Various unified models and their identification

- **Isotropic/kinematic hardening in non-pro loading**
- **The most common effects in real world material**
  - ⋆ **Cyclic hardening curve**
  - ⋆ **Plastic effects: criterion, hardening rules**
  - ⋆ **Viscous effects**
- **Case study: identification on a GS cast iron**

–Browse behaviors of real world material–

Various unified models and their identification

- *Isotropic/kinematic hardening in non-pro loading*
  MORE on http://mms.ensmp.fr/Quizz/TTC-BIAX/index.html (sorry, still in french only)

- **The most common effects in real world material**
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Tension and shear, isotropic hardening

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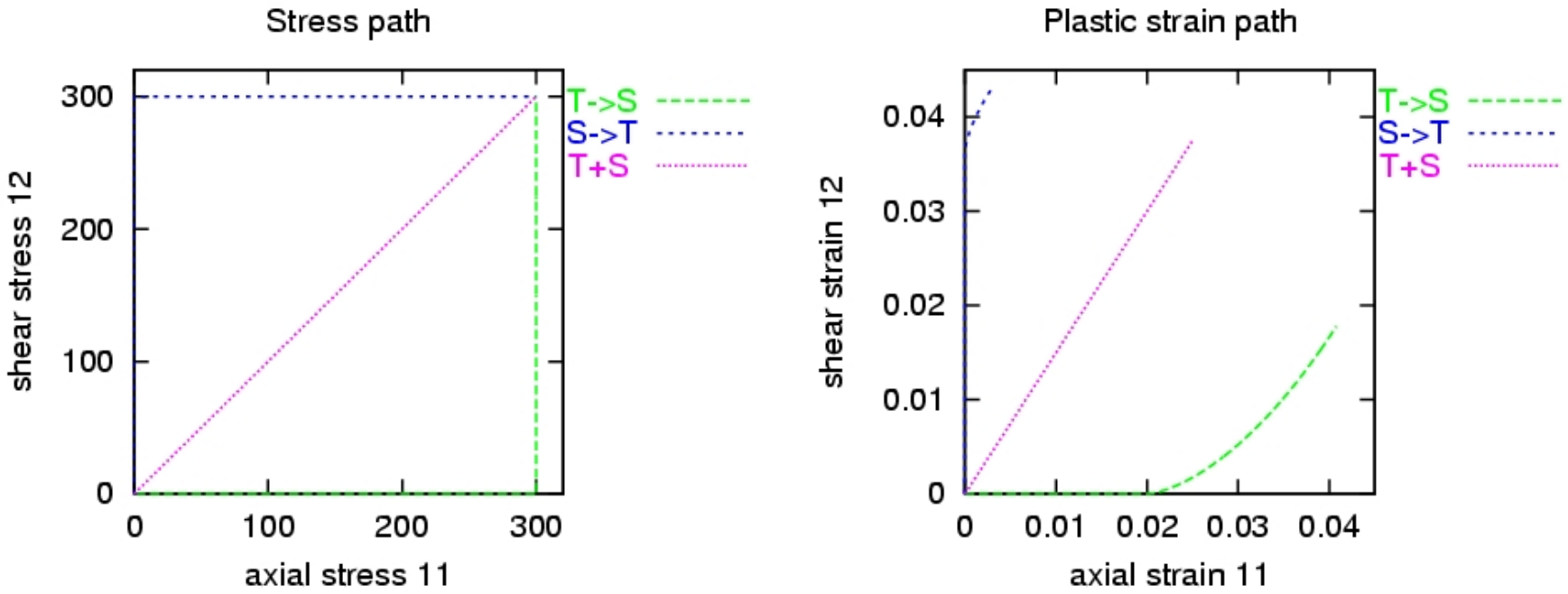
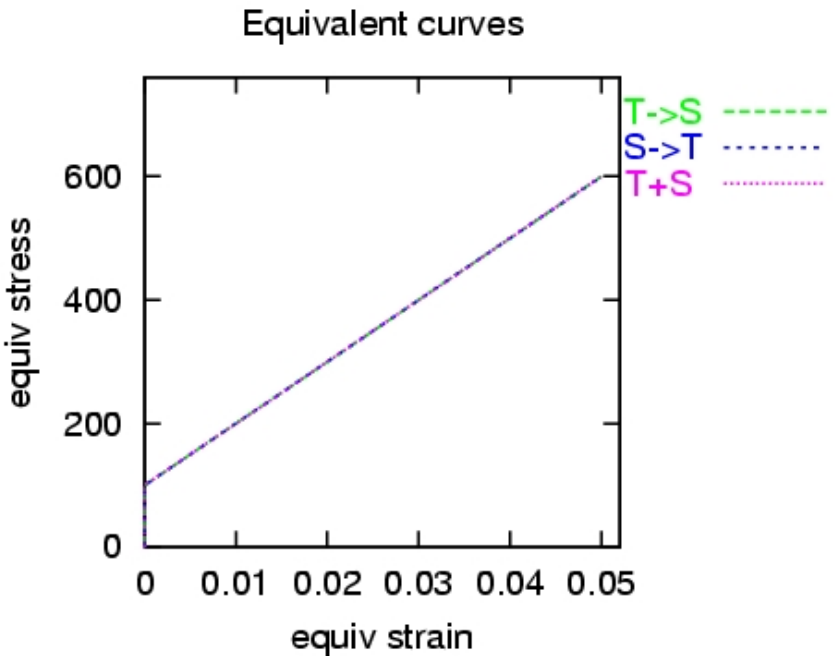
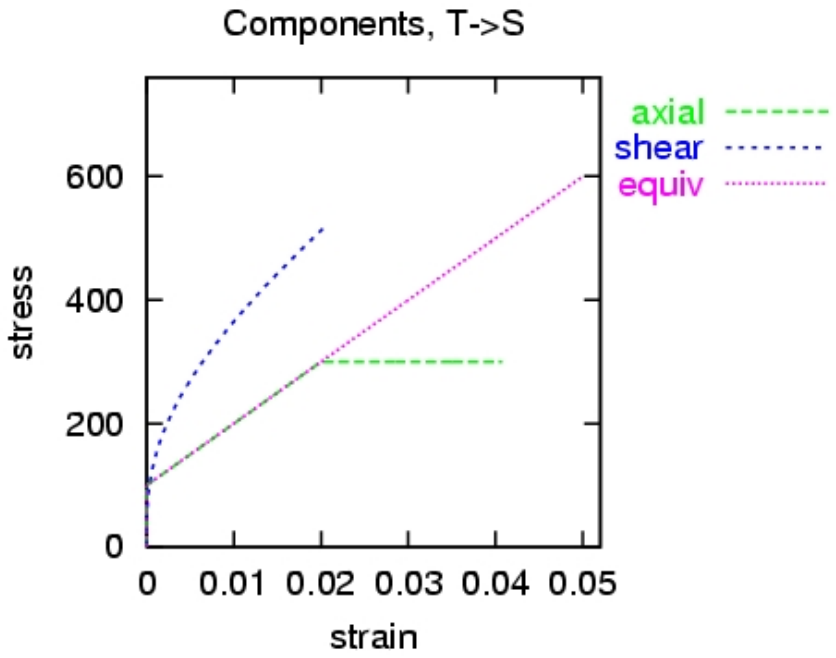
Stress and plastic strain paths

