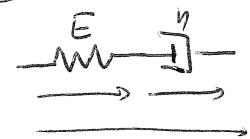
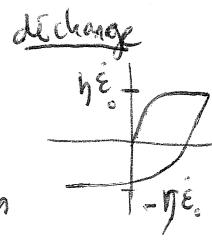


a) Viscoélasticité linéaire

modèle de Maxwell



$$\dot{\varepsilon} = \frac{\sigma}{E} + \frac{\sigma}{\eta}$$



à t_0 $\sigma = y \dot{\varepsilon}_0$

$$-\dot{\varepsilon}_0 = \frac{\sigma}{E} + \frac{\sigma}{\eta}$$

$$\sigma = -y \dot{\varepsilon}_0 + A e^{-\frac{E}{\eta}(t-t_0)}$$

$$\sigma(t_0) = y \dot{\varepsilon}_0 = -y \dot{\varepsilon}_0 + A$$

$$\sigma(t) = y \dot{\varepsilon}_0 (2 e^{-\frac{E}{\eta}(t-t_0)} - 1)$$

traction simple $\varepsilon = \dot{\varepsilon}_0$

$$\sigma = y \dot{\varepsilon}_0 (1 - \exp(-t/\tau_R)) \quad \tau_R = \eta/E$$

$$t=0, \varepsilon=0 \Rightarrow \sigma=0$$

flage $\varepsilon = \frac{\sigma}{y} t + \frac{\sigma_0}{E}$ non borné (fluide)

fluage II

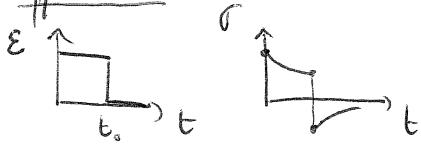
recourance

$$\varepsilon(t_0^-) = \frac{\sigma}{y} t_0^- + \frac{\sigma_0}{E}$$

$$\varepsilon(t_0^+) = \frac{\sigma}{y} t_0^+ \text{ ne recouvre pas toute la déformation}$$

relaxation $\sigma = E \varepsilon_0 (\exp(-t/\tau_R))$

effacement



$$\sigma(t_0^-) = E \varepsilon_0 e^{-\frac{t_0}{\tau_R}}$$

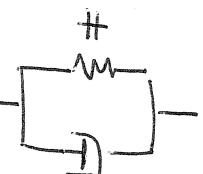
$$\sigma(t_0^+) = E \varepsilon_0 (e^{-\frac{t_0}{\tau_R}} - 1) \leq 0$$

→ fluide viscoélastique

$$\sigma(t > t_0^+) = E \varepsilon_0 (e^{-\frac{t}{\tau_R}} - 1) \exp(-\frac{t-t_0}{\tau_R})$$

flage non borné + effacer \Rightarrow fluide

Kelvin-Voigt I



$$\sigma = H \varepsilon + \eta \dot{\varepsilon}$$

traction $\sigma = H \dot{\varepsilon}_0 t + \eta \dot{\varepsilon}_0$

$$\text{flage } \varepsilon = \frac{\sigma_0}{H} + A e^{-t/\tau} \quad \text{à } t=0 \quad \varepsilon=0 \quad \varepsilon = \frac{\sigma_0}{H} (1 - e^{-\frac{t}{\tau}})$$

sol. part.

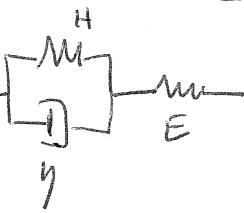
$$\text{recourance } \varepsilon = A e^{-t/\tau} = \frac{\sigma_0}{H} (1 - e^{-\frac{t}{\tau}}) e^{-\frac{t-t_0}{\tau}}$$

flage I

recourance totale

relaxation

Kelvin-Voigt II



$$\begin{cases} \sigma = H \varepsilon^v + \eta \dot{\varepsilon}^v & = H \left(\varepsilon - \frac{\sigma}{E} \right) + \eta \left(\dot{\varepsilon} - \frac{\dot{\sigma}}{E} \right) \\ \dot{\varepsilon} = \frac{\dot{\sigma}}{E} + \dot{\varepsilon}^v \end{cases}$$

$$\dot{\varepsilon} + \frac{H}{\eta} \varepsilon = \frac{\dot{\sigma}}{E} + \left(1 + \frac{H}{E} \right) \frac{\sigma}{\eta}$$

