

Thermodynamics



- **First and second principles of thermodynamics**
- **Clausius-Duhem, dissipation**
- **Thermoelasticity**
- **Standard models**
- **Kinematic nonlinear hardening**
- **Heat dissipation under cyclic loadings**

–Track the energy in the plastic process–

First and second principle of the thermodynamics

- first principle :

$$\frac{dE}{dt} = \int_D \rho \frac{de}{dt} dV = \dot{W}_e + \dot{Q}$$

(\dot{W}_e = power of the external load, \dot{Q} = heat received, E (e) specific internal energy, kinetic energy neglected).

With $\underline{\mathbf{q}}$ the heat flux vector and r the volumic heat :

$$\rho \frac{de}{dt} = \sigma_{ij} \dot{\epsilon}_{ij} + (r - \text{div}(\underline{\mathbf{q}}))$$

- second principle :

$$\frac{dS}{dt} \geq \int_D \frac{r}{T} dV - \int_{\partial D} \frac{q_i n_i}{T} dS$$

then :

$$\int_D \left(\rho \frac{ds}{dt} - \frac{r}{T} + \text{div} \left(\frac{\underline{\mathbf{q}}}{T} \right) \right) dV \geq 0$$

(entropy (specific) S (s), $\underline{\mathbf{n}}$ outward normal to the surface ∂D of the domain)

Dissipation

Clausius-Duhem inequality

(the specific free energy is $\psi = e - Ts$)

$$\sigma_{ij}\dot{\epsilon}_{ij} - \rho \frac{d\psi}{dt} - \rho s\dot{T} - \frac{1}{T} \underline{\mathbf{q}} \cdot \underline{\mathbf{grad}}(T) \geq 0$$

Application of the *method of local state*:

ψ depend on temperature and *state variables* α_i

$$\frac{d\psi}{dt} = \frac{\partial\psi}{\partial T}\dot{T} + \frac{\partial\psi}{\partial\alpha_i}\dot{\alpha}_i$$

$$s = -\frac{\partial\psi}{\partial T}$$

$$\sigma_{ij}\dot{\epsilon}_{ij} - \rho \frac{\partial\psi}{\partial\alpha_i}\dot{\alpha}_i - \frac{1}{T} \underline{\mathbf{q}} \cdot \underline{\mathbf{grad}}(T) \geq 0 \quad (4)$$

Intrinsic dissipation Φ_1 , thermal dissipation Φ_2 :

$$\Phi_1 = \sigma_{ij}\dot{\epsilon}_{ij} - \rho \frac{\partial\psi}{\partial\alpha_i}\dot{\alpha}_i \quad \Phi_2 = -\frac{1}{T} \underline{\mathbf{q}} \cdot \underline{\mathbf{grad}}(T)$$

Heat equation

Decoupling of intrinsic and thermal dissipations, each of them must be positive

- Fourier's law :

$$\underline{\mathbf{q}} = -k(T, \alpha_i) \underline{\mathbf{grad}}(T)$$

- heat equation in presence of mechanical strain:

$$\text{div} (k \underline{\mathbf{grad}}(T)) = \rho C_\varepsilon \dot{T} - r - \sigma_{ij} \dot{\varepsilon}_{ij} + \rho \left(\frac{\partial \psi}{\partial \alpha_i} - T \frac{\partial^2 \psi}{\partial T \partial \alpha_i} \right) \dot{\alpha}_i \quad (5)$$

(with $C_\varepsilon = T \frac{\partial s}{\partial T}$ specific heat at a constant deformation)

Thermo-elasticity (1)

- Zero elastic strain for $\underline{\sigma}^I$, zero thermal strain for T^I
- The state variable is elastic strain: $\alpha_I \equiv \underline{\xi}^e$ in intrinsic dissipation
- No dissipation for an elastic isothermal perturbation around equilibrium, then

$$\Phi_1 = \underline{\sigma} : \dot{\underline{\xi}}^e - \rho \frac{\partial \psi}{\partial \underline{\xi}^e} : \dot{\underline{\xi}}^e = 0$$

This provides a definition for stress:

- Two state variables, two energetically conjugated variables:

State variable	Conjugated variable
T	entropy $s = -\frac{\partial \psi}{\partial T}$
$\underline{\xi}^e$	stress $\underline{\sigma} = \frac{\partial \psi}{\partial \underline{\xi}^e}$

ψ is a thermodynamic potential which characterizes reversible processes

Thermo-elasticity (2)

Assume the following form for free energy:

$$\rho\psi = \underline{\underline{\sigma}}^I : \underline{\underline{\xi}}^e + \frac{1}{2}\lambda (\text{Tr } \underline{\underline{\xi}}^e)^2 + \mu \underline{\underline{\xi}}^e : \underline{\underline{\xi}}^e - 3K\alpha \text{Tr } \underline{\underline{\xi}}^e (T - T^I) - \frac{1}{2} \frac{\rho C_\varepsilon}{T^I} (T - T^I)^2 \quad (1)$$

The stress is then:

$$\underline{\underline{\sigma}} = \rho \frac{\partial \psi}{\partial \underline{\underline{\xi}}^e} = \underline{\underline{\sigma}}^I + \lambda \text{Tr } \underline{\underline{\xi}}^e \underline{\underline{I}} + 2\mu \underline{\underline{\xi}}^e - 3K\alpha (T - T^I) \underline{\underline{I}}$$
$$\underline{\underline{\sigma}} - \underline{\underline{\sigma}}^I = \lambda (\text{Tr } \underline{\underline{\xi}}^e - 3\alpha (T - T^I)) \underline{\underline{I}} + 2\mu \underline{\underline{\xi}}^e$$

Variation of specific entropy:

$$\rho s = -\rho \frac{\partial \psi}{\partial T} = 3K\alpha \text{Tr } \underline{\underline{\xi}}^e + \frac{\rho C_\varepsilon}{T^I} (T - T^I)$$

Temperature changes for elastic loadings

Temperature variation related to volume change:

$$(T - T^I) = -\frac{3K\alpha T^I}{\rho C_\varepsilon} \varepsilon_{ll}^e$$

Cooling during tension

Loading curves

— isothermal : $\underline{\underline{\sigma}} - \underline{\underline{\sigma}}^I = \lambda \text{Tr} \underline{\underline{\varepsilon}}^e \underline{\underline{\mathbf{I}}} + 2\mu \underline{\underline{\varepsilon}}^e$

— adiabatic : $\underline{\underline{\sigma}} - \underline{\underline{\sigma}}^I = \left(\lambda + \frac{9K^2\alpha^2 T^I}{\rho C_\varepsilon} \right) \text{Tr} \underline{\underline{\varepsilon}}^e \underline{\underline{\mathbf{I}}} + 2\mu \underline{\underline{\varepsilon}}^e$

No change on shear modulus, variation of the axial component

Extension to dissipative processes

- Definition of state variables α_I and of hardening variables Y_I

State variable	Conjugated variable	
T	$s = -\frac{\partial \psi}{\partial T}$	entropy
$\underline{\underline{\xi}}^e$	$\underline{\underline{\sigma}} = \rho \frac{\partial \psi}{\partial \underline{\underline{\xi}}^e}$	stress
α_I	$Y_I = \rho \frac{\partial \psi}{\partial \alpha_I}$	state variables

- The intrinsic dissipation can be rewritten:

$$\Phi_1 = \underline{\underline{\sigma}} : \dot{\underline{\underline{\xi}}}^p - Y_I \dot{\alpha}_I = Z \dot{z}$$

with : $Z = \{ \underline{\underline{\sigma}}, Y_I \}$; $z = \{ \underline{\underline{\xi}}^p, -\alpha_I \}$

Standard models (1)

- A model is *standard* if one can find a potential $\Omega \equiv \Omega(Z)$ such as:

$$\dot{z} = \frac{\partial \Omega}{\partial Z}$$

- If Ω is a convex function of Z which includes the origin, the dissipation is automatically positive, since:

$$\phi_1 = Z \frac{\partial \Omega}{\partial Z}$$

(all the points of the surface Ω are on the same side of the tangent plane defined by $\frac{\partial \Omega}{\partial Z}$))

- One can also define (through the Legendre-Fenchel transform) a companion potential in terms of \dot{z} :

$$\Omega^*(\dot{z}) = \max_Z (Z\dot{z} - \Omega(Z))$$

- A new potential, expressed as $\Omega^*(\dot{z})$ or $\Omega(Z)$ must be introduced to characterize the dissipative processes.