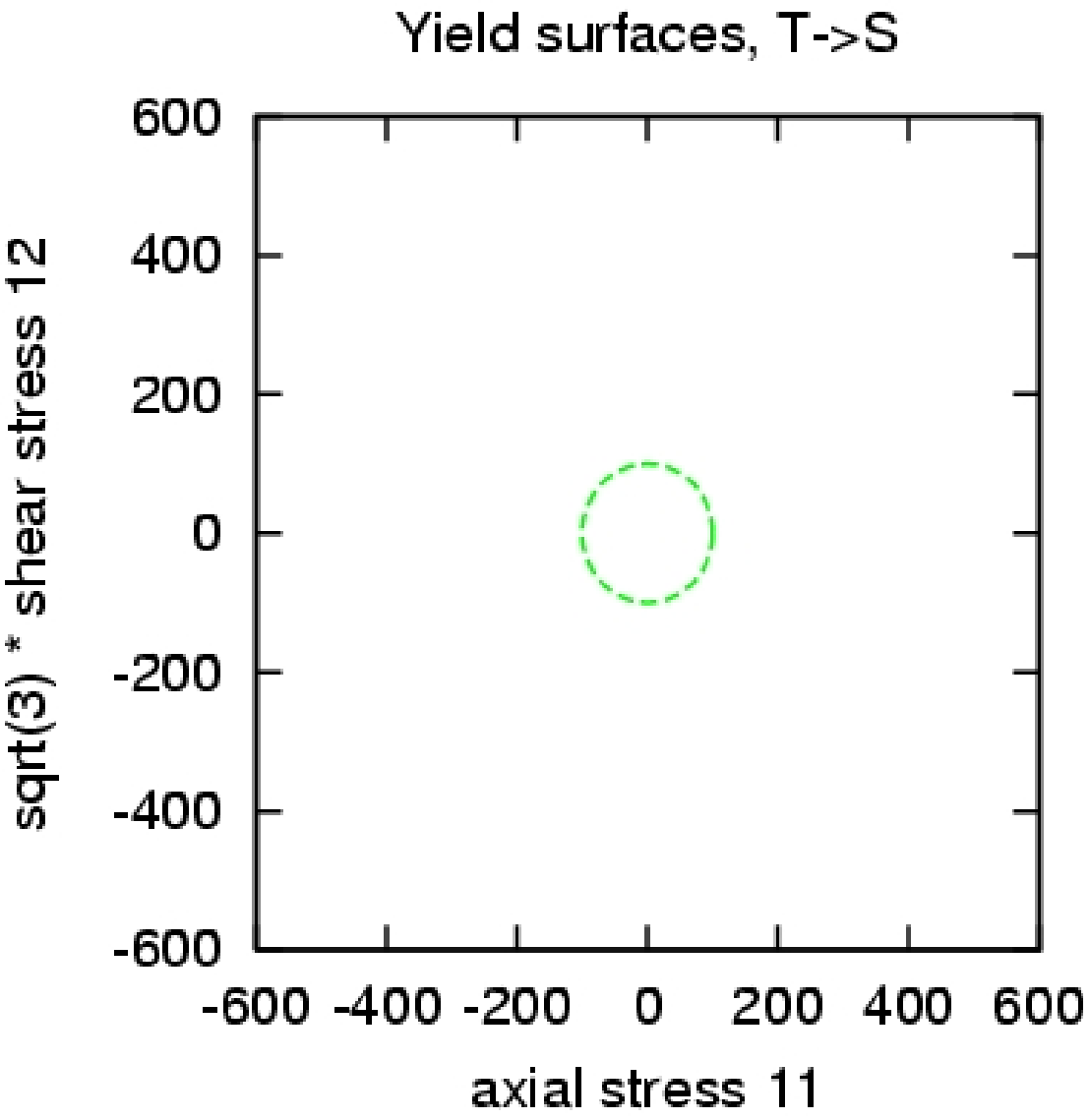


T-S Stress and plastic strain paths





T-S Yield surface evolution



summary

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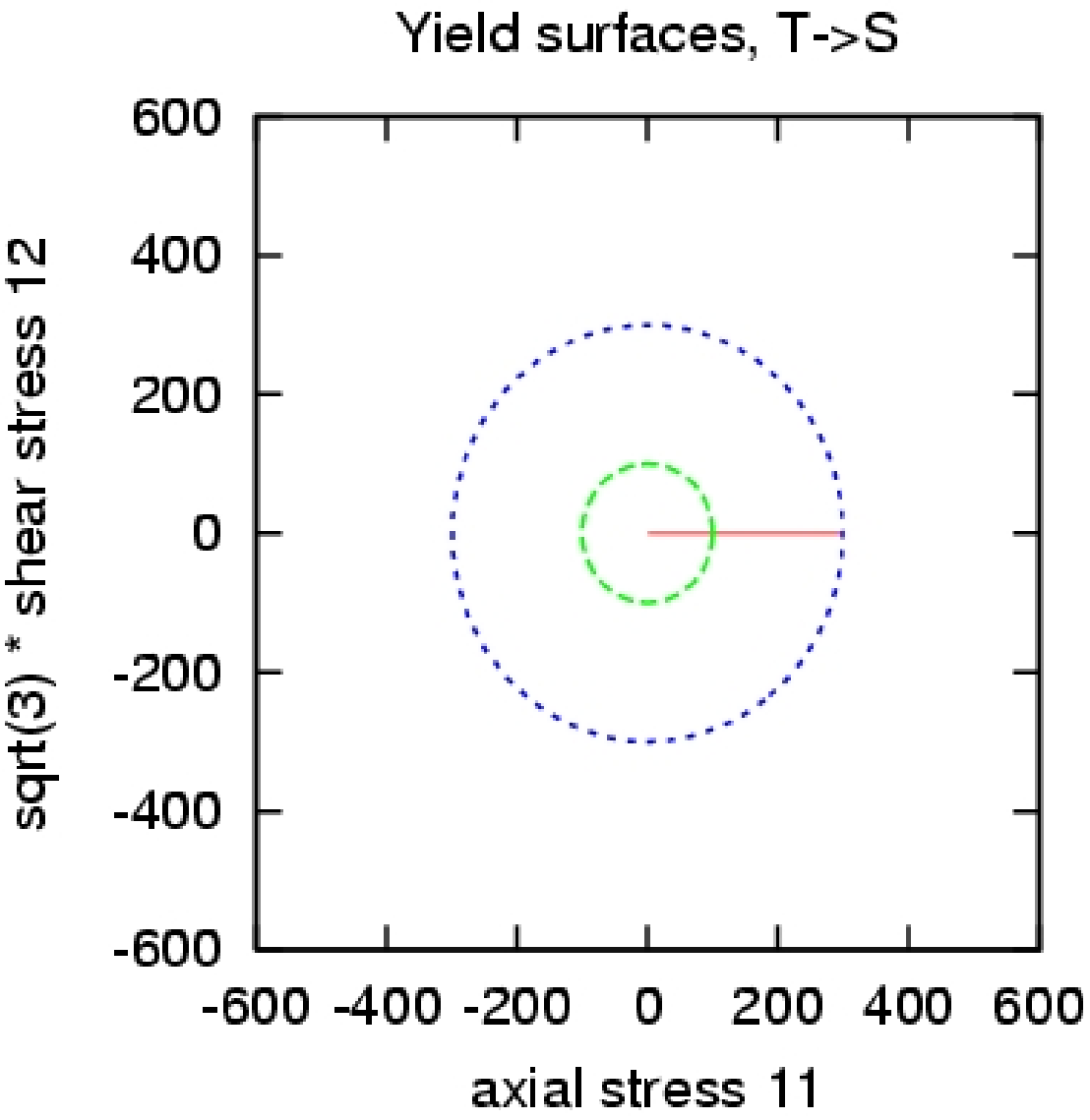