traction $\sigma = A e^{-\frac{E+H}{\eta} t} + \underbrace{B t + C}_{\text{pol. part.}} + \text{realiste}$

$$B = \frac{EH}{E+H} \dot{\varepsilon}_0 \quad \frac{B}{E} + \frac{C}{\eta} \left(\frac{H+E}{H} \right) = \dot{\varepsilon}_0$$

$$\sigma(0) = \eta \dot{\varepsilon}_0$$

flage $\dot{\varepsilon} = \dot{\varepsilon}^v \quad \sigma_0 = H \varepsilon + \eta \dot{\varepsilon}^v$

$$\dot{\varepsilon}^v = \frac{\sigma_0}{H} + A e^{-\frac{H}{\eta} t}$$

$$\varepsilon(0) = \frac{\sigma_0}{E}$$

$$\varepsilon = \frac{\sigma_0}{H} + \sigma_0 \left(\frac{1}{E} - \frac{1}{H} \right) e^{-\frac{H}{\eta} t}$$

flage I

recourrage $\varepsilon(t_0^-) = \frac{\sigma_0}{H} + \sigma_0 \left(\frac{1}{E} - \frac{1}{H} \right) e^{-\frac{H}{\eta} t_0}$

$$\varepsilon(t_0^+) = \left(\frac{\sigma_0}{H} - \frac{\sigma_0}{E} \right) \left(1 - e^{-\frac{H}{\eta} t_0} \right)$$

$$\varepsilon(t > t_0^+) = \left(\frac{\sigma_0}{H} - \frac{\sigma_0}{E} \right) \left(1 - e^{-\frac{H}{\eta} (t-t_0)} \right) \quad \text{recourrage relevé}$$

relaxation $\dot{\varepsilon}^v = -\frac{\dot{\sigma}}{E} \Rightarrow \varepsilon^v = -\frac{\sigma}{E} + \text{const.} \quad \varepsilon^v(0) = 0 \Rightarrow \varepsilon^v = \varepsilon_0 - \frac{\sigma}{E}$

$$\sigma = H \varepsilon_0 - \frac{H}{E} \sigma - \frac{\eta}{E} \dot{\sigma}$$

$$\left(1 + \frac{H}{E} \right) \sigma + \frac{\eta}{E} \dot{\sigma} = H \varepsilon_0$$

$$\sigma = \frac{H E \varepsilon_0}{E+H} + A e^{-\frac{E+H}{\eta} t} \quad \sigma(0) = E \varepsilon_0 = \frac{H E \varepsilon_0}{E+H} + A$$

pol. part.

$$\sigma = \frac{E \varepsilon_0}{E+H} \left(H + E e^{-\frac{E+H}{\eta} t} \right) \quad \text{pas de relaxation complète}$$

$\rightarrow \sigma \text{ tq } \varepsilon_0 = \frac{\sigma}{E} + \frac{\sigma}{H}$

éffacement admettant que l'état

à symétrie $\sigma = \frac{EH}{E+H} \varepsilon_0$ ait été atteint $= \sigma(t_0^-)$

$$\sigma(t_0^+) = \frac{EH}{E+H} \varepsilon - E \varepsilon_0 = \frac{E^2 \varepsilon_0}{E+H}$$

$$\left| \begin{array}{l} \sigma = \varepsilon^v + \frac{\sigma}{E} \\ \sigma = -H \frac{\sigma}{E} + \eta \frac{\dot{\sigma}}{E} \end{array} \right.$$

$$\sigma = A e^{-\frac{E+H}{\eta} (t-t_0)}$$

$$\sigma = \frac{E^2 \varepsilon_0}{E+H} e^{-\frac{E+H}{\eta} (t-t_0)}$$

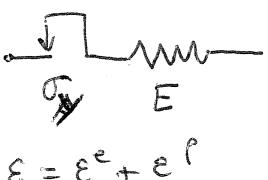
ex: relaxation sur du verre / élément de Norton-Hoff

