5e08	2.7e08	6.6e08	1.1e09
μ (Pa)	2.3e07	3.1e07	1.0e08	2.7e08	6.5e09	9.5e08
μ_c (Pa)	4.3e04	8.2e04	3.2e05	7.5e05	2.8e06	5.5e06
α (Pa m ²)	2.0e04	-4.e03	7.1e03	-2.e03	2.9e04	-2.e04
β (Pa m ²)	4.5e04	5.6e04	1.4e05	2.4e05	6.1e05	8.5e05
γ (Pa m ²)	5.2e04	5.8e04	1.5e05	2.6e05	6.3e05	8.7e05
l (μ m)	0.75	0.59	0.41	0.41	0.33	0.28
l' (μ m)	0.81	0.57	0.39	0.39	0.33	0.27

Figure 1 : Mesh of a random beam network (top left) and values of the cosserat elastic parameters.

4 OVERALL PROPERTIES OF CONTINUOUS HETEROGENEOUS COSSERAT MEDIA

Let us now consider continuous Cosserat media with locally varying constitutive properties. A dislocated single crystal is an example of a homogeneous Cosserat continuum, for which lattice curvature is due to so-called geometrically necessary dislocations [14]. Engineering materials like polycrystals, metal matrix composites, usually are aggregates of such constituents [15]. The problem is then to work out the effective properties and the resulting characteristic length of such an aggregate. That is why the homogenization problem of Cosserat media is addressed in this section. Homogenization techniques now are well-developed in the case of classical media in the linear regime but also in the non-linear regime [16]. The extension of some aspects of homogenization theory to heterogeneous Cosserat continua is proposed. The application is restricted to heterogeneous linear elasticity. The RVE is denoted by Ω .

4.1 Boundary value problem with Dirichlet boundary conditions.

The determination of the effective properties goes through the resolution of an initial boundary value problem on Ω . In particular, some boundary conditions must be specified in order to have a well-posed micro-mechanical problem. The field equations of the considered boundary value problem on Ω are given by (2) to (4). Constitutive equations are then necessary but need not be specified yet. The proposed Dirichlet boundary conditions read :

$$\forall \underline{x} \in \partial\Omega, \quad \underline{u} = \underline{E} \underline{x} \quad \text{and} \quad \underline{\phi} = \underline{K} \underline{x}. \tag{9}$$

Tensors \underline{E} and \underline{K} are given and constant. The origin of the coordinate system is assumed to be the geometric center of gravity.

The following properties are derived using Gauss' theorem :

$$\langle \underline{u} \otimes \underline{\nabla} \rangle = \frac{1}{|\Omega|} \int_{\Omega} \underline{u} \otimes \underline{\nabla} \, d\Omega = \underline{E} \quad \text{and} \quad \langle \underline{\chi} \rangle = \underline{K}. \tag{10}$$

Similarly, the field equations and boundary conditions enable one to work out the following form of the work of internal forces :

$$\langle \underline{\sigma} : \underline{e} + \underline{\mu} : \underline{\chi} \rangle = \langle \underline{\sigma} \rangle : \underline{E} + \langle \underline{\mu} + (\underline{\epsilon} \underline{\sigma}) \otimes \underline{x} \rangle : \underline{K}. \tag{11}$$

If \underline{E} and \underline{K} are taken as the macroscopic deformation and curvature, the right term can be interpreted as the expression of the internal work of the macroscopic medium considered as a Cosserat continuum. This leads to the definition of the macroscopic force and couple stress tensors :

$$\underline{\Sigma} = \langle \underline{\sigma} \rangle \quad \text{and} \quad \underline{M} = \langle \underline{\mu} + (\underline{\epsilon} \underline{\sigma}) \otimes \underline{x} \rangle. \tag{12}$$

It must be noticed that $\underline{M} \neq \langle \underline{\mu} \rangle$ contrary to what may have been expected at first glance. Furthermore, the boundary conditions imply that \underline{e} is not homogeneous near the boundary as soon as $\underline{K} \neq 0$. For compatibility reasons, it can be shown that there exist no purely homogeneous boundary conditions for Cosserat continua contrary to the classical case. Finally, one may think of dual boundary conditions of the form

$$\underline{t} = \underline{\Sigma} \underline{n} \quad \text{and} \quad \underline{m} = \underline{M} \underline{n}, \tag{13}$$

where \underline{t} and \underline{m} are the traction and moment vectors on $\partial\Omega$. But in general the balance of moment of momentum is not satisfied using them. It means that there seems to be no simple dual formulation of the previous boundary value problem contrary to the classical case.