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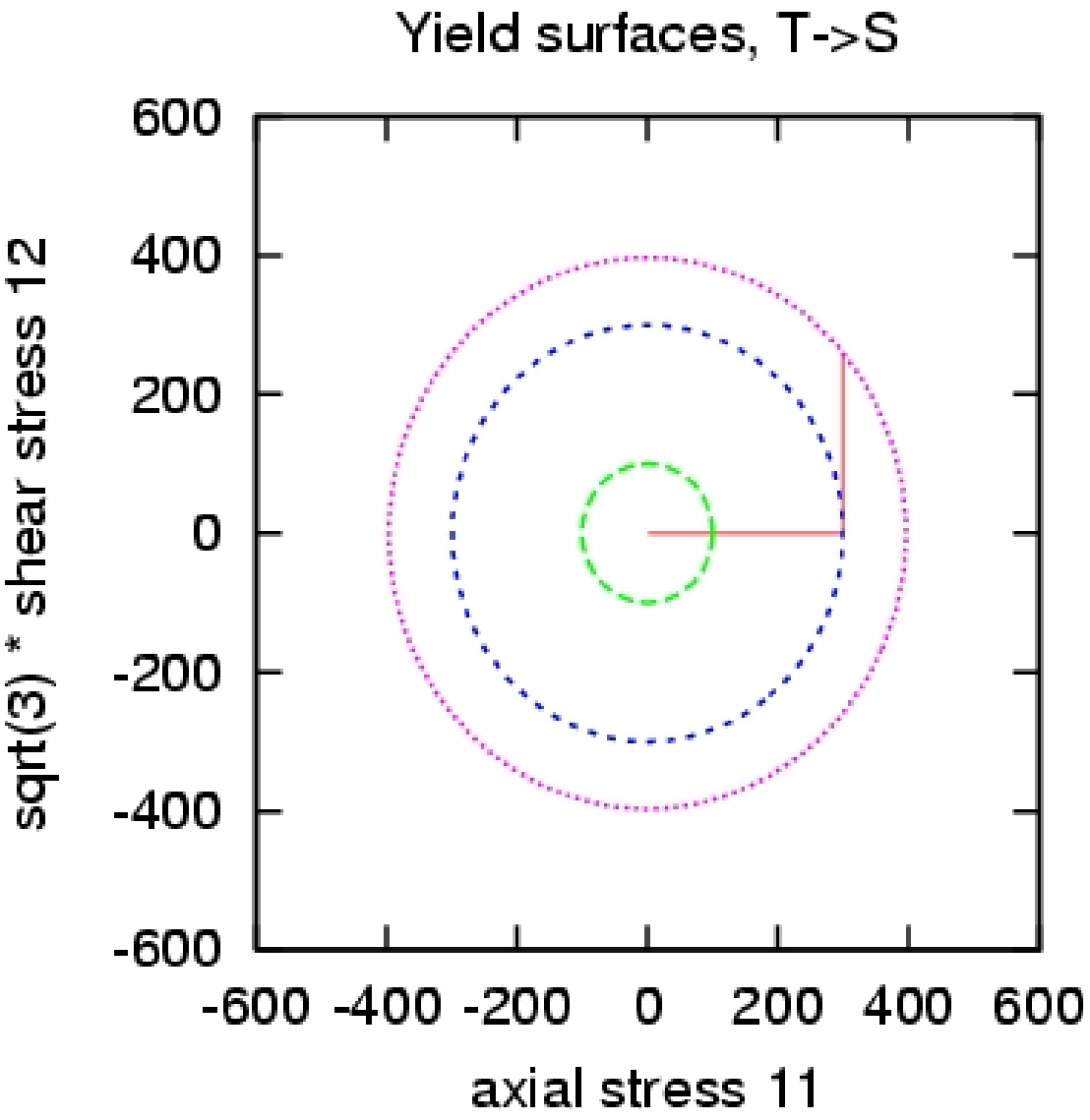

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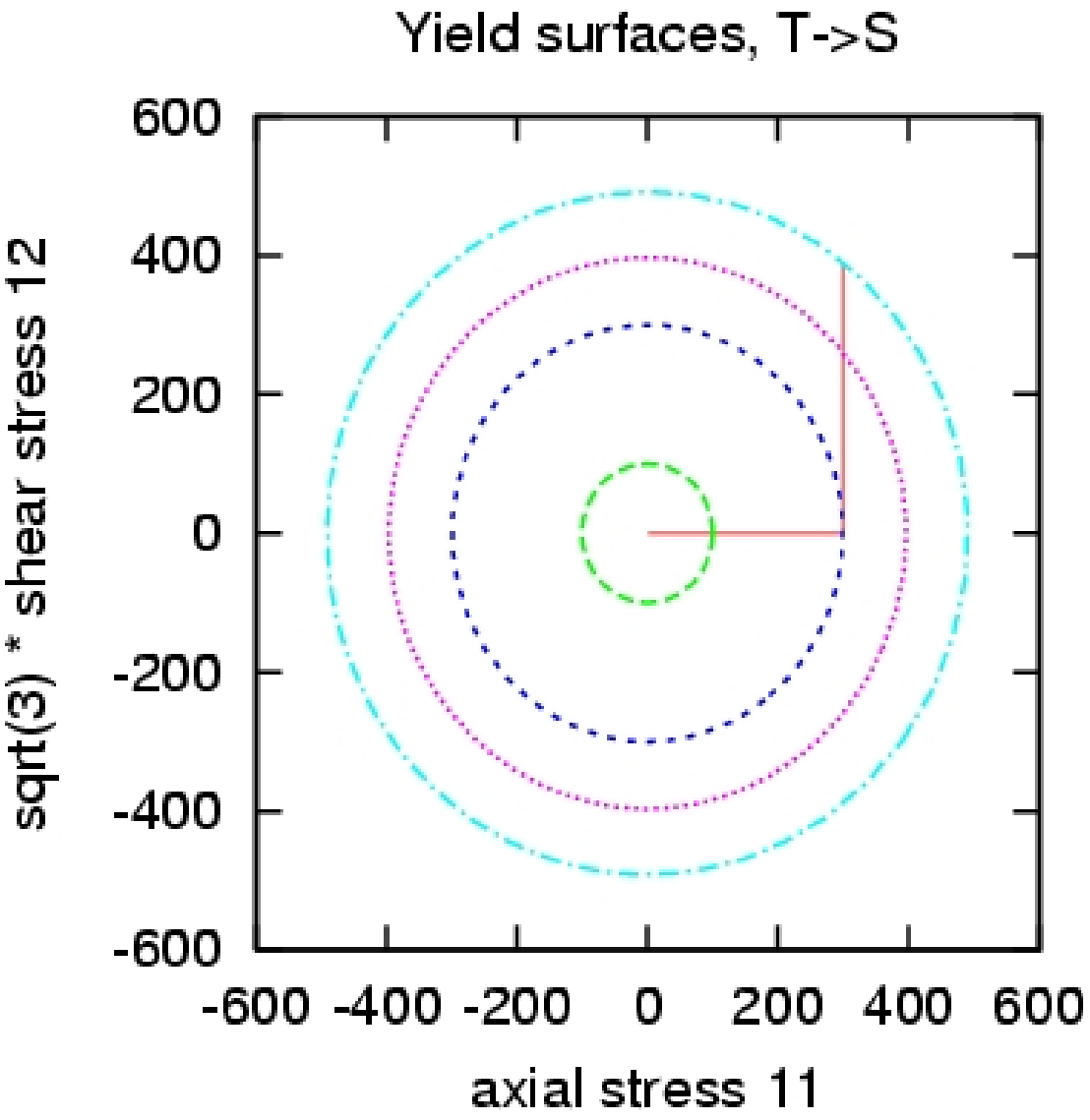

