

Figure 3: Overall and local longitudinal shear stresses in the 0° ply of a (0/90) laminate with sliding fibers

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MODELLING THE CYCLIC BEHAVIOUR OF TWO-PHASE SINGLE CRYSTAL NICKEL-BASE SUPERALLOYS

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1. Introduction

The modelling of monotonic and cyclic behaviour of single crystal superalloys using crystal elastoviscoplasticity has proved successful in the last ten years. Within this framework slip processes involving octahedral and cubic slip systems are taken into account. However the two-phase microstructure of the material plays a significant role at high temperature. Coherent cuboidal precipitates of phase γ' are distributed in the matrix γ . It appears that the viscoplastic deformation takes place mainly in the γ channels and γ' precipitates are only occasionally sheared for low strain rates. The aim of the present work is to include this microstructural information into the modelling. But the model must remain simple enough to enable its implementation in a finite element (FE) code for structural calculations. For this reason we resort to homogenization techniques and consider only average stress and strain in each phase. The concentration law which gives the local mean stress when the macroscopic stress is known, should be explicit. Some explicit concentration laws in elastoplasticity and elastoviscoplasticity are tested numerically using the FE method.

Approximate self-consistent constitutive equations for the homogeneous equivalent medium (HEM) are then proposed.

2. Explicit concentration laws in elastoplasticity according to the self-consistent scheme

Hill (1965) has proposed a rigorous treatment of the self-consistent scheme to describe the elastoplastic behaviour of polycrystals or, more generally, of multiphase materials. Berveiller and Zaoui (1979) derived then an explicit concentration law using the approximation of global isotropy (μ : shear modulus, ν : Poisson's ratio) and for radial and monotonous loading:

$$\underline{\underline{\sigma}}_i = \underline{\underline{\Sigma}} + 2\mu\alpha(1 - \beta)(\underline{\underline{E}}_i^p - \underline{\underline{\xi}}_i^p) \quad (1)$$

where $\beta = 2(4 - 5\nu)/15(1 - \nu)$ and α decreases from 1 to 0 with increasing strain. Homogeneous elasticity is assumed. $\underline{\underline{\sigma}}_i$ and $\underline{\underline{\xi}}_i^p$ denote respectively the average stress and plastic strain in each phase, whereas $\underline{\underline{\Sigma}}$ and $\underline{\underline{E}}_i^p$ are the overall macroscopic quantities. Pilvin (1990) proposed a modification for cyclic loading and introduced interphase accommodation variables $\underline{\underline{\beta}}_i$. In (C) f_i is the volume fraction of phase i and the function $\underline{\underline{G}}_i$ is introduced to describe the non linear interaction between the HEM and each phase.

$$\begin{cases} \underline{\underline{\sigma}}_i = \underline{\underline{\Sigma}} + 2\mu(1 - \beta)(\underline{\underline{B}} - \underline{\underline{\beta}}_i) & \text{with } \underline{\underline{B}} = \langle \underline{\underline{\beta}} \rangle = \sum_i f_i \underline{\underline{\beta}}_i \\ \underline{\underline{\beta}}_i = \underline{\underline{\xi}}_i^p - \underline{\underline{G}}_i(\underline{\underline{\xi}}_i^p, \beta_i, \underline{\underline{\xi}}_i^p) \end{cases} \quad (C)$$

3. An optimization procedure for the development of quasi-self-consistent models

It is possible to check the validity of the previous approximate concentration laws using FE calculations. For the sake of simplicity we first considered a two-phase isotropic elastoplastic material, each phase obeying a von Mises criterion with linear isotropic hardening (J_2 -theory). The following constitutive equations for the HEM medium are proposed