4.2 Boundary value problem with periodicity conditions

The material is now supposed to have a periodic microstructure. The geometry of the representative volume element is then completely determined. A unit cell Ω can be defined like in the classical periodic homogenization theory [16]. We look for displacement and micro-rotation fields satisfying the field equations (2) to (4) and having the following form on one unit cell Ω :

$$\underline{u}(\underline{x}) = \underline{E} \underline{x} + \underline{v}(\underline{x}) \tag{14}$$

$$\underline{\phi}(\underline{x}) = \underline{K} \underline{x} + \underline{\psi}(\underline{x}) \tag{15}$$

where \underline{E} and \underline{K} are given and constant. The vector fields \underline{v} and $\underline{\psi}$ are assumed to be periodic, which means that they take equal values on opposite sides of $\partial\Omega$. Furthermore we require the traction and moment vectors to be anti-periodic, which means that they are opposite on opposite sides of $\partial\Omega$ (where the external normal vectors \underline{n} are opposite). If the origin of the coordinate system does not coincide with the geometric center of gravity, the previous conditions are written

$$\underline{u}(\underline{x}) = \underline{E}(\underline{x} - \underline{x}_G) + \underline{v}(\underline{x}) \tag{16}$$

$$\underline{\phi}(\underline{x}) = \underline{K}(\underline{x} - \underline{x}_G) + \underline{\psi}(\underline{x}) \tag{17}$$

This problem is defined on a unit cell Ω and if the neighbouring cell is considered, the conditions (16) and (17) must be applied using the center of the new cell as the origin of the coordinate system. At the common boundary, deformation, curvature, force and couple stresses are continuous but their gradient are not in general. For linear elasticity, this problem admits then a solution like in the classical case, as a consequence of Lax-Milgram theorem. This solution is unique up to a rigid body motion and to a micro-rotation which is equal to the rotation of the rigid body motion. Note that in the classical case the solution is unique up to a translation [16]. In the finite element analysis of this problem presented in the next section, we will therefore have to fix the displacement and the micro-rotation of one node.

Conditions (14) and (15) imply that

$$\langle \underline{u} \otimes \underline{\nabla} \rangle = \underline{E} \tag{18}$$

$$\langle \underline{\phi} \otimes \underline{\nabla} \rangle = \underline{K} \tag{19}$$

Since $(\underline{v}, \underline{\psi})$ and $(\underline{\sigma} \underline{n}, \underline{\mu} \underline{n})$ respectively are periodic and skew-periodic, it can be shown that relation (11) still holds in the periodic case. This leads to the same definition of the macroscopic force and couple stress tensors as in (12).

4.3 Hill-Mandel's lemma for Cosserat media.

Let $(\underline{\sigma}^*, \underline{\mu}^*)$ be self-equilibrated force and couple stress fields on Ω , which means that they fulfill equations (2). Let then $(\underline{\epsilon}', \underline{\chi}')$ be two compatible deformation and curvature fields $((\underline{u}', \underline{\phi}')$ are the associated displacement and micro-rotation fields). Note that $(\underline{\sigma}^*, \underline{\mu}^*)$ and $(\underline{\epsilon}', \underline{\chi}')$ are not necessarily related to each other by the constitutive relations. Then if $(\underline{\epsilon}', \underline{\chi}')$ satisfy the boundary conditions (9) or (14, 15), the following relation holds :

$$\langle \underline{\sigma}^* : \underline{\epsilon}' + \underline{\mu}^* : \underline{\chi}' \rangle = \langle \underline{\sigma}^* : \underline{\epsilon}' \rangle + \langle \underline{\mu}^* : \underline{\chi}' \rangle + \langle \underline{\sigma}^* \otimes \underline{\mu}^* \rangle \otimes \underline{\mathbb{X}} \rangle > \langle \underline{\chi}' \rangle. \quad (20)$$