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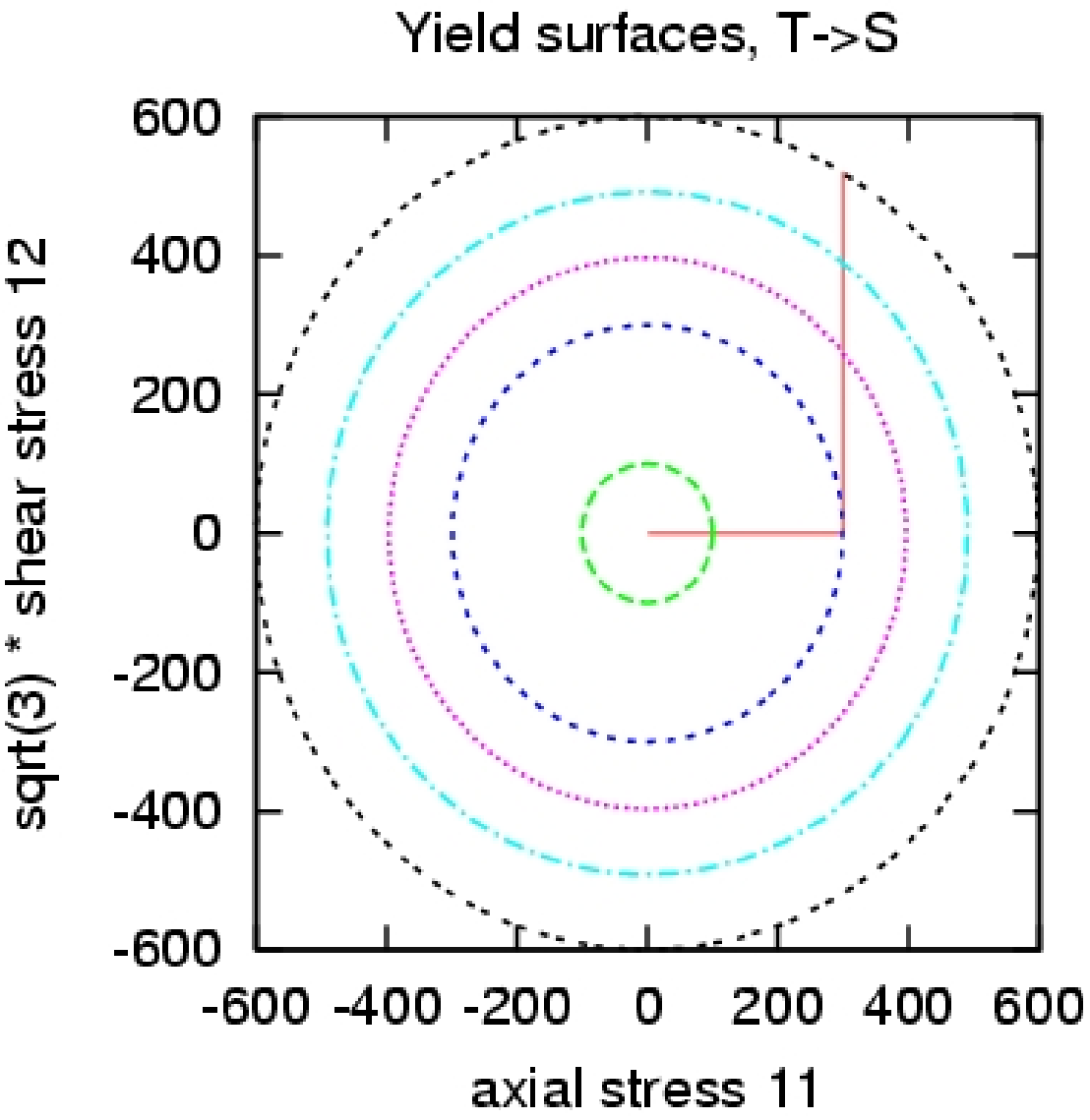

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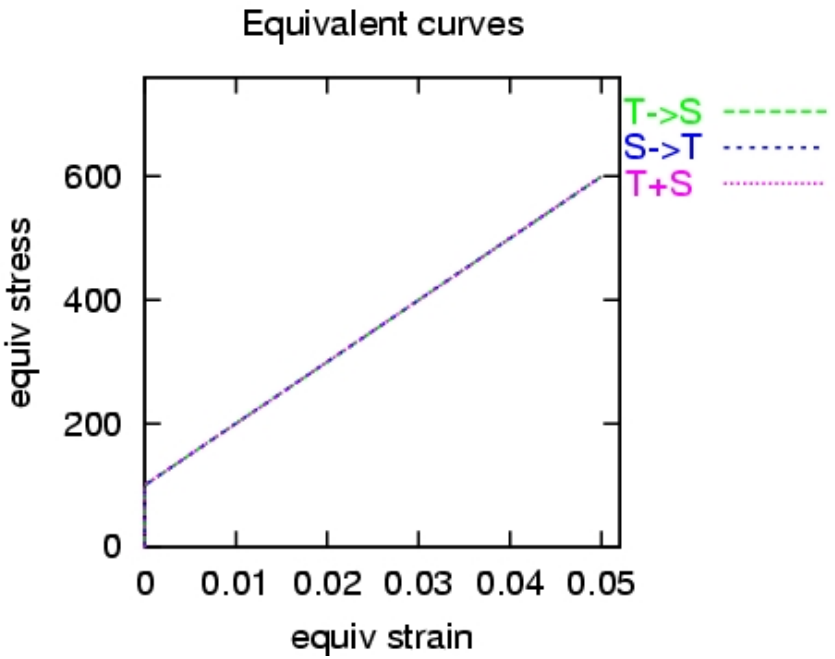
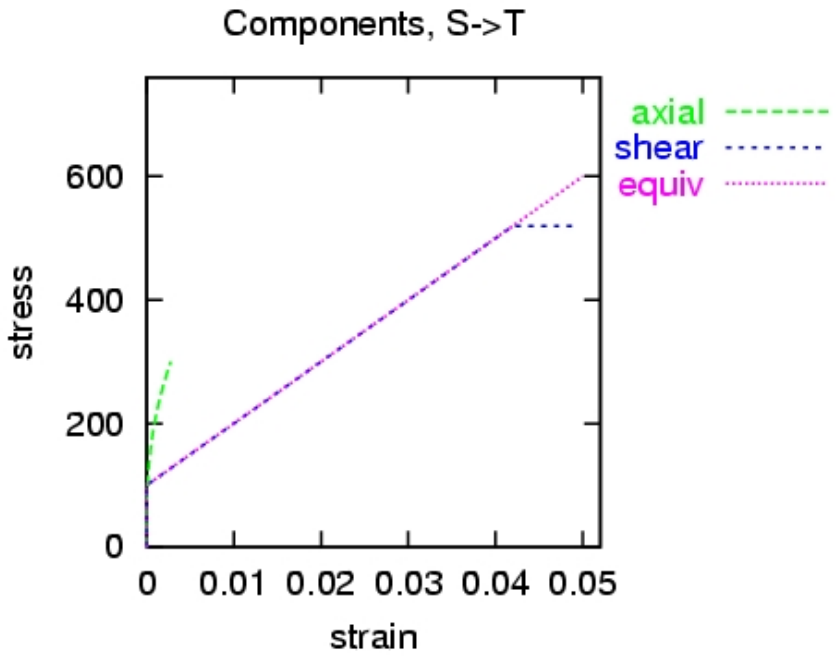
S-T Stress and plastic strain paths