$$\begin{cases} \underline{\underline{\sigma}}_i = \underline{\underline{\Sigma}} + 2\mu(1 - \beta)(\underline{\underline{B}} - \underline{\underline{\beta}}_i) & i = 1, 2 \\ \underline{\underline{\beta}}_i = \underline{\underline{\xi}}_i^p - D\underline{\underline{\beta}}_i \sqrt{\frac{2}{3}\underline{\underline{\xi}}_i^p : \underline{\underline{\xi}}_i^p} \\ \underline{\underline{\xi}}_i^p = \dot{p}_i \frac{\partial F_i}{\partial \underline{\underline{\sigma}}_i} = \dot{p}_i \frac{3}{2} \underline{\underline{J}} : \underline{\underline{\sigma}}_i / J_2(\underline{\underline{\sigma}}_i) \\ F_i = J_2(\underline{\underline{\sigma}}_i) - Y_i - H_i p_i & \text{(yield function)} \\ \underline{\underline{E}}_i^p \stackrel{\text{def}}{=} \sum_i f_i \underline{\underline{\xi}}_i^p \\ \underline{\underline{\Sigma}} = (3k\underline{\underline{K}} + 2\mu\underline{\underline{J}}) : \underline{\underline{E}}^e \end{cases} \quad (S)$$

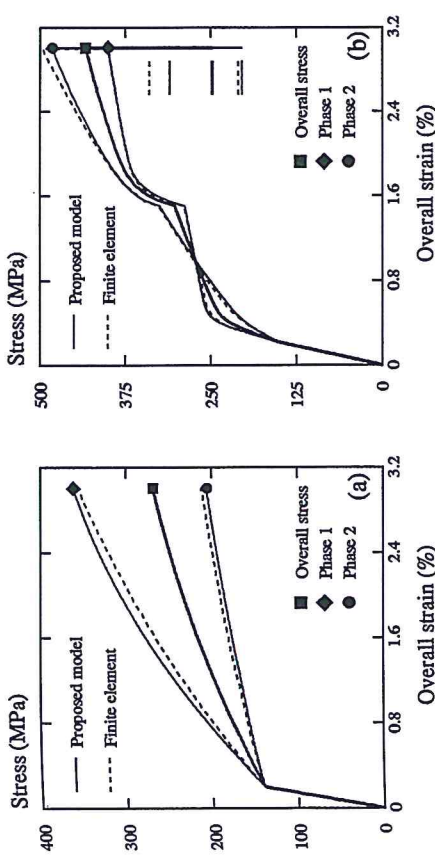


Figure 1. Prediction of the homogenized constitutive equations compared with FE simulations (a) tensile test (elastoplasticity), (b) tension with strain rate jump followed by relaxation (elastoviscoplasticity)

where $3\underline{\underline{K}} = \underline{\underline{1}} \otimes \underline{\underline{1}}$ and $\underline{\underline{J}} = \underline{\underline{1}} - \underline{\underline{K}}$. They can be regarded as an approximation of the self-consistent scheme. To check its validity, the stress-strain state of a spherical inclusion of phase i ($i = 1, 2$) embedded in the HEM with the constitutive equations (S), is computed using the FE method. Axisymmetric calculations are performed with homogeneous boundary conditions. The average stress in phase i can then be compared with the value obtained after direct application of the concentration law (C). However (C) involves a parameter D which must firstly be determined. Its value must be such that the self-consistency condition holds at each loading step for each loading path

$$f_1 \underline{\underline{\xi}}_1(\underline{\underline{E}}) + f_2 \underline{\underline{\xi}}_2(\underline{\underline{E}}) = \underline{\underline{E}} \quad (2)$$

$\underline{\underline{\xi}}_1$ and $\underline{\underline{\xi}}_2$ are the average strains in each inclusion and $\underline{\underline{E}}$ is prescribed at infinity. The FE code is now coupled with an optimization code to find a parameter D such that the previous condition is best fulfilled for the given form of the constitutive equations of the HEM. Using the following parameters for the individual phases: $Y_1 = Y_2 = 130$ MPa, $H_1 = 1000$ MPa and $H_2 = 20000$ MPa, the optimization process provides $D = 55$. The self-consistency condition is then fulfilled up to 0.01%, at least for the tension test that has been chosen for the identification. As a result, the stress state of each phase is given by direct application of the concentration law with very good accuracy (figure 1a).

4. Use of the explicit concentration law out of its a priori validity domain

The explicit concentration law (1) has been worked out by Berveiller and Zaoui (1979) within the framework of elastoplasticity and for radial monotonous loading paths only. However concentration law (C) can also be used in the cyclic case. Comparison with FE computations similar to the previous ones and presented in (Forest *et al.*, 1995) show that the self-consistency condition is still fulfilled with enough precision during a cyclic test with the proposed model (S).