$\rightarrow HT \rightarrow$ Maxwell

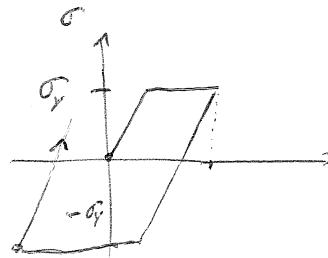
$$\sigma = K |\dot{\varepsilon}|^{1/m} \operatorname{sign} \dot{\varepsilon}$$

relaxation complète: effacement complet

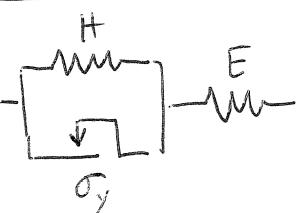
2) b)

modèle de St Venant / Prandtl

traction / cycle



critère $f(\sigma) = |\sigma| - \sigma_y$

 $f > 0$ inélasticité $f < 0$ élastique $f = 0$ $\dot{f} < 0$ élastique décharge $f = 0$ $\dot{f} = 0$ charge plastiquemodèle de Prager

contrainte intime $X = H \epsilon^p$

$\sigma = E \epsilon^e$

$\sigma_{\text{plast}} = \sigma - X$

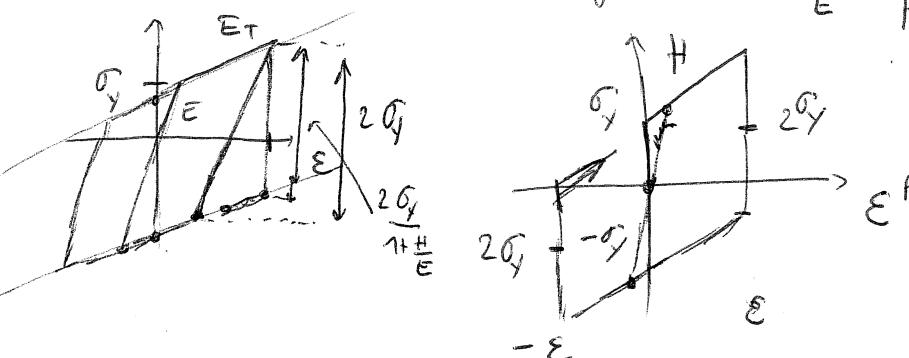
si $|\sigma - X| < \sigma_y$ $\dot{\epsilon}^p = 0$

si $|\sigma - X| = \sigma_y$ $\sigma = \sigma_y + X$ en traction

$\sigma = -\sigma_y + X$ en compression

$\dot{\epsilon}^p = \frac{\dot{\sigma}}{H}$

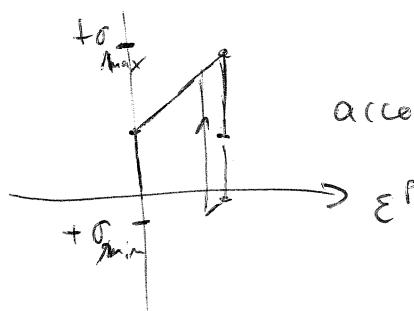
module target $\dot{\epsilon} = \frac{\dot{\sigma}}{E} + \frac{\dot{\sigma}}{H} = \left(\frac{1}{H} + \frac{1}{E} \right) \dot{\sigma} = \frac{1}{E_T} \dot{\sigma}$



effet Bauschinger

écrasage cinétique linéaire stabilisé immédiatement

cycle en contraintes



accommodation élastoplastique immédiate

écrasage intime

$\sigma = \sigma_y + H \dot{\epsilon}^p$

patin du cisaillement

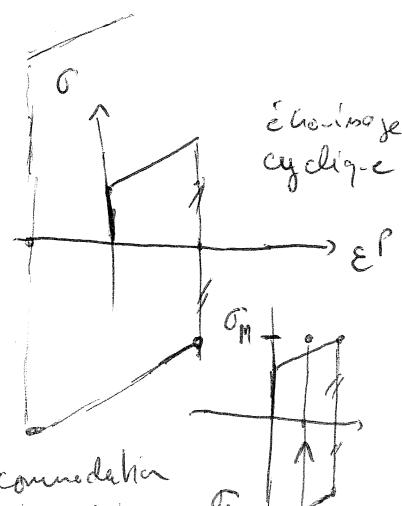
def. plastique cumulée $\dot{\epsilon}^p = 1 \dot{\epsilon}^p$