Operational way for material model development

- Define a set of two potentials, ψ , Ω
- Derive the relation between state variables and hardening variables from ψ
- Derive the *nature* of the hardening variables and their evolution rules from Ω

In the following, example of isotropic and kinematic nonlinear hardenings; the choice for the sets (Y_I, α_I) is:

Type of hardening	State variable	Conjugated variable
Isotropic hardening	r	R
Kinematic hardening	$\underline{\alpha}$	\underline{X}

NOTE: Previously, p for isotropic hardening, $\underline{\epsilon}^p$ for linear kinematic hardening

Viscoplastic standard model with isotropic and kinematic hardening (1)

Example using the von Mises criterion:

$$f(\underline{\boldsymbol{\sigma}}, \underline{\boldsymbol{X}}, R) = J(\underline{\boldsymbol{\sigma}} - \underline{\boldsymbol{X}}) - R - \sigma_y = \left((3/2)(\underline{\boldsymbol{s}} - \underline{\boldsymbol{X}}) : (\underline{\boldsymbol{s}} - \underline{\boldsymbol{X}}) \right)^{0,5} - R$$

Assume the following free energy:

$$\psi(T, \underline{\boldsymbol{\varepsilon}}^e, r, \underline{\boldsymbol{\alpha}}) = \frac{1}{2} \underline{\boldsymbol{\varepsilon}}^e : \underline{\boldsymbol{\Lambda}} : \underline{\boldsymbol{\varepsilon}}^e + \frac{1}{2} b Q r^2 + \frac{1}{3} C \underline{\boldsymbol{\alpha}} : \underline{\boldsymbol{\alpha}}$$

Viscoplastic potential:

$$\Omega(\underline{\boldsymbol{\sigma}}, \underline{\boldsymbol{X}}, R) = \frac{n+1}{K} \left(\frac{f}{K} \right)^{n+1}$$

Viscoplastic flow:

$$\underline{\dot{\boldsymbol{\varepsilon}}}^p = \frac{\partial \Omega}{\partial \underline{\boldsymbol{\sigma}}} = \frac{\partial \Omega}{\partial f} \frac{\partial f}{\partial \underline{\boldsymbol{\sigma}}} = \dot{p} \underline{\boldsymbol{n}}$$

with

$$\dot{p} = \frac{\partial \Omega}{\partial f} = \left(\frac{f}{K} \right)^n \quad \text{and} \quad \underline{\boldsymbol{n}} = \frac{\partial f}{\partial \underline{\boldsymbol{\sigma}}} = \frac{3}{2} \frac{\underline{\boldsymbol{s}} - \underline{\boldsymbol{X}}}{J(\underline{\boldsymbol{\sigma}} - \underline{\boldsymbol{X}})}$$

Viscoplastic standard model with isotropic and kinematic hardening (2)

The state variable for isotropic hardening is found from:

$$\dot{r} = -\frac{\partial \Omega}{\partial R} = -\frac{\partial \Omega}{\partial f} \frac{\partial f}{\partial R} = \dot{p}$$

The state variable for kinematic hardening is found from:

$$\dot{\alpha} = -\frac{\partial \Omega}{\partial \underline{\mathbf{X}}} = -\frac{\partial \Omega}{\partial f} \frac{\partial f}{\partial \underline{\mathbf{X}}} = \dot{p} \underline{\mathbf{n}} = \underline{\boldsymbol{\epsilon}}^p$$

It is then *mandatory* to choose accumulated strain for isotropic hardening and plastic strain for kinematic hardening

$$R = bQp \qquad \underline{\mathbf{X}} = \frac{2}{3}C\underline{\boldsymbol{\alpha}} = \frac{2}{3}C\underline{\boldsymbol{\epsilon}}^p$$

The choices made in the free energy provide *linear* hardening rules.

Generalization of Hill's principle

Maximize of the intrinsic dissipated power instead of just maximize $\underline{\sigma} : \underline{\dot{\epsilon}}^p$

$$\Phi_1 = \underline{\sigma} : \underline{\dot{\epsilon}}^p - Y_I \dot{\alpha}_I = \underline{\sigma} : \underline{\dot{\epsilon}}^p - \dot{\psi}_p = \dot{Zz}$$

With Z including stress and the hardening variables Y_I ,
 z including plastic strain and the state variables ($-\alpha_I$):

$$(Z - Z^*)\dot{z} \geq 0$$

... with the constraint $f \leq 0$.

Let us define $\mathbb{F}(Z) = Z\dot{z} - \dot{\lambda} f$ and search for the zero of $\partial\mathbb{F}/\partial Z$

$$\dot{z} = \dot{\lambda} \frac{\partial f}{\partial Z} \quad \text{then:} \quad \underline{\dot{\epsilon}}^p = \dot{\lambda} \frac{\partial f}{\partial \underline{\sigma}} = \dot{\lambda} \underline{n} \quad ; \quad \dot{\alpha}_I = -\dot{\lambda} \frac{\partial f}{\partial Y_I}$$

Dissipation for a standard model

(1) Z actual maximizes the intrinsic dissipation;

equivalent to:

(2) the flow direction \dot{z} is normal to the surface defined by f ; the domain defined by f is convex.

If the domain defined by f includes $(f=0)$, the dissipation Φ_1 is automatically positive.

Nature of the hardening variables

Example using the von Mises criterion:

$$f(\underline{\boldsymbol{\sigma}}, \underline{\boldsymbol{X}}, R) = J(\underline{\boldsymbol{\sigma}} - \underline{\boldsymbol{X}}) - R - \sigma_y = \left((3/2)(\underline{\boldsymbol{s}} - \underline{\boldsymbol{X}}) : (\underline{\boldsymbol{s}} - \underline{\boldsymbol{X}}) \right)^{0,5} - R$$

- *kinematic hardening*, $\underline{\boldsymbol{\alpha}} \equiv \underline{\boldsymbol{\varepsilon}}^p$

$$\dot{\underline{\boldsymbol{\alpha}}} = -\dot{\lambda} \frac{\partial f}{\partial \underline{\boldsymbol{X}}} = \dot{\lambda} \frac{\partial f}{\partial \underline{\boldsymbol{\sigma}}} = \dot{\underline{\boldsymbol{\varepsilon}}}^p$$

- *isotropic hardening*, p

$$\dot{p} = -\dot{\lambda} \frac{\partial f}{\partial R} = \dot{\lambda}$$

p , *accumulated plastic strain*, length of the strain path:

$$\left((2/3) \dot{\underline{\boldsymbol{\varepsilon}}}^p : \dot{\underline{\boldsymbol{\varepsilon}}}^p \right)^{0,5} = \left((2/3) \dot{\lambda} \underline{\boldsymbol{n}} : \dot{\lambda} \underline{\boldsymbol{n}} \right)^{0,5} = \dot{\lambda}$$

Under onedimensional loading, $\dot{p} = |\dot{\varepsilon}^p|$

Non linear hardening (1)

kinematic and isotropic hardening both are needed for a realistic modeling of the behavior

- standard framework

$$f(\underline{\sigma}, \underline{X}, R) = J(\underline{\sigma} - \underline{X}) - \sigma_y - R + \frac{D}{2C} J^2(\underline{X}) + \frac{R^2}{2Q}$$

$$\text{with } J(\underline{X}) = \left(\frac{3}{2} \underline{X} : \underline{X} \right)^{0,5}$$

- associated framework

$$f(\underline{\sigma}, \underline{X}, R) = J(\underline{\sigma} - \underline{X}) - \sigma_y - R$$

Non linear hardening (2)

$$\dot{\underline{\alpha}} = -\dot{\lambda} \frac{\partial f}{\partial \underline{\alpha}} = \left(\underline{n} - \frac{3D}{2C} \underline{\alpha} \right) \dot{\lambda}$$

$$\dot{r} = -\dot{\lambda} \frac{\partial f}{\partial R} = \left(1 - \frac{R}{Q} \right) \dot{\lambda}$$

The state variables $\underline{\alpha}$ and r , define the hardening variables \underline{X} et R :

$$\psi = \dots + \frac{1}{2} b Q r^2 + \frac{1}{3} C \underline{\alpha} : \underline{\alpha}$$

$$\underline{X} = \frac{2}{3} C \underline{\alpha} \quad ; \quad R = b Q r$$

- plastic multiplier \equiv accumulated plastic strain rate, but $r \neq p$

- for the case where the coefficients are constants :

$$\dot{\underline{X}} = \frac{2}{3} C \dot{\underline{\epsilon}}^p - D \underline{X} \dot{p}$$

$$\dot{R} = b(Q - R)\dot{p} \quad \text{then } R = Q(1 - \exp(-b p))$$

Dissipated energy

$$\begin{aligned}\Phi_1 &= \underline{\underline{\sigma}} : \underline{\underline{\dot{\xi}}}^p - R \dot{r} - \underline{\underline{X}} : \underline{\underline{\dot{\alpha}}} \\ &= \left(\underline{\underline{\sigma}} : \underline{\underline{n}} - R + \frac{R^2}{Q} - \underline{\underline{X}} : \underline{\underline{n}} + \frac{D}{C} J^2(\underline{\underline{X}}) \right) \dot{\lambda}\end{aligned}$$

$$\text{since} \quad \underline{\underline{\sigma}} : \underline{\underline{n}} - \underline{\underline{X}} : \underline{\underline{n}} = J(\underline{\underline{\sigma}} - \underline{\underline{X}})$$

$$\begin{aligned}\Phi_1 &= \left(J(\underline{\underline{\sigma}} - \underline{\underline{X}}) + \frac{R^2}{Q} + \frac{D}{C} J^2(\underline{\underline{X}}) \right) \dot{\lambda} \\ &= \left(f + \sigma_Y + \frac{R^2}{2Q} + \frac{D}{2C} J^2(\underline{\underline{X}}) \right) \dot{\lambda}\end{aligned}$$