We can now wonder whether the result holds if elastoviscoplastic constitutive equations are substituted in system (S) for the local behaviour of each phase

$$\dot{p}_i = < \frac{J_2(\underline{\sigma}_i) - Y_i - H_i p_i}{K_i} >^{n_i} \quad (3)$$

FE computations have been performed in the elastoviscoplastic case with $K_1 = 850 \text{ MPa.s}^{1/n_1}$, $K_2 = 150 \text{ MPa.s}^{1/n_2}$, and $n_1 = n_2 = 5$. Surprisingly, the explicit concentration law still gives a good description of the stress state in each phase taken as an inclusion in the HEM, as shown in figure 1b for a tension test with a strain rate jump and followed by relaxation. Although the rigorous treatment of the self-consistent scheme in elastoviscoplasticity is rather tedious (Rougier *et al.*, 1994), the proposed equations still provide an acceptable and useful approximation. This statement also holds for non-proportional loading as demonstrated numerically in (Forest *et al.*, 1995). The previous calculations have been performed with the value of D identified in the last section and represent therefore a validation of the model.

The optimization procedure presented in section 3 can also be applied to make the concentration law (C) also work for more complicated morphological patterns. The notion of representative morphological pattern modelling has been introduced in (Bornert *et al.*, 1993). The generalized self-consistent scheme for instance accounts for the inclusion-matrix morphology. Phases 1 and 2 are now limited by two concentric spheres, phase 2 being the inclusion, phase 1 filling the shell. The composite assembly is embedded in the HEM (figure 3a). The evolution law of $\underline{\beta}_i$ (C) seems to be rather inadequate for this three-phase model since both phases play a symmetric role in the expression. However this symmetry can be broken to some extent if parameter D is determined so that the self-consistency condition

$$< \underline{\varepsilon} >_{\text{shell-inclusion}} = \underline{\underline{E}} \quad (4)$$

is optimally fulfilled. The optimization procedure proposed in section 3 then gives $D = 85$. Another simple way of breaking the symmetry of relation (C) is to introduce a value of D for each phase, as done in (Pilvin *et al.*, 1995).

5. Generalization to the anisotropic case : modelling the elastoviscoplastic behaviour of two-phase single crystal nickel-base superalloys

In the anisotropic case and when homogeneous elasticity is still assumed, we propose the following extension of the concentration law

$$\underline{\underline{g}}_i = \underline{\underline{\Sigma}} + \underline{\underline{L}}_i^{\dagger} : (\underline{\underline{B}} - \underline{\underline{\beta}}_i) \quad (5)$$

$\underline{\underline{L}}_i^{\dagger} = \underline{\underline{C}} : (\underline{\underline{I}} - \underline{\underline{S}})$ is the elastic accommodation tensor, $\underline{\underline{C}}$ and $\underline{\underline{S}}$ are respectively the (anisotropic) elasticity tensor and the Eshelby tensor associated with it. For cubic elasticity a simple form of the concentration law can be derived. Like in (Walpole, 1985), the four-rank tensor $\underline{\underline{N}}$ is introduced

$$\underline{\underline{N}} = \sum_{k=1}^3 \underline{\underline{a}}_k \otimes \underline{\underline{a}}_k \otimes \underline{\underline{a}}_k \otimes \underline{\underline{a}}_k \quad (6)$$

where unit vectors $\underline{\underline{a}}_k$ lie on the edges of the cubic cell. We can then define

$$\underline{\underline{J}}_1 = \underline{\underline{N}} - \underline{\underline{K}}, \quad \underline{\underline{J}}_2 = \underline{\underline{I}} - \underline{\underline{N}} \quad (7)$$

As a result, the cubic elasticity and associated Eshelby tensor have the form

$$\underline{\underline{C}} = 3k\underline{\underline{K}} + 2\mu_1\underline{\underline{J}}_1 + 2\mu_2\underline{\underline{J}}_2, \quad \underline{\underline{S}} = \alpha\underline{\underline{K}} + \beta_1\underline{\underline{J}}_1 + \beta_2\underline{\underline{J}}_2 \quad (8)$$

α, β_1, β_2 being computed numerically after (Mura, 1987), so that

$$\underline{\underline{g}}_i = \underline{\underline{\Sigma}} + 2\mu_1(1 - \beta_1)\underline{\underline{J}}_1 : (\underline{\underline{B}} - \underline{\underline{\beta}}_i) + 2\mu_2(1 - \beta_2)\underline{\underline{J}}_2 : (\underline{\underline{B}} - \underline{\underline{\beta}}_i) \quad (9)$$

In the isotropic case, $\mu_1 = \mu_2 = \mu$, $\beta_1 = \beta_2 = \beta$ and (C) is retrieved.