5 APPLICATION TO A TWO-PHASE LINEAR ELASTIC COSSERAT MATERIAL

We now consider a plane cubic assemblage of cubic heterogeneities embedded in a homogeneous matrix. The RVE is chosen as a sample of four cube heterogeneities. Note that we could also have retained only one cube in Ω or more than four. In principle, the resulting effective properties will depend on this choice since the absolute-size L of the RVE is different. Periodicity conditions are well-suited for this analysis. The volume fraction is $f = 40\%$. The elastic properties of the matrix and cubes are isotropic and are respectively

$$E^m = 70000 \text{ MPa}; \nu^m = 0.3; \mu_c^m = 100000 \text{ MPa}; l_c^m = 1 \text{ m}$$

for the matrix and

$$E^i = 600000 \text{ MPa}; \nu^i = 0.4; \mu_c^i = 100000 \text{ MPa}; l_c^i = 1 \text{ m}$$

for the inclusions. In the two-dimensional case (plane strain in our case), the constant α does not intervene and we have taken $\beta = \gamma$. A simplified characteristic length is then defined :

$$l_c = \sqrt{\frac{\beta}{\mu}}. \quad (21)$$

The resulting properties have plane cubic symmetry. We want to determine the following components of the effective elasticity matrix :

$$\begin{bmatrix} \Sigma_{11} \\ \Sigma_{22} \\ \Sigma_{12} \\ \Sigma_{21} \\ M_{31} \\ M_{32} \end{bmatrix} = \begin{bmatrix} D_{1111} & D_{1122} & 0 & 0 & 0 & 0 \\ D_{1122} & D_{1111} & 0 & 0 & 0 & 0 \\ 0 & 0 & D_{1212} & D_{1221} & 0 & 0 \\ 0 & 0 & D_{1221} & D_{1212} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{3131} & C_{3132} \\ 0 & 0 & 0 & 0 & C_{3132} & C_{3131} \end{bmatrix} \begin{bmatrix} E_{11} \\ E_{22} \\ E_{12} \\ E_{21} \\ K_{31} \\ K_{32} \end{bmatrix} \quad (22)$$

The following quantities are defined :

$$\mu^{12} = (D_{1212} + D_{1221})/2 \quad (23)$$

$$\mu_c^{12} = (D_{1212} - D_{1221})/2 \quad (24)$$

$$l_c^h = \sqrt{\frac{C_{3131}}{2\mu^{12}}} \quad (25)$$

The definite-positiveness of the overall elasticity tensors implies that μ^{12} and μ_c^{12} are positive and non-vanishing.

The effective moduli are derived from the solution of four elementary loading conditions of the volume element :

$$E_{11} = 1; \quad E_{12} = E_{21} = \frac{1}{2} \quad (26)$$

$$E_{12} = -E_{21} = 1 \quad (27)$$

$$K_{31} = 1 \quad (28)$$

the remaining components of \underline{E} and \underline{K} being zero in each case. These problems are solved using the finite element method. The calculations have been performed with the object-oriented FE code ZeBuLoN [18]. The Dirichlet boundary conditions (9) and the associated periodic boundary conditions have been implemented. The elements used are quadratic two-dimensional plane strain Cosserat elements with full integration.

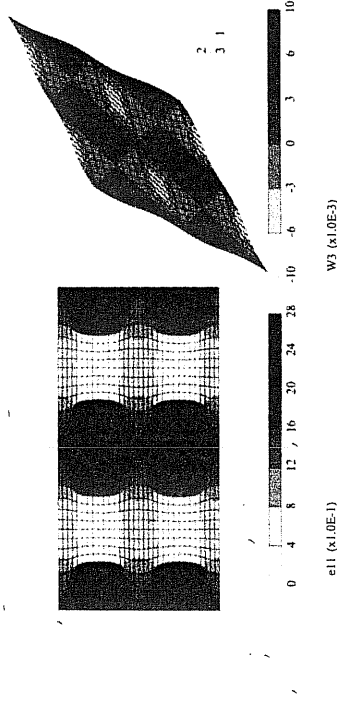


Figure 2 : Extension and simple shear on a heterogeneous Cosserat material with periodicity conditions (deformation in direction 1 (left) and microrotation (right)) ($L = 1$).