Dissipation :

- $f \dot{\lambda}$, viscous dissipation;
- $\sigma_Y \dot{\lambda}$, dissipation (due to friction) related to initial threshold;
- quadratic terms, non-linearity of the hardening;

Trapped energy

(\equiv variation of free energy) :

$$\begin{aligned}\dot{\psi} &= R \dot{r} + \underline{\underline{\mathbf{X}}} : \underline{\underline{\dot{\alpha}}} \\ &= Q(1 - e^{-bp}) e^{-bp} \dot{p} + \underline{\underline{\mathbf{X}}} : \left(\underline{\underline{\mathbf{n}}} - \frac{3D}{2C} \underline{\underline{\mathbf{X}}} \right) \dot{p}\end{aligned}$$

Dissipation for time independent plasticity (1)

$$\Phi_1 = \underline{\underline{\sigma}} : \dot{\underline{\underline{\varepsilon}}}^p - A_I \dot{\alpha}_I$$

Perfect plasticity

no hardening variables

$$f(\sigma) = |\sigma| - \sigma_y$$

$$\psi \equiv \psi_e$$

$$\Phi_1 = \sigma \dot{\varepsilon}^p = \sigma_y \dot{p}$$

$$\Phi_1 = \underline{\underline{\sigma}} : \underline{\underline{\dot{\varepsilon}}}^p = \sigma_y \dot{p}$$

since $\underline{\underline{\sigma}} : \underline{\underline{n}} = J(\underline{\underline{\sigma}})$

linear kinematic hardening

$$\underline{\underline{\alpha}} \equiv \underline{\underline{\varepsilon}}^p$$

$$f(\sigma, X) = |\sigma - X| - \sigma_y$$

$$\psi_p = \frac{1}{2} C \alpha^2$$

$$\Phi_1 = \sigma \dot{\varepsilon}^p - X \dot{\alpha} = \sigma_y \dot{p}$$

$$\Phi_1 = \underline{\underline{\sigma}} : \underline{\underline{\dot{\varepsilon}}}^p - \underline{\underline{X}} : \underline{\underline{\dot{\alpha}}} = \sigma_y \dot{p}$$

since $(\underline{\underline{\sigma}} - \underline{\underline{X}}) : \underline{\underline{n}} = J(\underline{\underline{\sigma}} - \underline{\underline{X}})$

Dissipation for time independent plasticity (1)

Nonlinear isotropic and kinematic hardening (1D), criterion :

$$f(\sigma, X, R) = |\sigma - X| - R - \sigma_y = 0$$

$$\Phi_1 = \sigma \dot{\varepsilon}^p - X \dot{\alpha} - R \dot{r}$$

$$\text{with } \dot{\alpha} = \dot{\varepsilon}^p - (D/C)X\dot{p} \quad \dot{r} = (1 - b\frac{R}{Q})\dot{p}$$

Then

$$\begin{aligned}\Phi_1 &= (\sigma - X)\dot{\varepsilon}^p - R\dot{p} + \frac{D}{C}X^2\dot{p} + \frac{R^2}{Q}\dot{p} \\ &= |\sigma - X|\dot{p} - R\dot{p} + \frac{D}{C}X^2\dot{p} + \frac{R^2}{Q}\dot{p} \\ &= \sigma_y\dot{p} + \frac{D}{C}X^2\dot{p} + \frac{R^2}{Q}\dot{p}\end{aligned}$$

Dissipation for time independent plasticity (1)

$$\Phi_1 = \sigma_y \dot{p} + \frac{D}{C} X^2 \dot{p} + \frac{R^2}{Q} \dot{p}$$

For 1D tension:

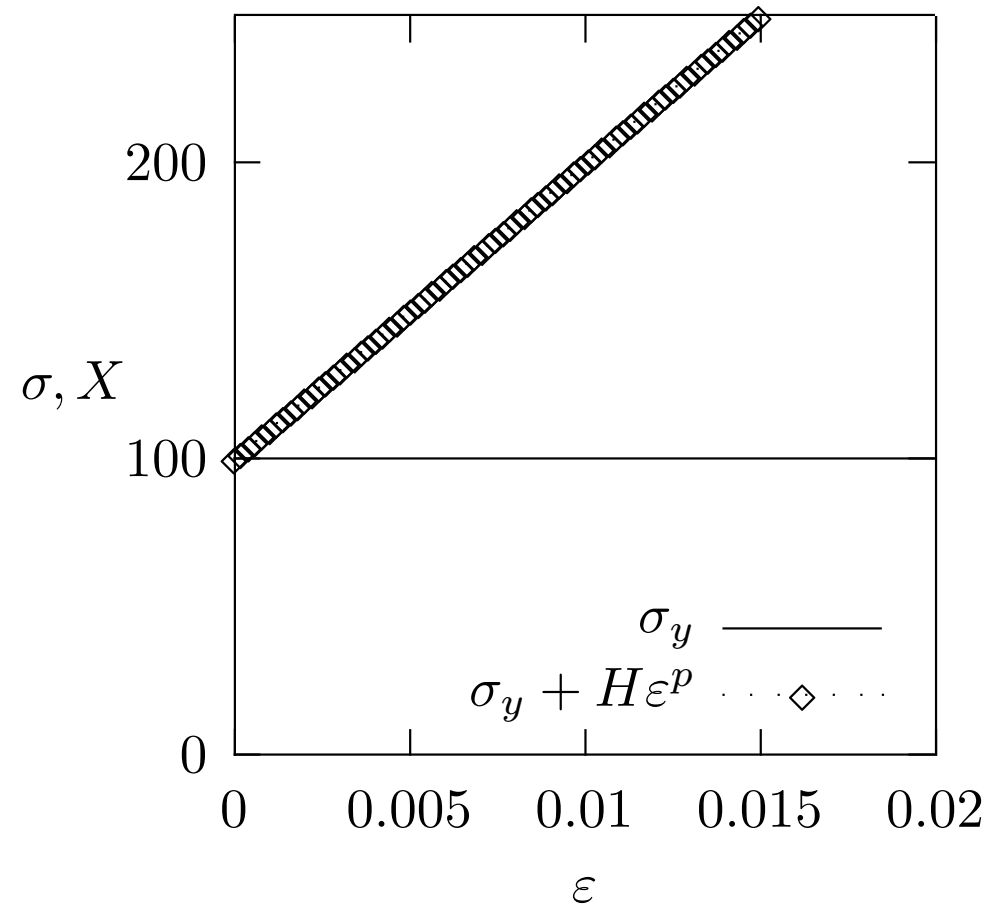
Stored forever by the isotropic mechanism

$$\int R \dot{r} = \int R \left(1 - \frac{R}{Q}\right) \dot{p} = \int Q (1 - \exp(-bp)) \exp(-bp) \dot{p} = \frac{R^2}{2Q}$$

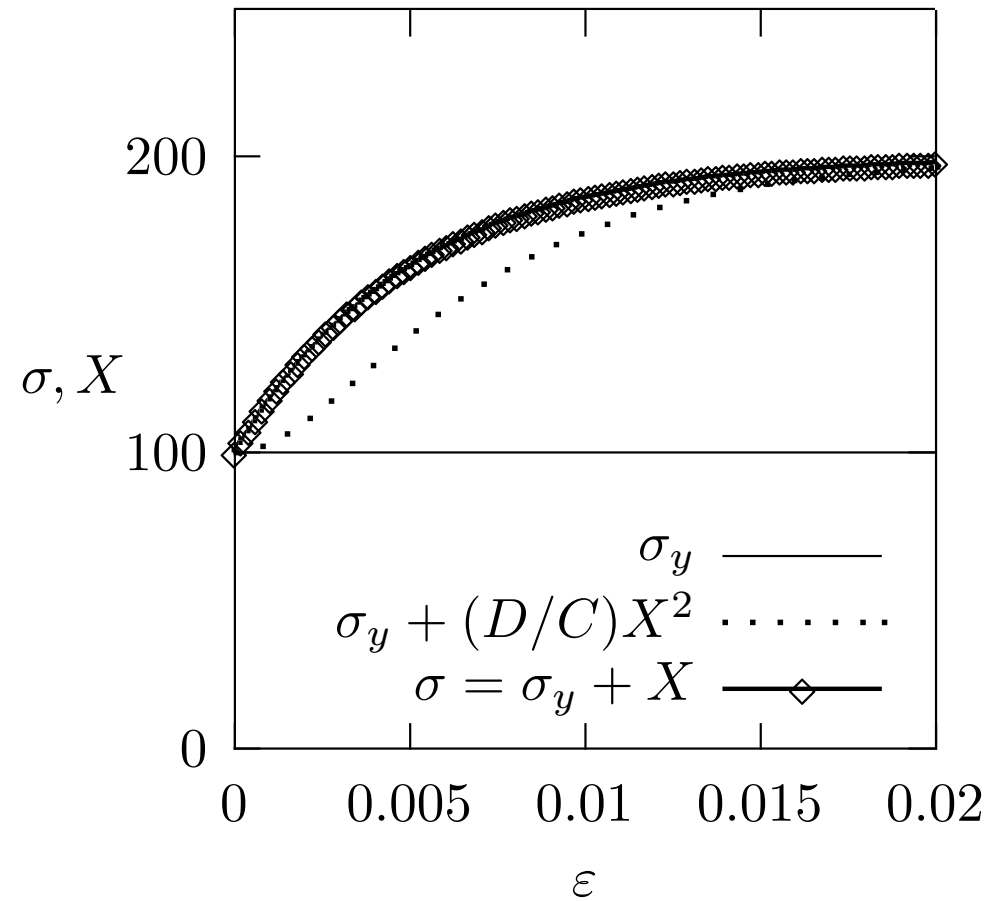
Temporary stored by the kinematic mechanism