To describe the anisotropic viscoplastic behaviour of each phase, we resort to the constitutive equations for single crystals proposed by (Cailletaud, 1987) and the fact that γ and γ' phases are coherent is taken into account. The equations of the model are then

macroscopic strain partition and elasticity law

$$\underline{\underline{E}} = \underline{\underline{E}}^e + \underline{\underline{E}}^p, \quad \underline{\underline{\Sigma}} = \underline{\underline{C}} : \underline{\underline{E}}^e \quad (10)$$

concentration law

$$\underline{\underline{g}}_i = \underline{\underline{\Sigma}} + \underline{\underline{L}}_i^{\dagger} : (\underline{\underline{B}} - \underline{\underline{\beta}}_i) \quad \text{with} \quad \underline{\underline{B}} = f_1\underline{\underline{\beta}}_1 + f_2\underline{\underline{\beta}}_2 \quad (11)$$

local behaviour

$$\dot{\xi}_i^p = \sum_{g=1}^{N_i} \dot{\gamma}_i^g \{ \underline{\xi}_i^g \otimes \underline{\xi}_i^g \}^{sym}, \quad \dot{\gamma}_i^g = < \frac{|\tau_i^g - \alpha_i^g| - r_i^g}{k_i} >^{n_i} \text{sign}(\tau_i^g - \alpha_i^g) \quad (12)$$

$$\tau_i^g = y_i + q_i^1 (1 - e^{-b_i^1 v_i^g}) + q_i^2 (1 - e^{-b_i^2 v_i^g}) \quad \text{with} \quad \dot{v}_i^g = |\dot{\gamma}_i^g| \quad (13)$$

$$\alpha_i^g = c_i^g \alpha_i^s \quad \text{with} \quad \dot{\alpha}_i^g = \dot{\gamma}_i^g - d_i \alpha_i^s \dot{v}_i^g \quad (14)$$

local interphase accommodation evolution law

$$\dot{\beta}_i = \xi_i^p - D_i \beta_i \sqrt{\frac{2}{3} \xi_i^p : \xi_i^p} \quad (15)$$

homogenization

$$\mathbb{E}^p \stackrel{\text{def}}{=} f_1 \xi_1^p + f_2 \xi_2^p, \quad (16)$$

6. Application to SC16 at 950°C

Among the parameters involved in the previous model, one should distinguish between "material constants" (like k_i , n_i , τ_i , q_i^1 , b_i^1 , q_i^2 , b_i^2 , c_i , d_i) that are responsible for the hardening behaviour, and "geometric parameters" (in this case, $D = D_1 = D_2$) that ensure the validity of the concentration law. The material parameters are determined from specific mechanical tests like tension and low cycle fatigue, whereas the geometric parameters are provided by the optimization procedure proposed in section 3. Since both sets of parameters are unknown, the identification process should simultaneously involve the experimental data and the FE simulation with the retained morphological pattern.

With its small γ' volume fraction (40%), and its bimodal microstructure made of 0.5 μm - γ' cuboids and smaller spherical precipitates, single crystal superalloy SC16 has been chosen for identification of the parameters of the previous model. Tension, low cycle fatigue and creep tests have been performed at the BAM in Berlin. The experimental data retained for the identification contains five tension tests with prescribed strain rate ranging from 10^{-6} to 10^{-2} s^{-1} , and five cyclic tests with various strain rates and amplitudes. Furthermore the FE computation of a tension test in direction [001] with the configuration of figure 3a is introduced in the optimization procedure. In this paper we restrict ourselves to octahedral slip so that no tests in direction [111] are considered. A more realistic description of the material behaviour involves cubic slip and will be presented elsewhere.