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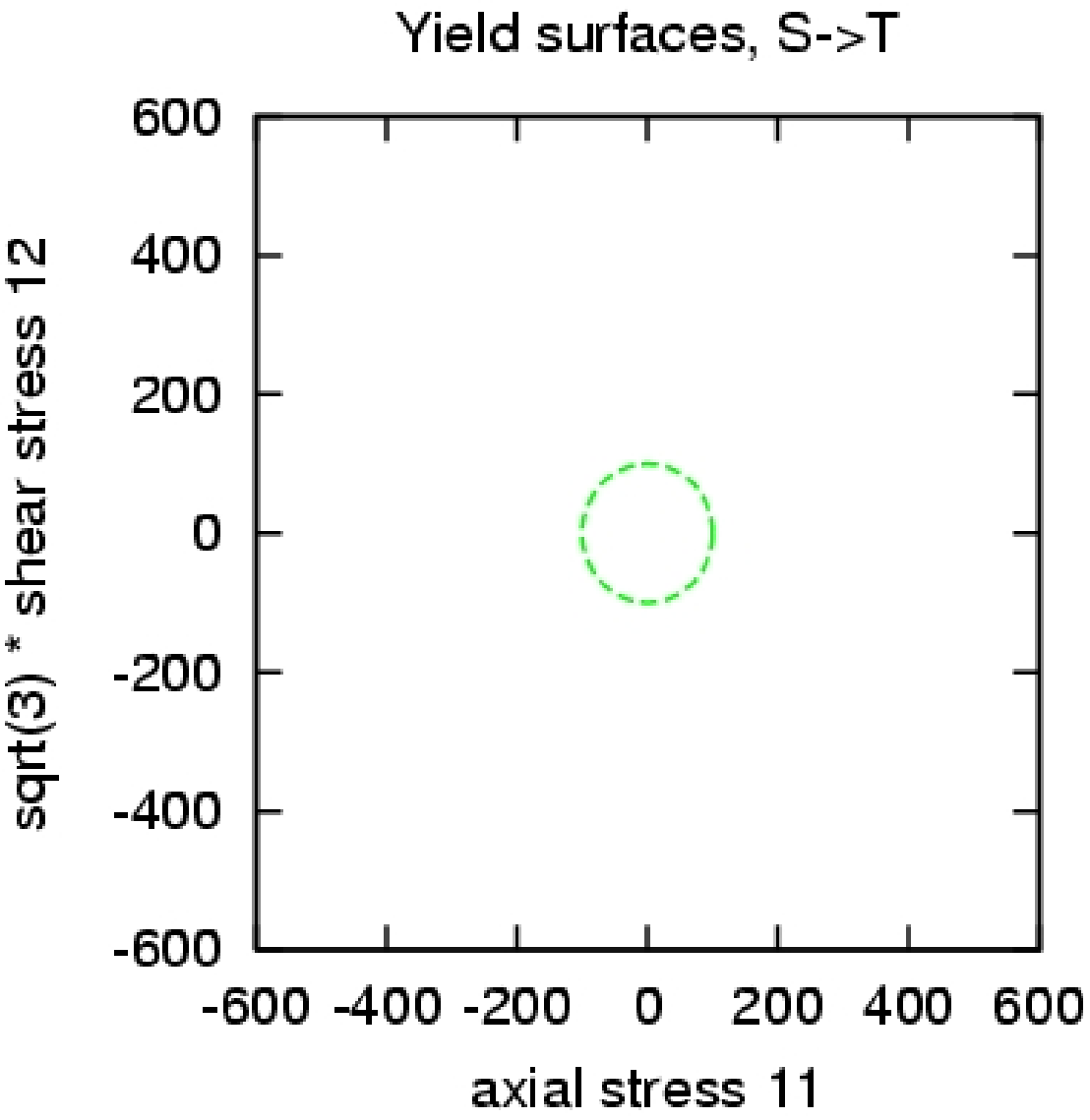

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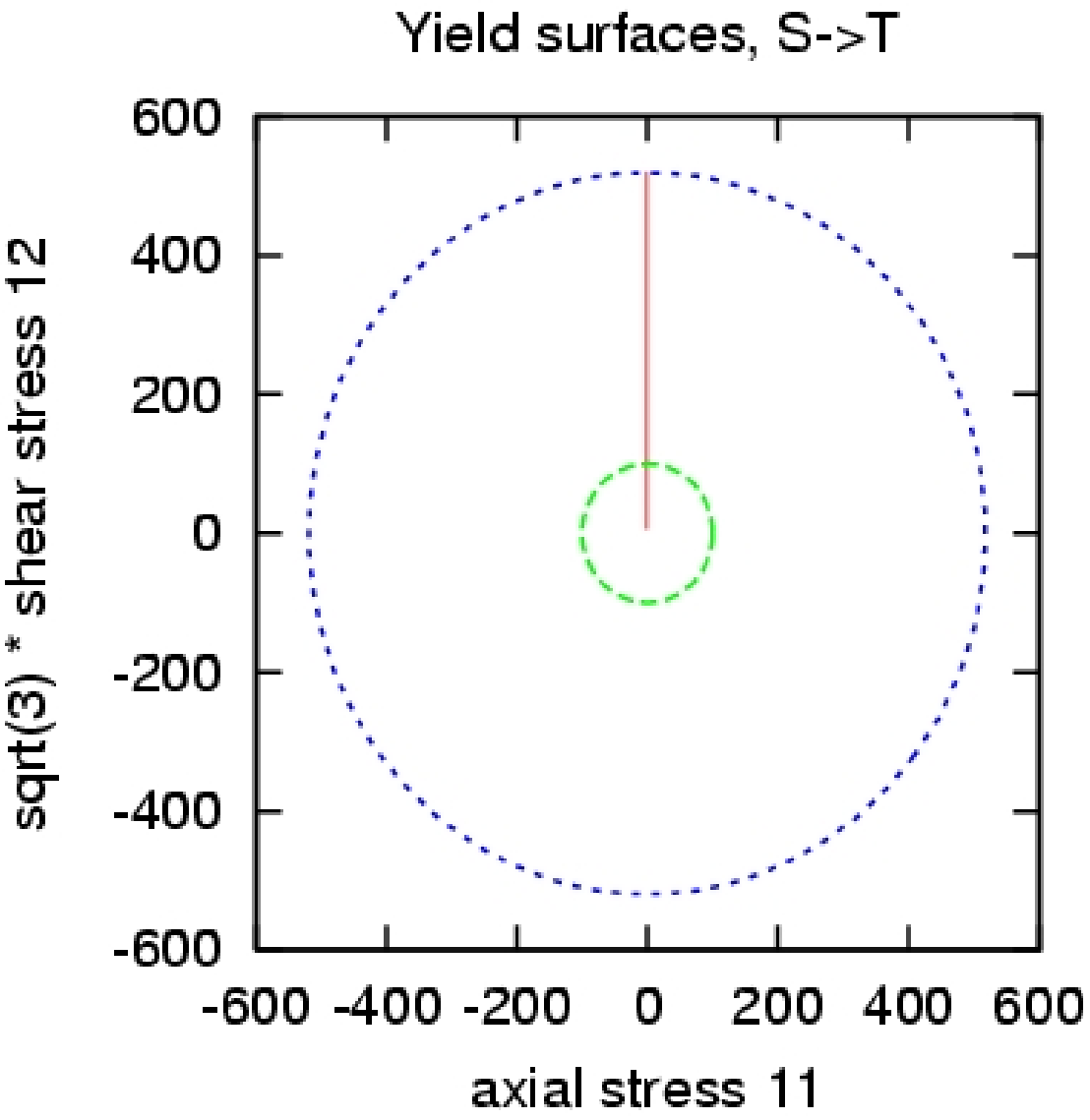