$$\int X \dot{\alpha} = \frac{X^2}{2C} = \frac{C}{2D^2} (1 - \exp(-D\varepsilon^p))^2$$

A view on dissipated and stored energy for linear kinematic hardening



A view on dissipated and stored energy for non linear kinematic hardening



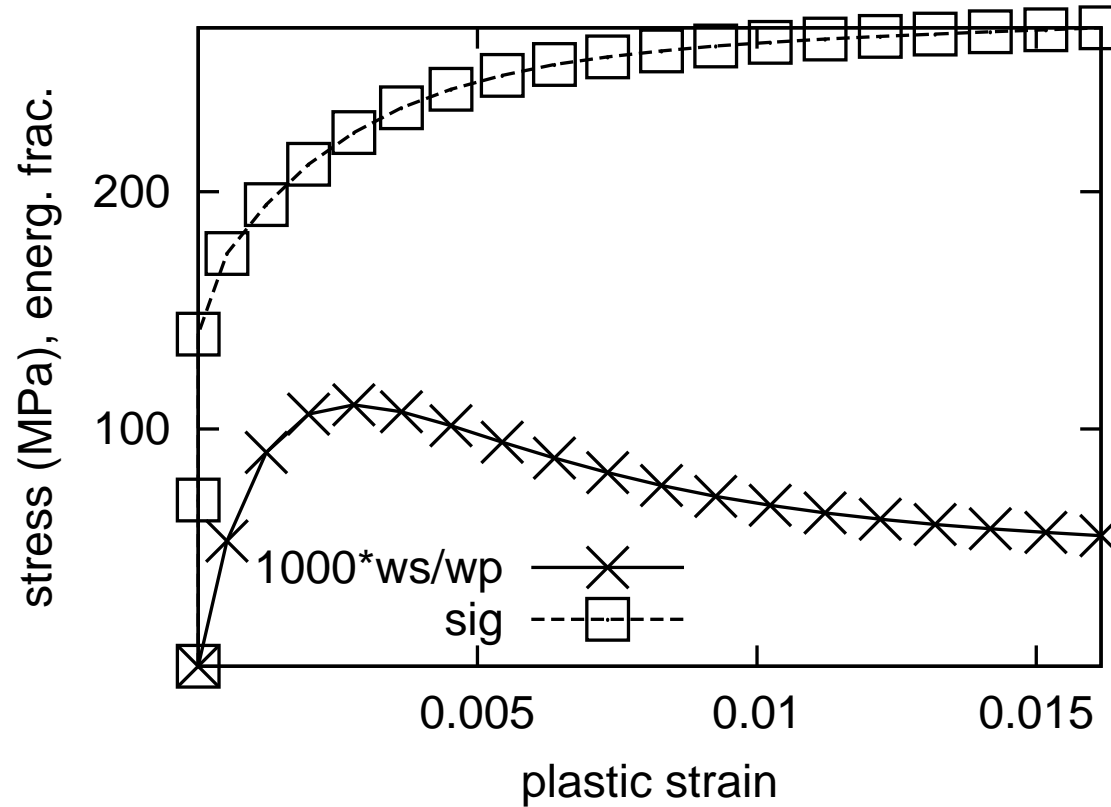
Adiabatic RVE

The dissipated energy will produce a temperature change $\dot{T} = \frac{\Phi_1}{\rho C}$

```
@Class DISSIP : SIMUL_MODEL {
  @Coefs  young, K, n, R0, Q, b, C, D, H, rhoCp;
  @VarInt  eel, evcum, alpha, evi, wp, ws, tpr;
  @VarAux  X, R, sigeff, f;
  @Observable sig, eto; };

@Derivative {
  eto = eel + evi; sig = young*eel;
  X = C*alpha; R = Q*(1.0-exp(-b*evcum));
  sigeff = sig-X-H*evi;
  f = fabs(sigeff) - R -R0;
  if (f>0.0){
    devcum = pow((f/K),n); devi = sign(sigeff)*devcum;
    if (C>0.0)  dalpha = devi - devcum*D*X/C;
    dwp=sig*devi; dws=X*dalpha+R*exp(-b*evcum)*devcum;
    dtpr=(dwp-dws)/rhoCp; }
  if      (load[0]=="sig") deel = dglobal[0]/young;
  else if (load[0]=="eto") deel = dglobal[0] - devi; };
```

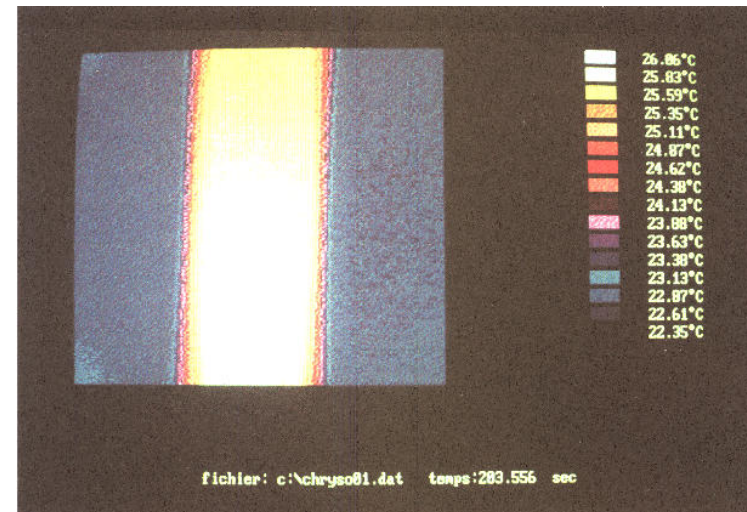
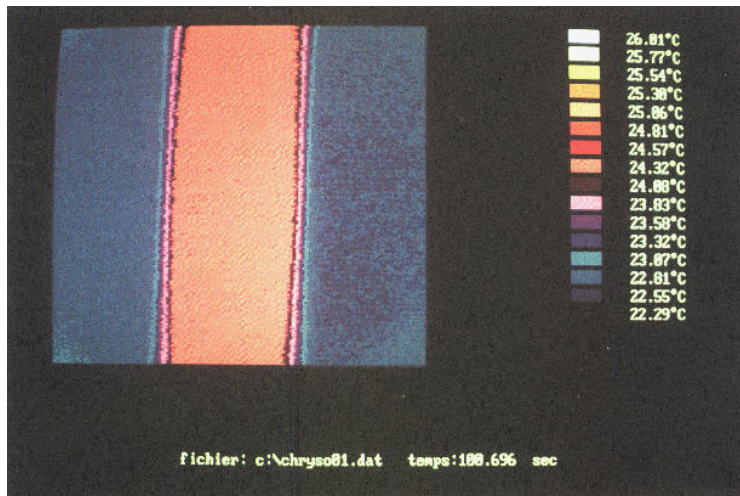
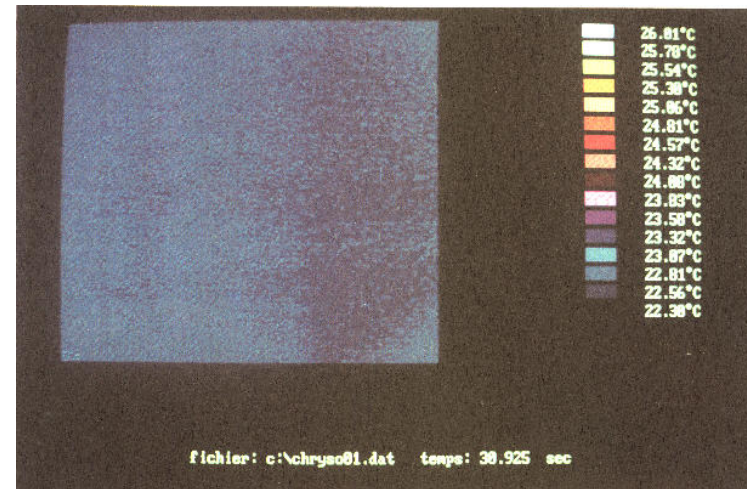
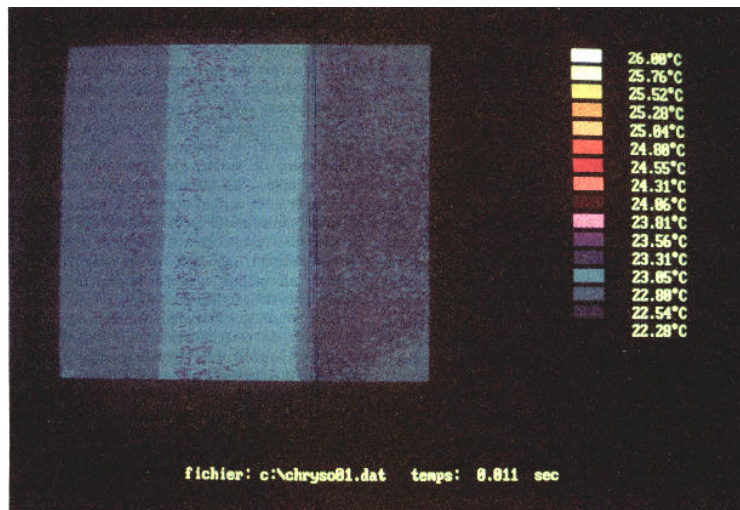
Energy distribution in a tensile test