The value of D obtained in section 4 for the general self-consistent scheme is used as a starting value for the identification of the two-phase

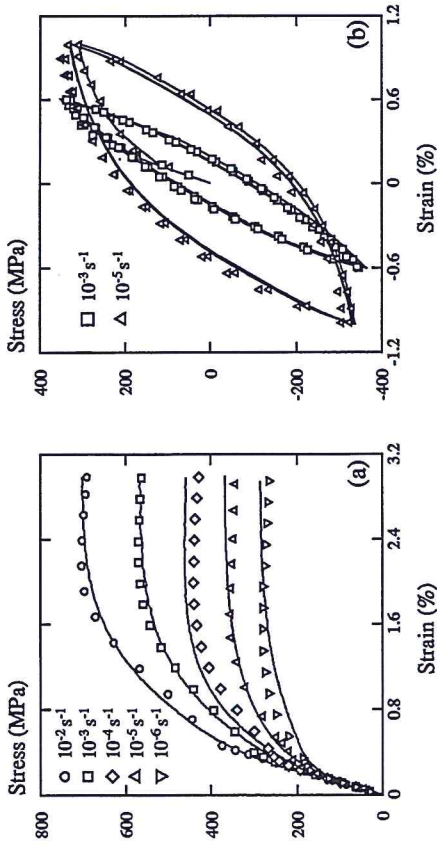


Figure 2. Simulation and experimental results, (a) tensile tests, (b) two fatigue tests (initial and stabilized loops)

single crystal model. The final value, $D = 110$, ensures that the self-consistency condition is fulfilled up to 2%. Material parameters were found that provide a good description of the tensile and fatigue tests (figure 2) :

coefficients	γ	γ'
k ($\text{MPa.s}^{1/n}$), n	560, 3.24	545, 4.76
y (MPa)	36	90
q^1 (MPa), b^1	33, 420	11, 280
q^2 (MPa), b^2	-68, 575	-83, 84
c (MPa), d	59200, 535	637000, 2880

Little is known about the elastoviscoplastic behaviour of each phase within the aggregate. At high temperatures deformation takes place almost exclusively in matrix γ for low strain rates. That is why some constraints were imposed on the material parameters during the identification process : $y\gamma' = 2.5 y\gamma$ and $c\gamma'/d\gamma' = 2 c\gamma/d\gamma$. Figure 3b shows the strain heterogeneity within the aggregate for a tension test simulated with the final set of parameters.

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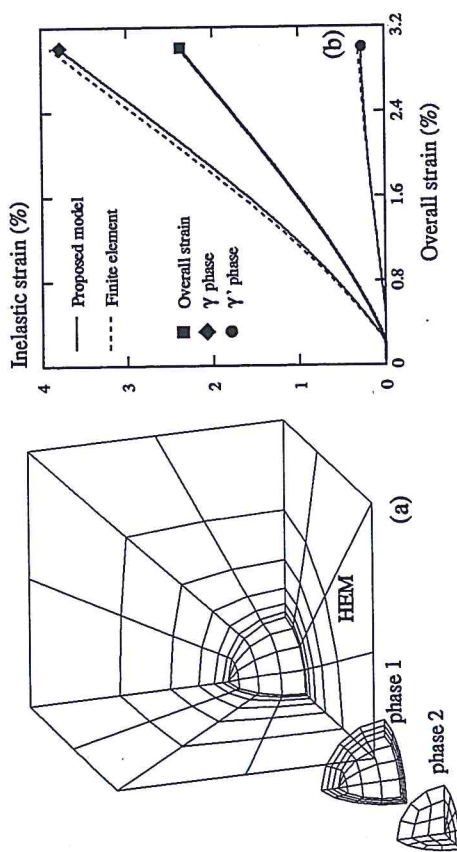


Figure 3. Exploded view of the mesh (a), comparison of the local strains predicted by the homogenized constitutive equations with FE computations (b)

CT92-0176. Experimental results have been obtained by the group of J. Ziebs at BAM.

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EXPERIMENTAL EVIDENCES OF THERMO-ELASTO-PLASTIC STRAIN INCOMPATIBILITIES IN MICROHETEROGENEOUS METAL BASED MATERIALS.