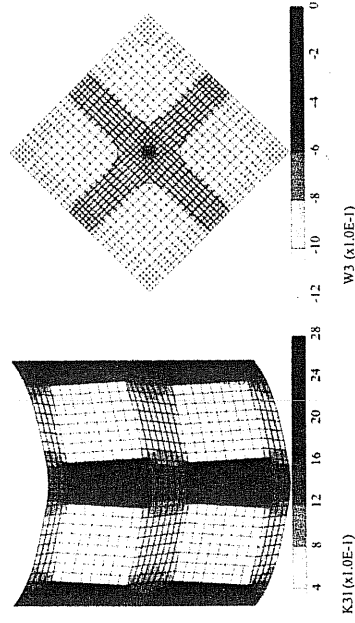


Figure 3 : Prescribed curvature (left) K_{31} and relative rotation $E_{12} = -E_{21}$ (right) on a cubic assemblage of hard cubes ($L = 1$).

The application of the elementary tests are shown on figures 2 and 3. The figures show the deformation and micro-rotation fields for the loading conditions (26), and the curvature and microrotation fields for the loading conditions (27) and (28), so that the periodicity requirements can be explicitly seen. The corresponding deformed state of the entire body should not be deduced from figure 3 (left) for instance, by an infinite number of translations, although this would lead by construction to a compatible deformation field and an equilibrated stress field. It is more relevant to think of a body undergoing strong overall strain gradients and curvature and the deformed cell of figure 3 is thought to approach the real state of an extracted isolated cell. Note that a more precise account of large overall strain gradients like in [17] would lead to a Cosserat effective medium involving second gradients of displacement, which has not been considered for a first estimation.

The dependence of the previous effective parameters on the RVE size L is given on figure 4. The constant C_{3132} is found to always be zero. When Cosserat effects are predominant ($L \ll l_c$), D_{1221} is negative so that μ_c^{12} is large and tends to μ_c . The effective characteristic length l_c^h is then close to

l_c , D_{1111} and D_{1122} display a slight dependence on L . They reach the classical values obtained for classical homogenization when L is much larger than l_c . The constant μ_c^{12} fades with increasing L but l_c has then the same order of magnitude as L .

6 CONCLUSION

Both approaches in the discrete and continuum cases lead to effective properties that depend on the size of the retained volume element L . The classical effective elastic constants display only a slight dependence. The additional Cosserat constants strongly depend on L : μ_c tends to zero when L becomes very large and the effective characteristic lengths increase with L . The final choice of the adequate L depends on the structural application one aims at.

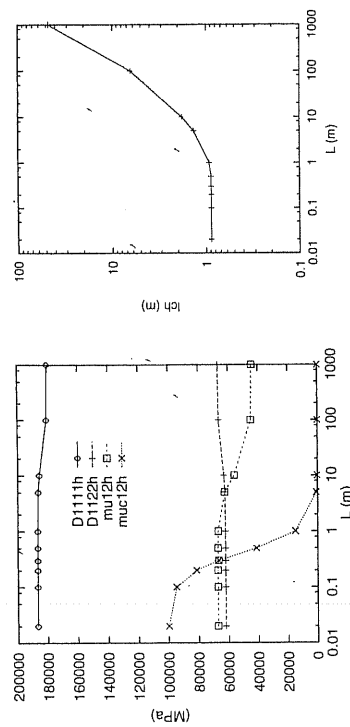


Figure 4 : Effective moduli (left) and characteristic length (right) as a function of the cell size.