$\sigma-\varepsilon$
 Dissipation/storage
 Dissipation/storage(zoom)
 Ratio

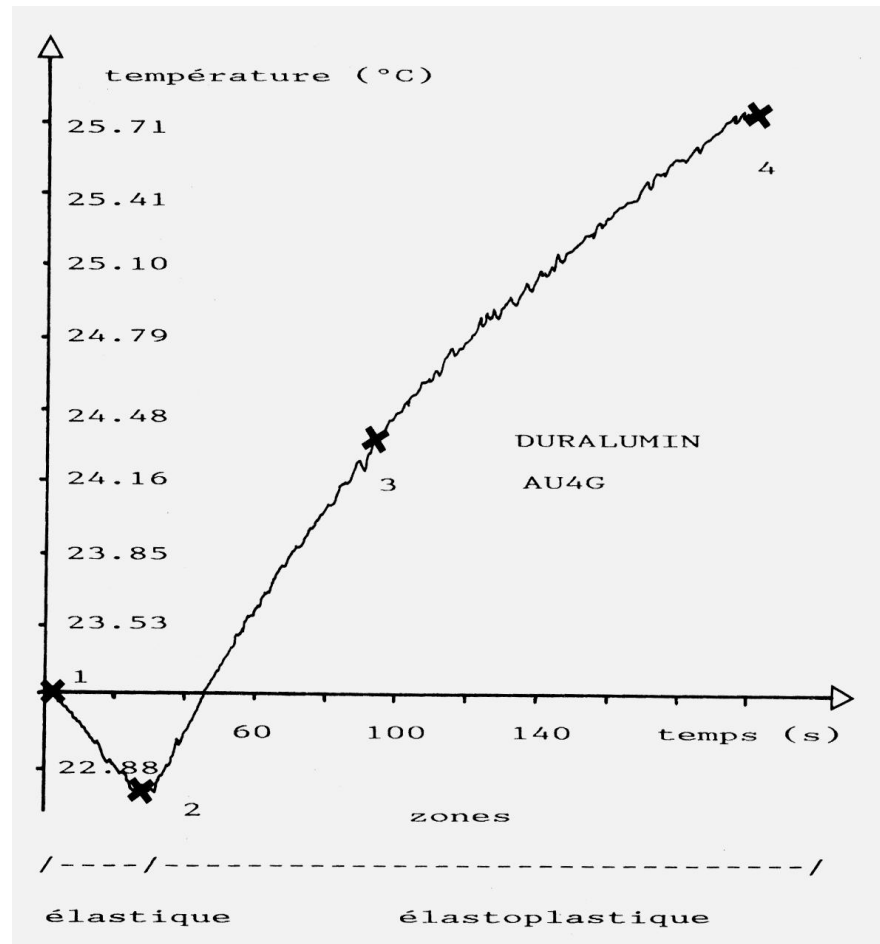
$$K=120, Q=50, b=20, C=40000, D=400, rhoCp=1.5$$

How big is the temperature change during loading ?



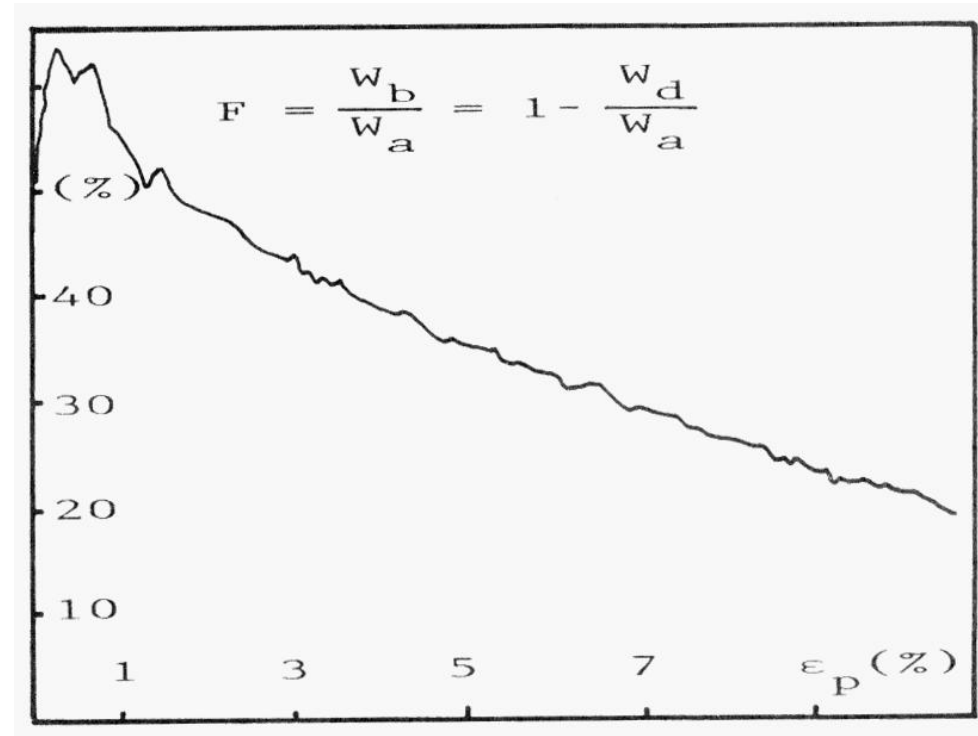
Low rate loadings : tests by Chrysochoos (Univ. Montpellier) on various materials, Al alloys, 316SS, SMA,...

Temperature evolution on a tensile specimen



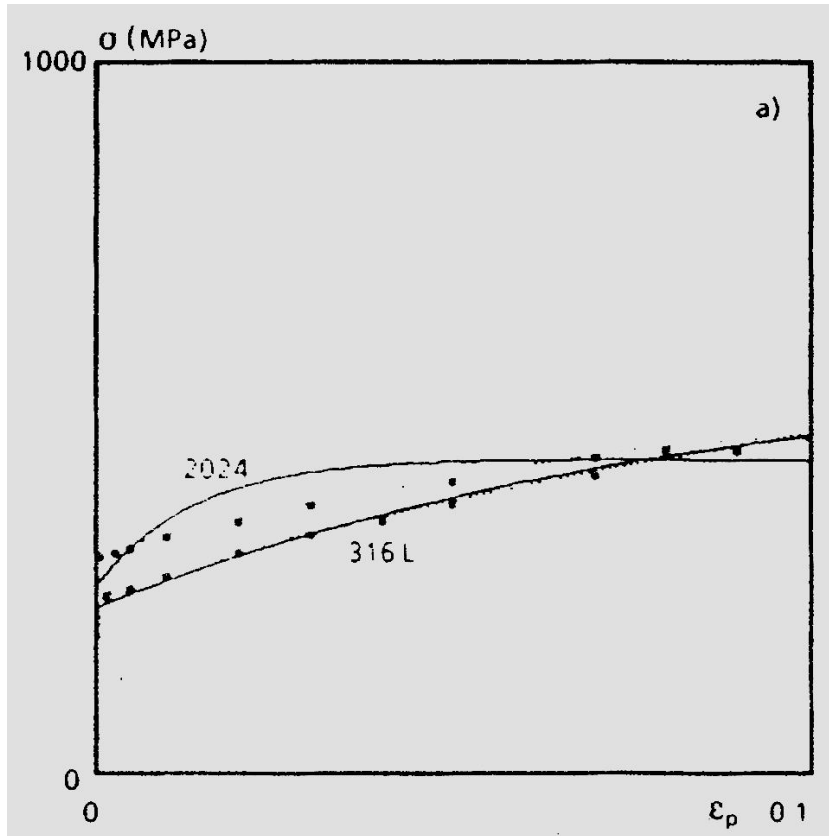
Experimental data on 2024-T4 alloy (Chrysochoos)