$\sigma_{\text{seuil}} = \sigma_y + H_p$

$\sigma = E \epsilon^e$

$|\sigma_s| < \sigma_y + H_p \quad \dot{\epsilon}^p = 0$

traction $\sigma = \sigma_y + H_p$



accommodation

Solide dans le viscoplastique

modèle de Bingham (extension)

$$\dot{\epsilon} = \dot{\epsilon}^e + \dot{\epsilon}^{vp}$$

$$\sigma_{\text{path}} = \sigma - H \dot{\epsilon}^{vp} - \gamma \dot{\epsilon}^{vp}$$

critère d'écoulement : $|\sigma_{\text{path}}| \leq \sigma_y$

$$\dot{\epsilon} \neq \dot{\epsilon}_{\text{path}}$$

si $|\sigma_{\text{path}}| < \sigma_y$ $\dot{\epsilon}^{vp} = 0$ $\sigma_{\text{path}} = \sigma - X$ $|X| < \sigma_y$

soit $\sigma_{\text{path}} = \sigma_y$
 $\Rightarrow \text{sign } \dot{\epsilon}^{vp} = 1$
 $(\text{car } \dot{\epsilon}^{vp} \neq 0)$

$$\sigma = X + \sigma_y + \gamma \dot{\epsilon}^{vp}$$

$$\dot{\epsilon}^{vp} = \frac{\sigma - X - \sigma_y}{\gamma} > 0$$

structure générale à connaître

si $\sigma > \sigma_y$ on dépasse la limite d'élasticité

soit $\sigma_{\text{path}} = -\sigma_y$ $\sigma = X - \sigma_y + \gamma \dot{\epsilon}^{vp}$ $\dot{\epsilon}^{vp} = \frac{\sigma - X + \sigma_y}{\gamma} \leq 0$

forme générale $\dot{\epsilon}^{vp} = \left\langle \frac{|X| - \sigma_y}{\gamma} \right\rangle \text{sign}(X - \sigma_y)$

en effet on vérifie sous charge ce que $\gamma \dot{\epsilon}^{vp} = \text{sign}(X - \sigma_y)$

Loi de Norton avec seuil et contrainte interne :

$$\dot{\epsilon}^{vp} = \left\langle \frac{|X| - \sigma_y}{K} \right\rangle^n \text{sign}(X - \sigma_y)$$

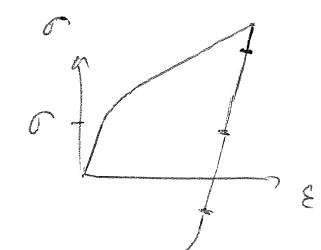
$$\sigma = H \dot{\epsilon}^{vp} + \sigma_y + \gamma \dot{\epsilon}^{vp}$$

$$\dot{\epsilon}^{vp} = \dot{\epsilon}_0 - \frac{\sigma}{E} \quad (1 + \frac{\gamma}{E}) \sigma - \gamma \frac{\sigma}{E} = (H t + \gamma) \dot{\epsilon}_0 + \sigma_y$$

$$\dot{\epsilon}^{vp} = \dot{\epsilon}_0 t - \frac{\sigma_y}{E} \quad \sigma = \frac{(H t + \gamma) E \dot{\epsilon}_0 + E \sigma_y}{H + E} + A e^{-\frac{E+H}{\gamma} t}$$

$$\Delta \sigma(t=0) = \sigma_y \quad A = \frac{H \sigma_y - (H t + \gamma) E \dot{\epsilon}_0}{H + E}$$

pente finale: $\frac{1}{E_f} = \frac{1}{H} + \frac{1}{E}$



charge $\sigma_0 > \sigma_y$

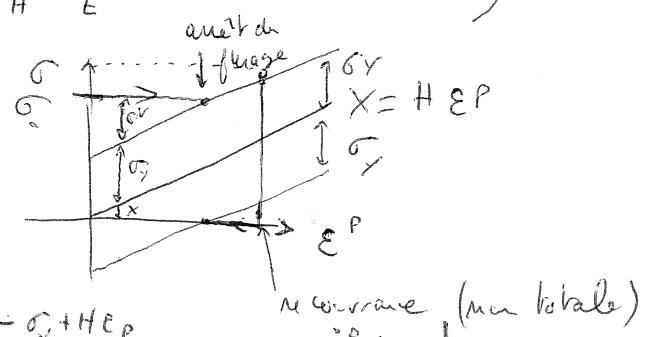
$$\sigma = H \dot{\epsilon}^p + \sigma_y + \gamma \dot{\epsilon}^p$$

$$\dot{\epsilon}^p = \frac{\sigma_0 - \sigma}{H} \left(1 - e^{-\frac{H}{\gamma} t}\right) \text{ avec } \dot{\epsilon}^p(0) = 0$$