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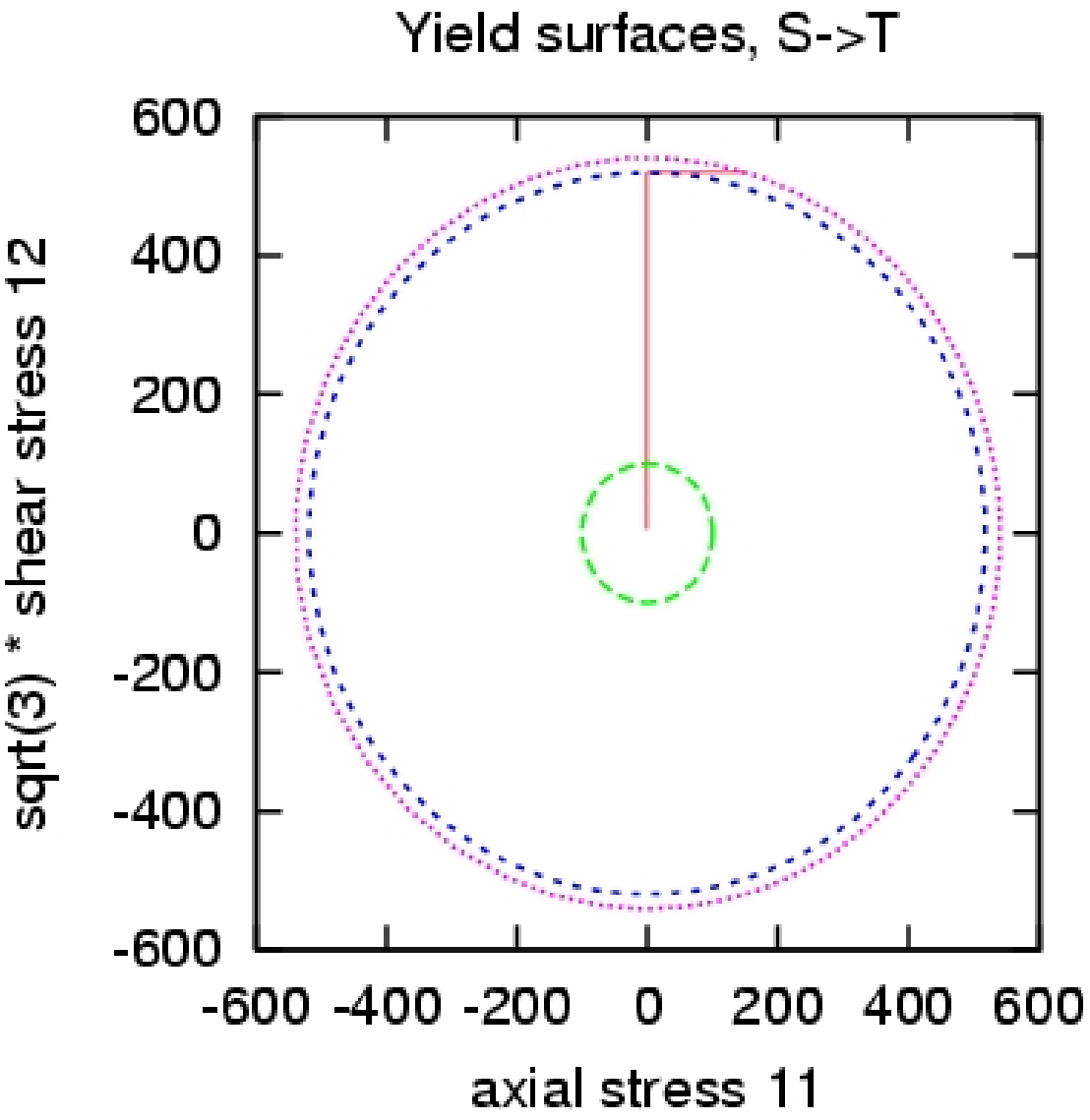

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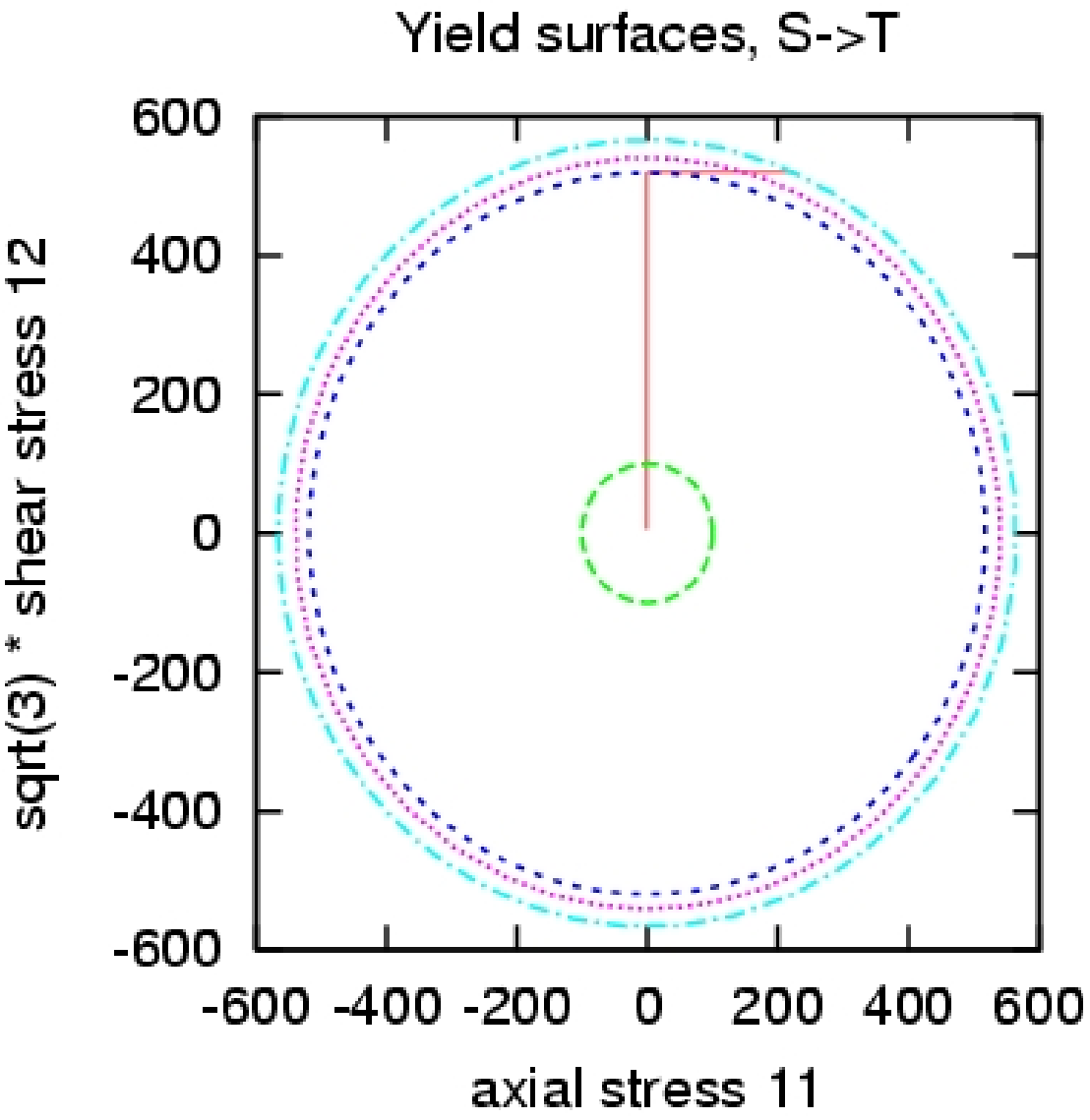