A view on stored energy: the ratio W_{stored}/W_{plast}

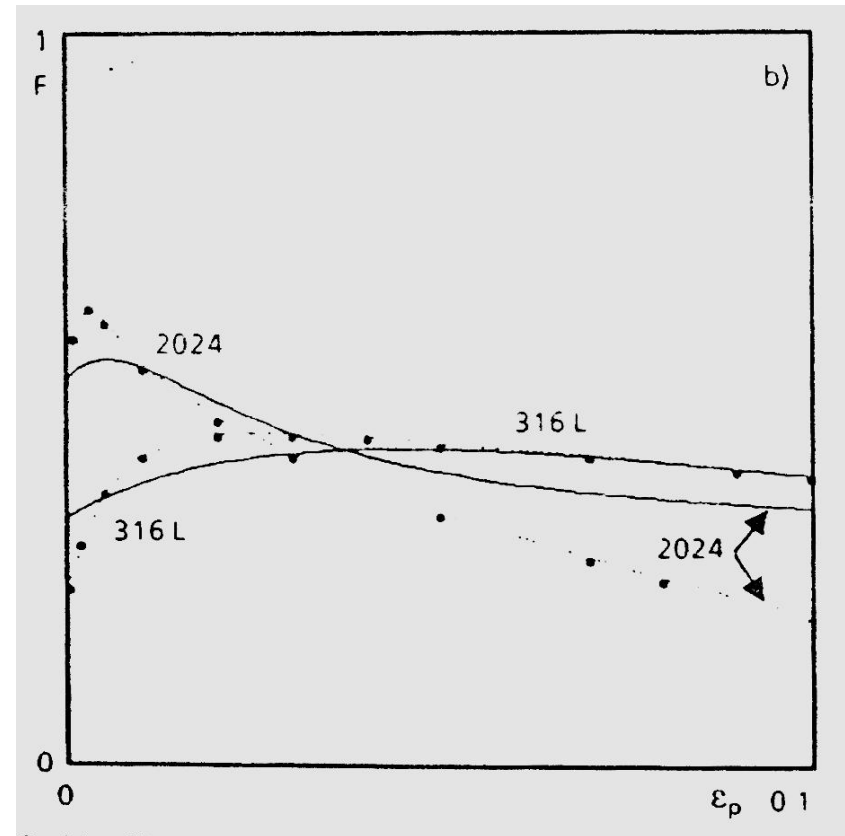


Experimental data on 2024-T4 alloy (Chrysochoos)

Simulation of hardening and dissipation (simple model)

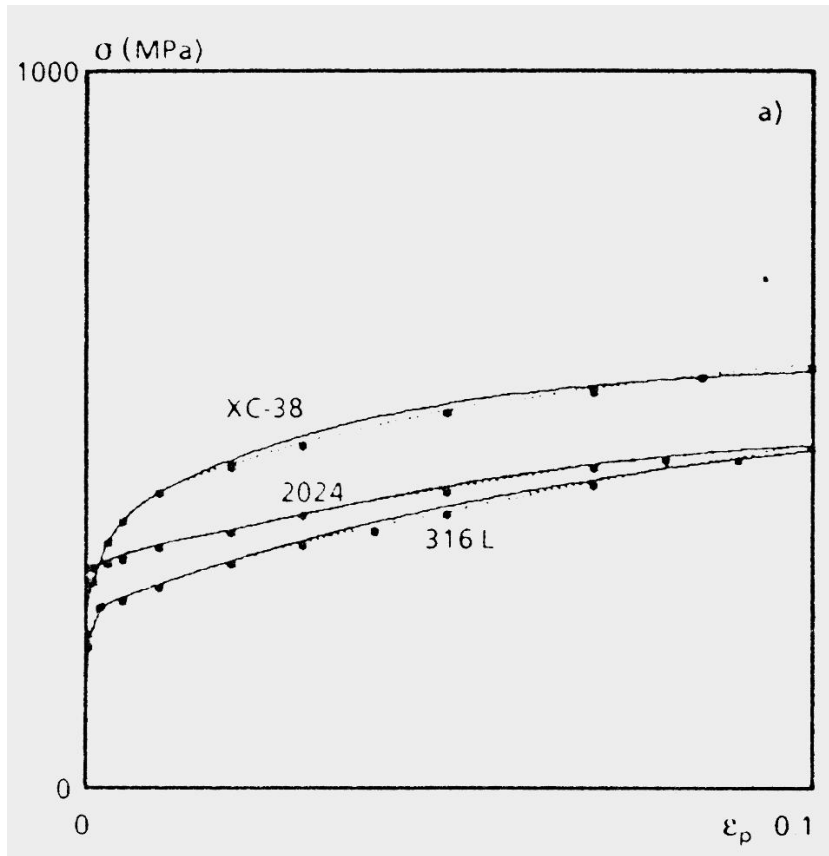


Tension curve

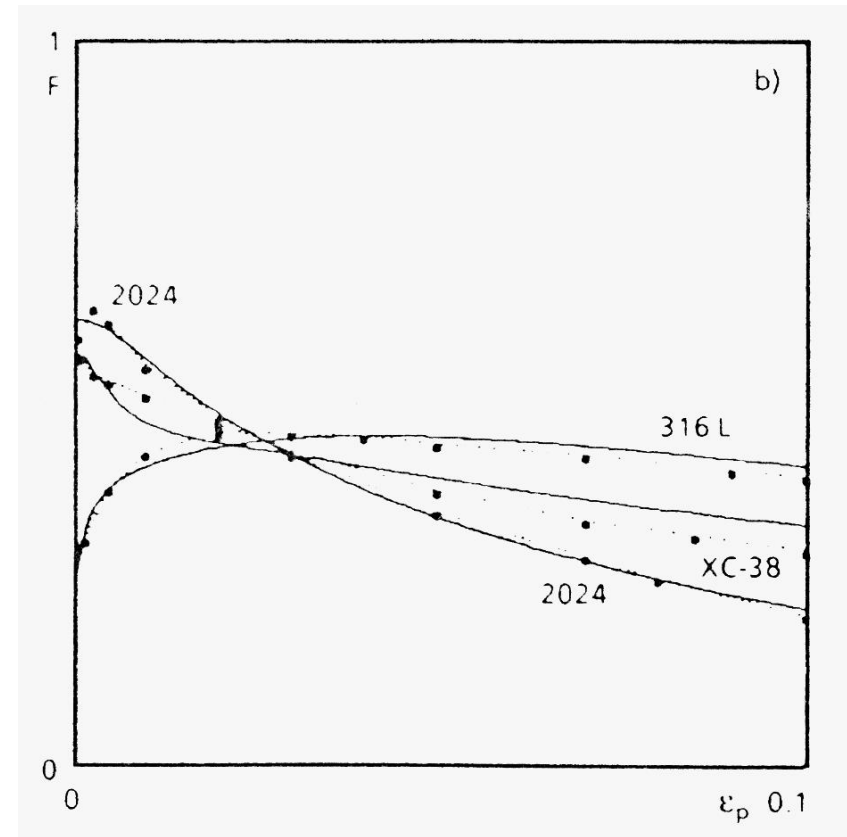


Fraction $F = W_{stored}/W_{plast}$

Simulation of hardening and dissipation (complex model)