$$\dot{\epsilon} = \frac{\dot{\epsilon}_0}{E} + \dot{\epsilon}^p$$

$$\text{à l'écoulement } \dot{\epsilon}^p = \frac{\sigma_0 - \sigma}{H}$$

$$\text{à l'écoulement } \sigma = \sigma_0 - \sigma_y + H \dot{\epsilon}_0$$



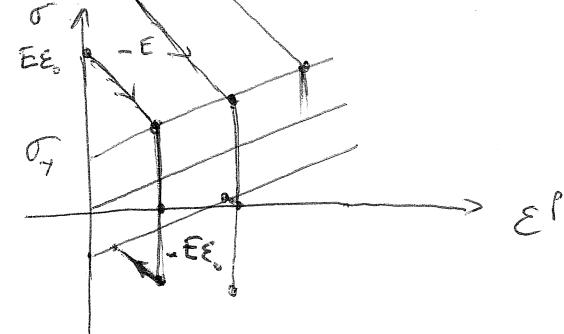
écoulement (non-légal)

relaxation $\epsilon = \epsilon_0$ $\sigma = E(\epsilon_0 - \epsilon')$ ($E\epsilon > \sigma_y$)

$$\sigma = H\epsilon' + \sigma_y + \gamma \dot{\epsilon}'$$

$$\sigma(1 + \frac{H}{E}) + \frac{\gamma}{E} \dot{\sigma} = \sigma_y + H\epsilon.$$

$$\sigma = E \frac{\sigma_y + H\epsilon_0}{H+E} + E \frac{H\epsilon_0 - \sigma_y}{H+E} e^{-\frac{E+H}{\gamma} t}$$

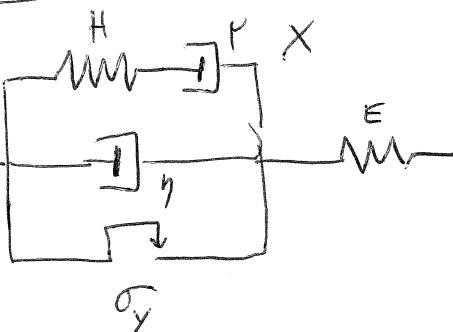


relaxation \rightarrow ~~$\sigma = \epsilon \frac{\sigma_y + H\epsilon_0}{E+H}$~~ intersection de la droite $-E$ et $X + \sigma_y$

effacement $\epsilon \rightarrow 0$ $E\epsilon_0 < 2\sigma_y \rightarrow$ encl. contrainte bégote

$E\epsilon_0 > 2\sigma_y \rightarrow$ effacement partiel

modèle avec restauration



$$\sigma = \sigma_y + X + \gamma \dot{\epsilon}^{vp} \quad \text{soit charge plastique}$$

$$f(\sigma, X) = |\sigma - X| - \sigma_y \quad \dot{\epsilon}^{vp} = \left\langle \frac{P}{\gamma} \right\rangle \text{sign}(\sigma - X)$$

$$X = \cancel{H\dot{\epsilon}} \quad H\dot{\epsilon}_X^e = P\dot{\epsilon}_X^e$$

$$\dot{X} = H\dot{\epsilon}_X^e = H(\dot{\epsilon}^{vp} - \dot{\epsilon}_X^e) = H\dot{\epsilon}^{vp} - \frac{H}{P}X$$

loi de Bailey - Orowan

érouvage cinétique non linéaire

restauration par le temps (stratique)

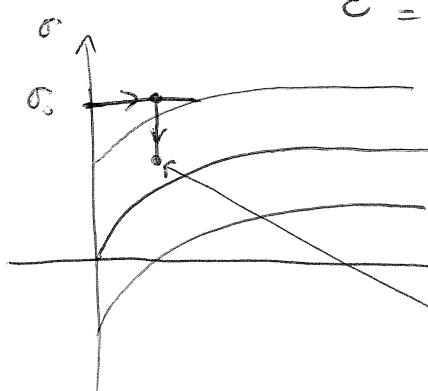
flage: $\dot{X} = \frac{H}{\gamma}(\sigma - X - \sigma_y) - \frac{H}{P}X$

car $\dot{\epsilon}^{vp} = \sigma - X - \sigma_y$

d'où $X = P \frac{\sigma_0 - \sigma_y}{\gamma + P} \left(1 - e^{-\frac{t}{\tau_F}} \right) \rightarrow$ contrainte stable stabilisée

on calcule ensuite

$$\dot{\epsilon}^{vp} = \frac{\sigma_0 - \sigma_y}{\gamma + P} \left(t + \frac{P^2}{H(\gamma + P)} \left(1 - e^{-t/\tau_F} \right) \right)$$



$$\dot{X} = -\frac{H}{P}X$$