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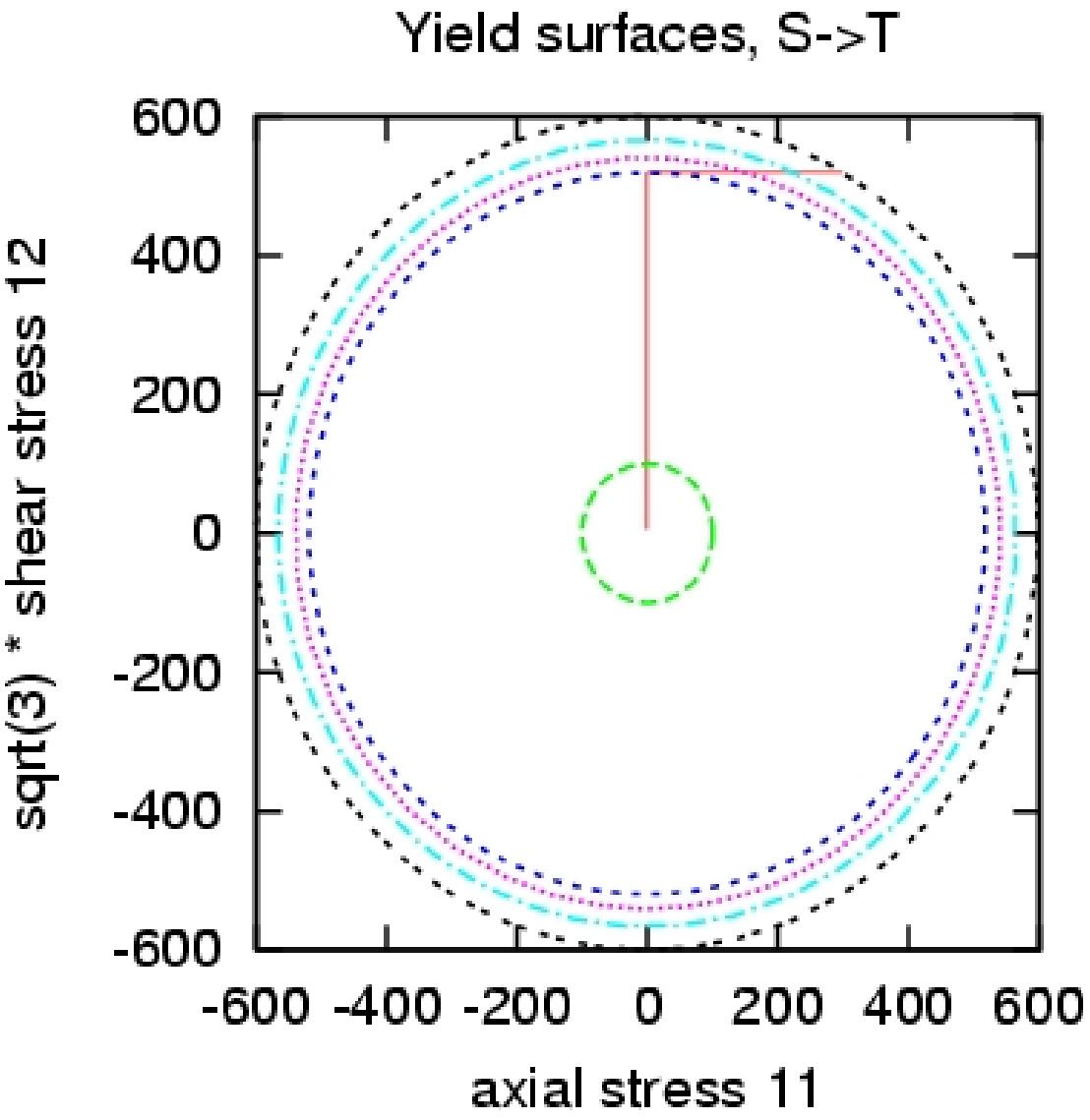
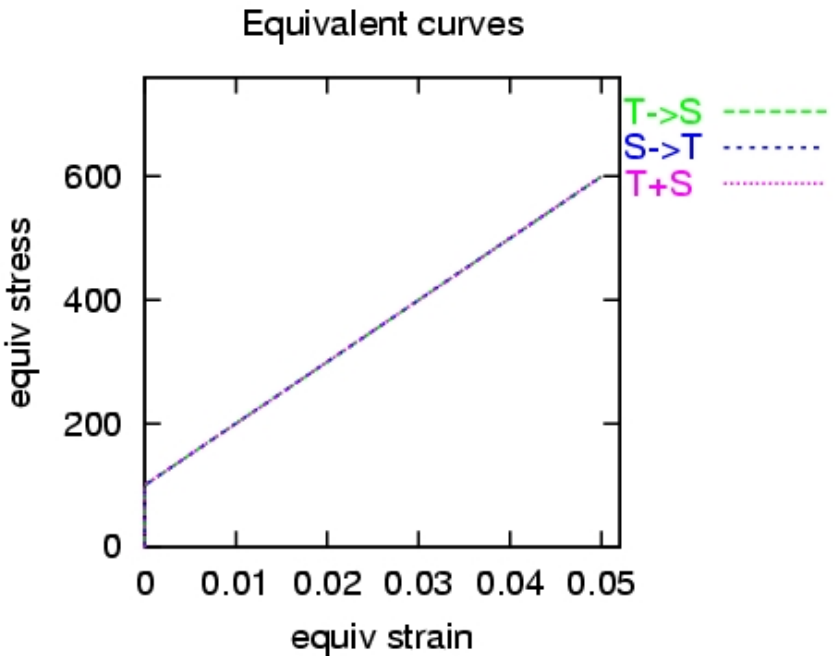
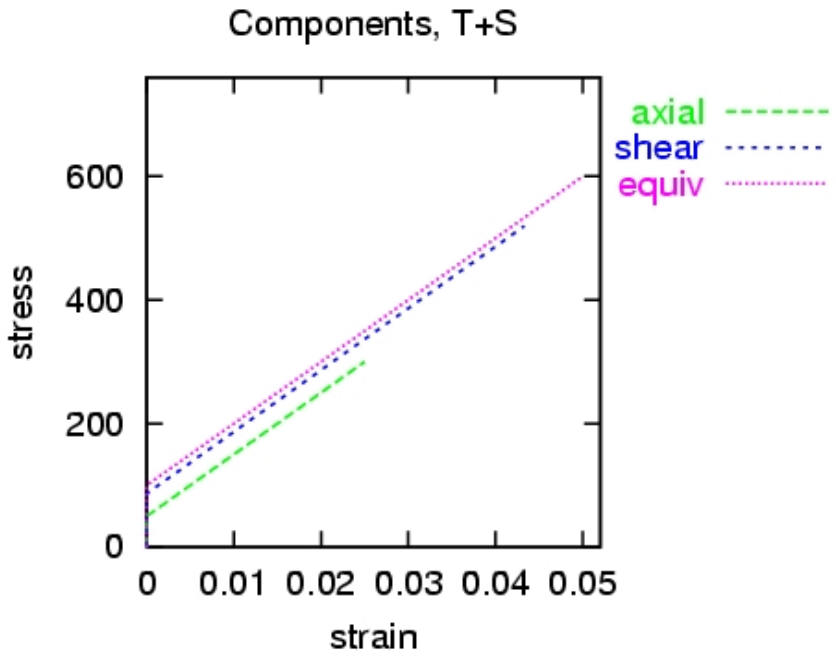