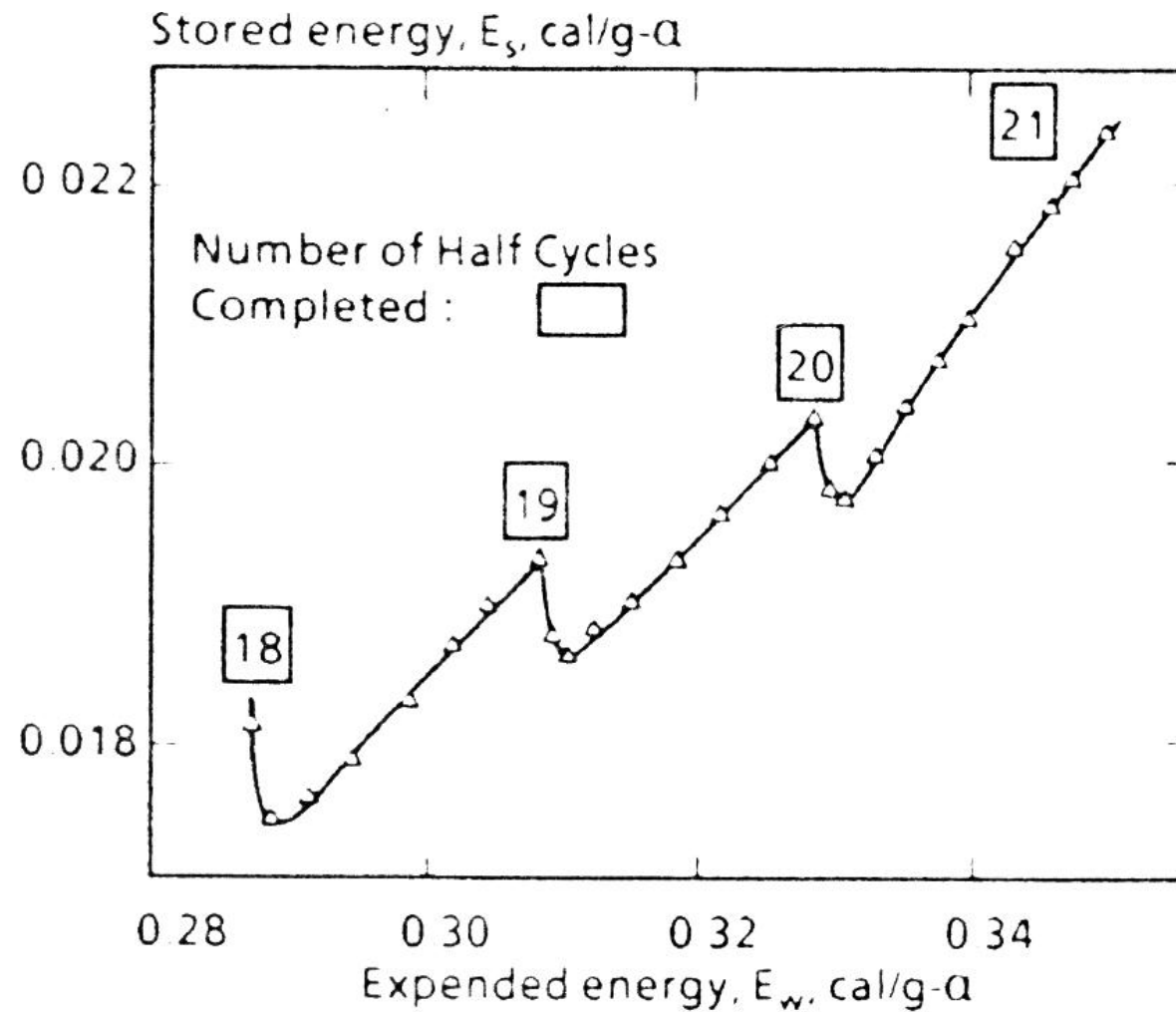


Tension curve

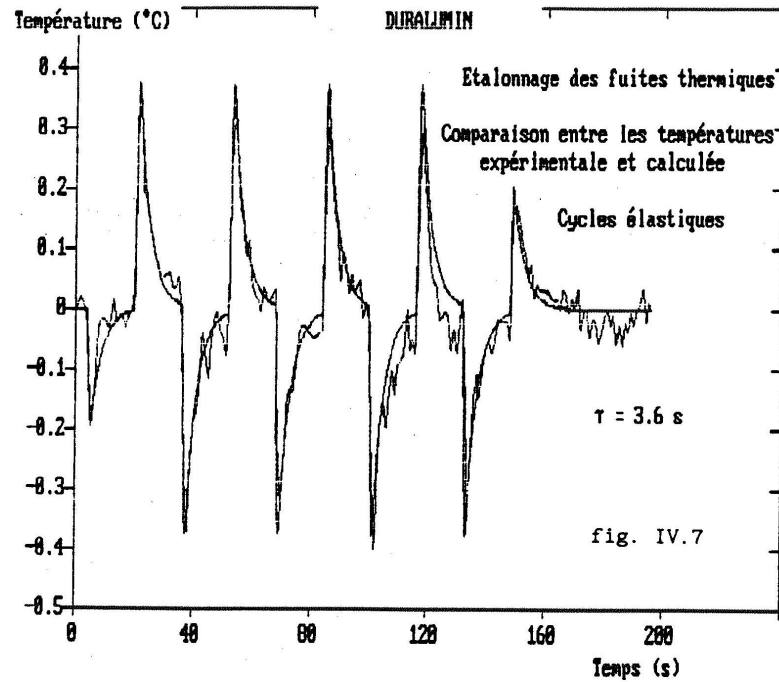


Fraction $F = W_{stored} / W_{plast}$

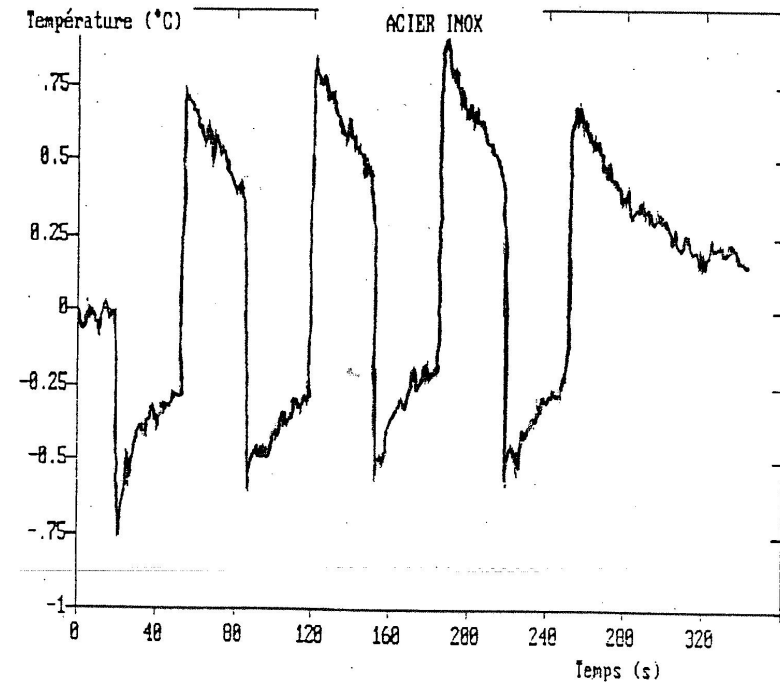
Cyclic loadings



Temperature change under cyclic loadings (1)



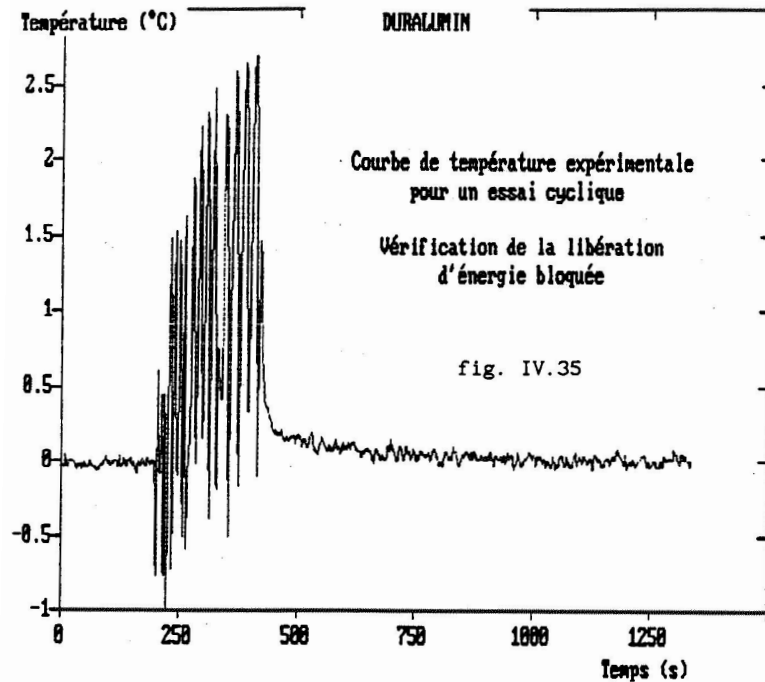
2024



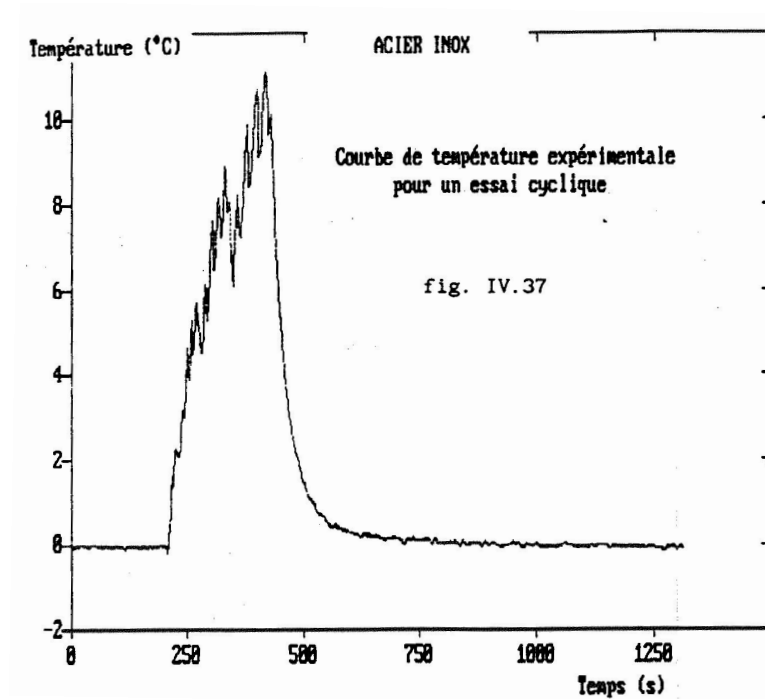
316 Stainless steel

Temperature variations for small elastic perturbations

Temperature change under cyclic loadings (2)