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1. Introduction

Microheterogeneous materials exhibit homogeneous macroscopic properties, but contain small volumes (size of the order of several micrometers) whose properties differ from those of the matrix. As a consequence, in metal based microheterogeneous materials such as metal matrix composites (containing reinforcing ceramic particles) or metallic alloys (containing coarse precipitates), strain incompatibilities arise during the thermo-elasto-plastic loading of the material. For example, the thermal expansion mismatch between matrix and particles induces a thermal stress field when the system is cooled. This stress field may be partially relaxed if plastic strain of the matrix occurs around the reinforcement. The corresponding thermally induced dislocations (TID) lead to a strong forest effect and contribute to the strengthening observed for the composite compared to the unreinforced material [1]. Several models have been proposed in the past for describing this process [2]. They are based on the assumption that the emitted dislocations are distributed randomly in the matrix, i.e. that the yield properties of the matrix are homogeneous. The present work aims at giving a new insight into this phenomenon on the experimental point of view. Firstly, thermal treatments during in situ TEM experiments show the emission and the motion of dislocations from the interface and through the matrix. Second, the TID are responsible for internal damping and microdeformation in torsion that appear on cooling and on heating microheterogeneous materials such as MMCs. Third, the consequences of thermal expansion and elastic modulus mismatches on the difference of behaviour between tension and compression tests are presented.

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IUTAM Symposium on Micromechanics of Plasticity and Damage of Multiphase Materials

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PREFACE

The IUTAM Symposium on "Micromechanics of Plasticity and Damage of Multiphase Materials" was held in Sèvres, Paris, France, 29 August - 1 September 1995. The Symposium was attended by 83 persons from 18 countries. In addition 17 young French students attended the meeting. During the 4 day meeting, a total of 35 papers were presented, including 24 papers in the poster sessions. The meeting was divided into 7 oral and 3 poster sessions.

The 7 oral sessions were the following:

- Plasticity and Viscoplasticity I and II;
- Phase transformations;
- Damage I and II;
- Statistical and geometrical aspects;
- Cracks and interfaces.

Each poster session was introduced by a Rapporteur, as follows:

- Session 1 (Plasticity and Viscoplasticity): G. Cailletaud;
- Session 2 (Damage): D. François;
- Session 3 (Phase transformation; statistical and geometrical aspects): D. Jeulin.

The main purpose of the Symposium was the discussion of the state of the art in the development of micromechanical models used to predict the macroscopic mechanical behaviour of multiphase solid materials. These materials consist of at least two chemically different phases, present either initially or formed during plastic deformation, when a strain-induced phase transformation takes place. One session was devoted to the latter case. Continuously strengthened composite materials, containing long fibers, were out of the scope of the Symposium. On the other hand, the definition of multiphase materials was extended to damaging materials in which the second phase corresponds to the formation of damaged zones (microvoids, microcracks etc.). Two oral sessions and one poster session were devoted to this topic. The statistical and the geometrical aspects in the mechanical behaviour of these multiphase materials were presented during another oral session and another poster session whereas an oral session was devoted to problems related to cracks and interfaces. In this Symposium, materials with a microstructure such that the usual concepts of continuum mechanics could be applied were essentially considered. Moreover, as a rule, one of the constituent phases of multiphase materials which were presented exhibited a nonlinear behaviour. Two oral sessions and one poster session were devoted to the mechanical behaviour of plastic or viscoplastic multiphase materials.

This Symposium clearly showed a growing interest in the development of micromechanical approaches to model the macroscopic behaviour of inhomogeneous materials. Several presentations emphasized the importance of local phenomena, of the spatial phase distribution and its modifications, as well as the importance of an adequate treatment of the nonlinear character of the overall response. The limitations of simplistic homogenization techniques in which the mechanical stress and strain fields are assumed uniform in each phase were also largely underlined. A significant number of presentations were devoted to the coupling effect between damage and constitutive equations. The statistical aspects in the modelling of the mechanical behaviour of multiphase materials, especially their damage behaviour, were also largely discussed.

The Editors particularly wish to thank the Bureau of IUTAM, the International Scientific Committee and the local organizing committee. This conference could not have been so successful without the excellent conference facilities provided by the "Centre International d'Etudes Pédagogiques" in the ancient Royal Sèvres Porcelain Factory. Thanks are also due to the authors and referees who have made this publication possible and to the session chairmen who have contributed to the success of the conference. Special thanks are due to M. Bornert, T. Bretheau and J. Besson for their very efficient help throughout the preparation and organization of the Symposium.

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