References

- [1] Cosserrat E., Cosserrat F., *Théorie des corps déformables*, Hermann, Paris, 1909.
- [2] Schäfer H., ZAMM, vol. 47, n8, pp. 485-498, 1967.
- [3] Schijve, L., J. Mech. Phys. Solids, Vol. 14, pp. 113-120, 1967.
- [4] Gauthier R.D., Jalsman, W.E., J. Appl. Mech., 1976.
- [5] Besdo, D. and Dorau, H.-U., ZAMM, vol. 68, T153-T155, 1988.
- [6] Muhliaus H.B., in 5th Int. Symp. on Plasticity and its current applications, Osaka, July 17-21, 1995 (S. Tamimura and A.S. Kahn eds, Gordon and Breach), pp. 427-430, 1996.
- [7] Lakes R. S., Int. J. Solids Structures, vol. 22, No. 1, pp. 55-63, 1986.
- [8] Adachi T., Yomita Y., Proceedings of the 8th Int. Symp. on Continuum Models and Discrete Systems, (June 11-16, 1995, Varana, Bulgaria), ed. K.Z. Markov, pp. 342-349, 1996.
- [9] Smyshtyayev V.P., Fleck N.A., J. Mech. Phys. Solids, Vol. 42, No. 12, pp. 1851-1882, 1994.
- [10] Banks, Sokolovski, Int. J. Solids and Structures, Vol. 4, pp. 15, 1968.
- [11] Lakes R.S., Cellular Polym., Vol. 12, pp. 17-30, 1993.
- [12] Adachi T., Tomita Y., Tanaka M., Proceedings of the 37th Japan Congress on Materials Research, Kyoto, Japan, pp. 215-221, 1994.
- [13] Favier V., Dendtevel R., Canova G., Cavaillé J.Y., Gilormini P., Acta mater., Vol. 45, No. 4, pp. 1557-1565, 1997.
- [14] Forest S., Caillaud G., Sievert R., Arch. Mech., Vol. 49, pp. 705-736, 1997.
- [15] Fleck N.A., Hutchinson J. W., J. Mech. Phys. Solids, No. 12, pp. 1825-1857, 1993.
- [16] Sanchez-Palencia E., Zaoui A., *Homogenization techniques for composite media*, Lecture notes in physics, No. 272, Springer-Verlag, 1985.
- [17] Forest S., 40^e Colloque de Métallurgie, Comportement Mécanique et Effets d'Echelle, INSTN, 24-26 juin 1997, to appear in Journal de Physique IV.
- [18] Besson J., Foerch R., Comp. Meth. Appl. Mech. Engng., Vol. 142, pp. 165-187, 1997.

On the characteristic scales involved in a fragmentation process

C. Denoual and F. Hild*

DGA/CTA, Département Surfaces et Matériaux en Conditions Sévères,
 16 bis avenue Prieur de la Côte d'Or, 94114 Arcueil cedex, France
 * ENS de Cachan, CNRS, Université Paris 6, 61 avenue du Président Wilson,
 94235 Cachan cedex, France

Abstract. Dynamic loadings produce high stress waves leading to the fragmentation of brittle materials such as ceramics. The main mechanism used to explain the change of the number of fragments with stress rate is an obscuration (or shielding) phenomenon. A probabilistic damage model is proposed and scalings are proposed. This approach allows an estimation of the characteristic size of structure involved in the transition between single fragmentation (quasi-static loading), and multiple fragmentation (dynamic loading). This criterion is used to understand the mechanism leading to damage localization for low impact velocity.

1. INTRODUCTION

Bilayered armor with ceramic as front plate and steel as back plate has been used for several years to improve the efficiency of light or medium armor. The high hardness of ceramic materials favors projectile failure [1] and spreads the kinetic energy on a large surface of a ductile back face. The weight of the armor is reduced in comparison to an armor made of steel only. In most impact configurations, the stress field associated with impact can be assumed to be spherical and an analogy can be made between real impact failure morphologies and soft recovery experiments of divergent spherical stress load [2]. Stress waves produce damage both in compressive and tensile modes in two different locations in the ceramic (Fig. 1). Damage in compression is produced near the impact surface when shear stresses reach a threshold value which can be dependent on pressure and strain rate. In the bulk of the ceramic, damage in tension is observed as well. With a projectile velocity under 1000 m/s, no significant perforation can be observed while damage grows. One can then uncouple the damage evolution phase from the complete penetration phase. The complete perforation is dependent on the way the ceramic fractures in terms of damage location and evolution, and in terms of anisotropic behavior due to cracking.

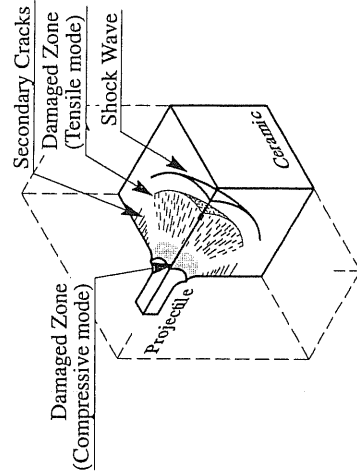


Figure 1: Morphology of damage in a ceramic specimen during impact.

Ceramic materials are known to break in a brittle way because of randomly distributed defects. This brittleness in tension is often related to the capability of supporting high loadings and breaking apart in a few