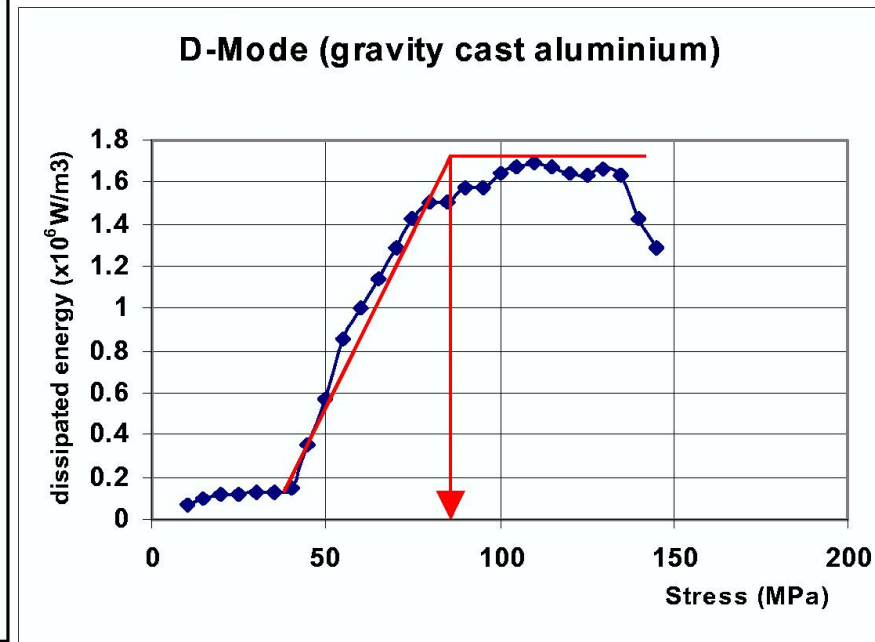
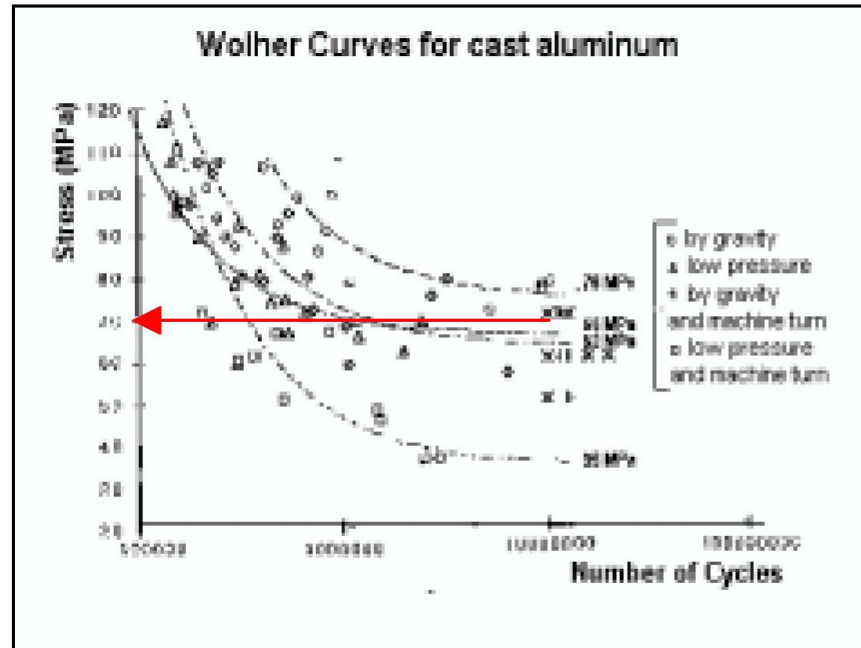
2024



316 Stainless steel

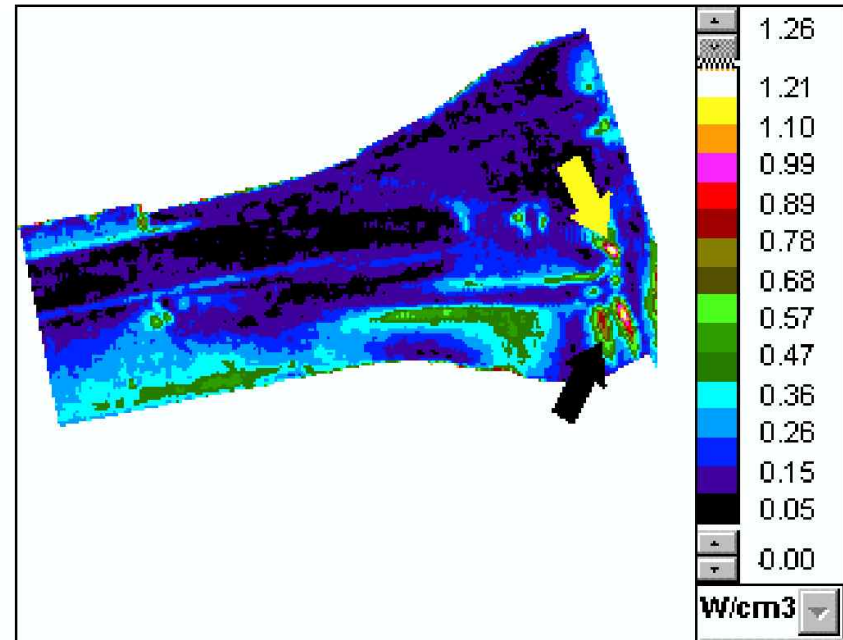
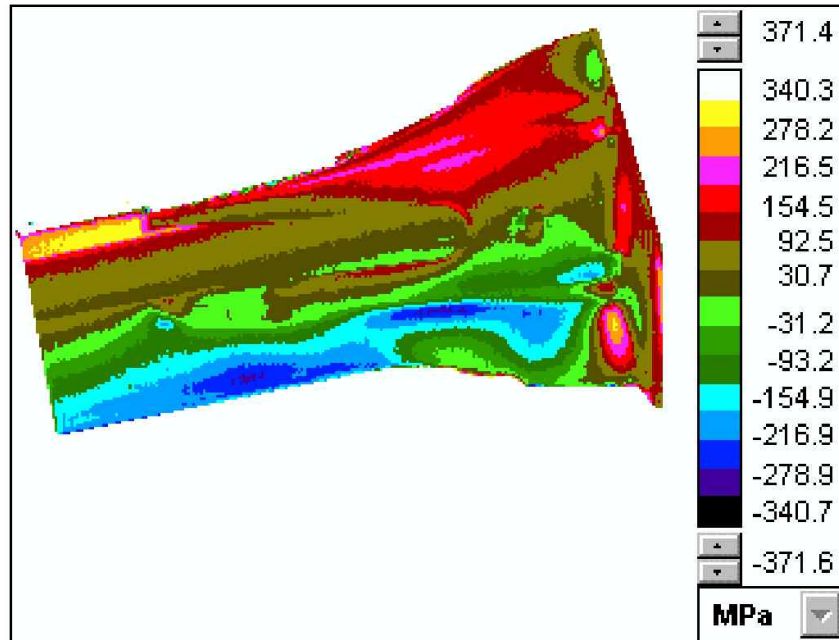
Temperature variations for cyclic elastoplastic loading under strain control

Determination of the fatigue limit



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Damage on a suspension arm



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Damage on a suspension arm



- Amount of plastic power providing heat generation

$$\Phi_1 = \underline{\underline{\sigma}} : \underline{\underline{\dot{\xi}}}^P - Y_I \dot{\alpha}_I = \beta \underline{\underline{\sigma}} : \underline{\underline{\dot{\xi}}}^P$$

- Physical meaning of hardening :
Hardening is the capability to store energy
- Stored forever: isotropic hardening
- Recoverable: kinematic hardening

–Track the energy in the plastic process–