summary

T+S Stress and plastic strain paths





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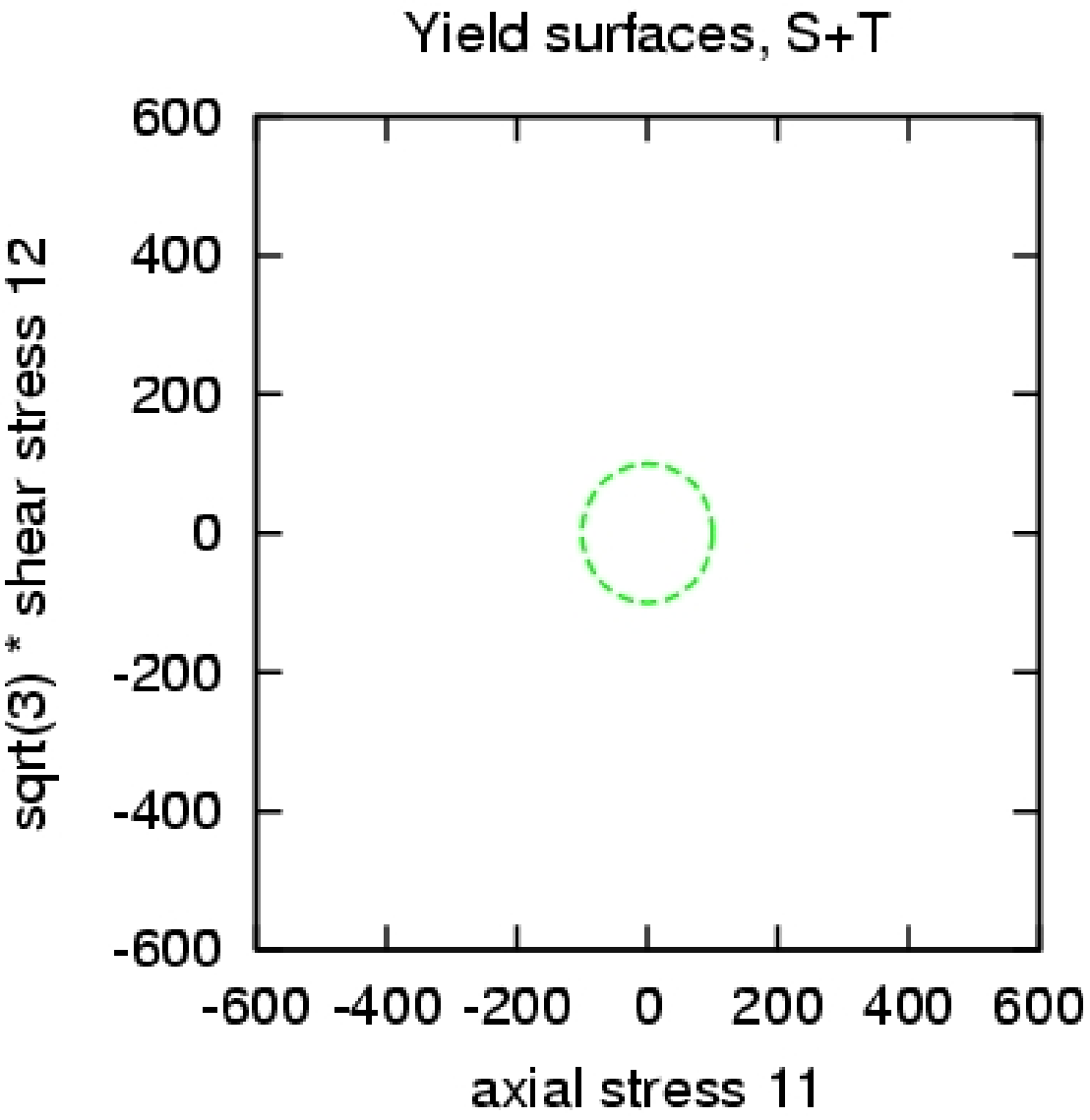

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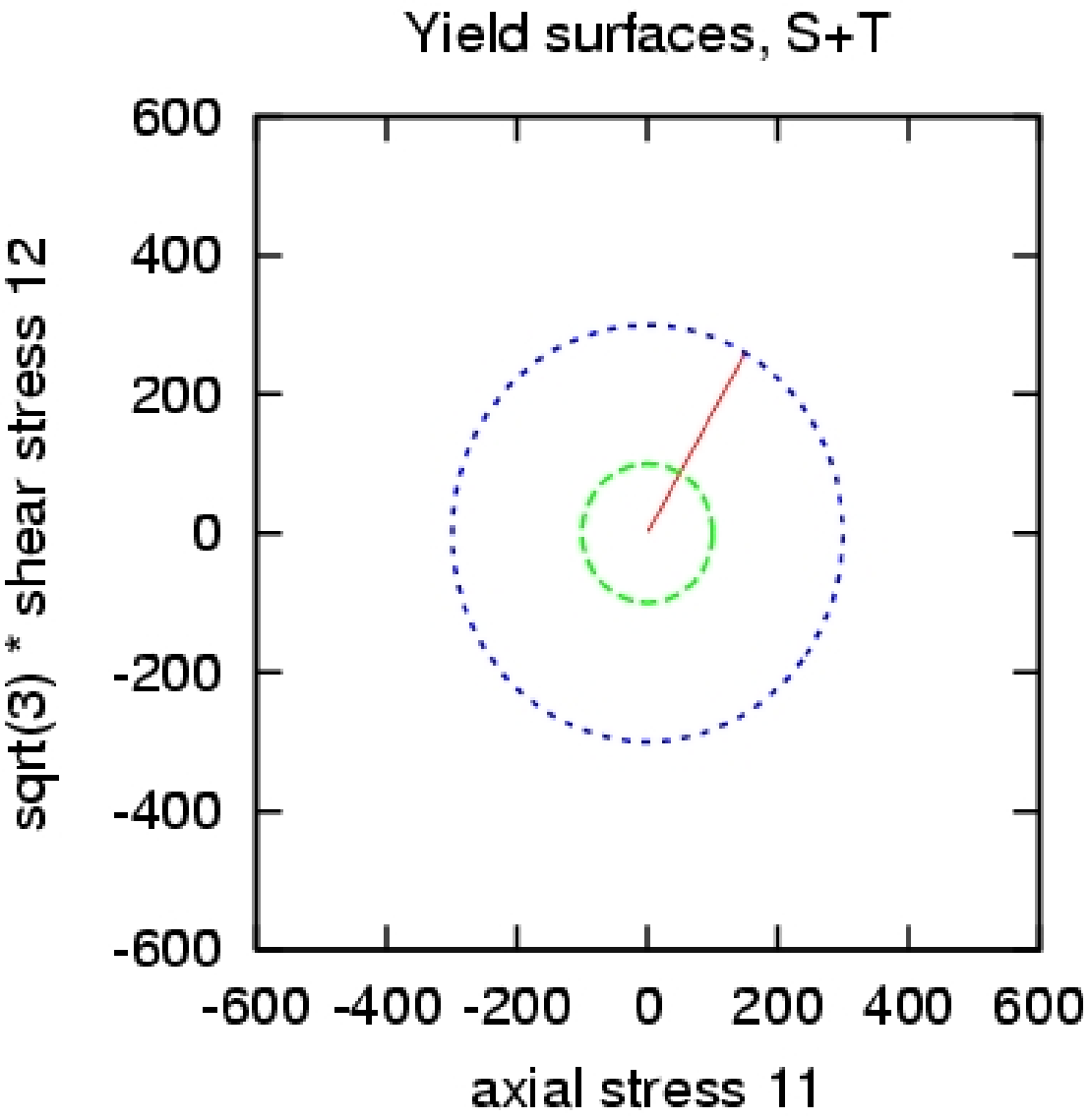

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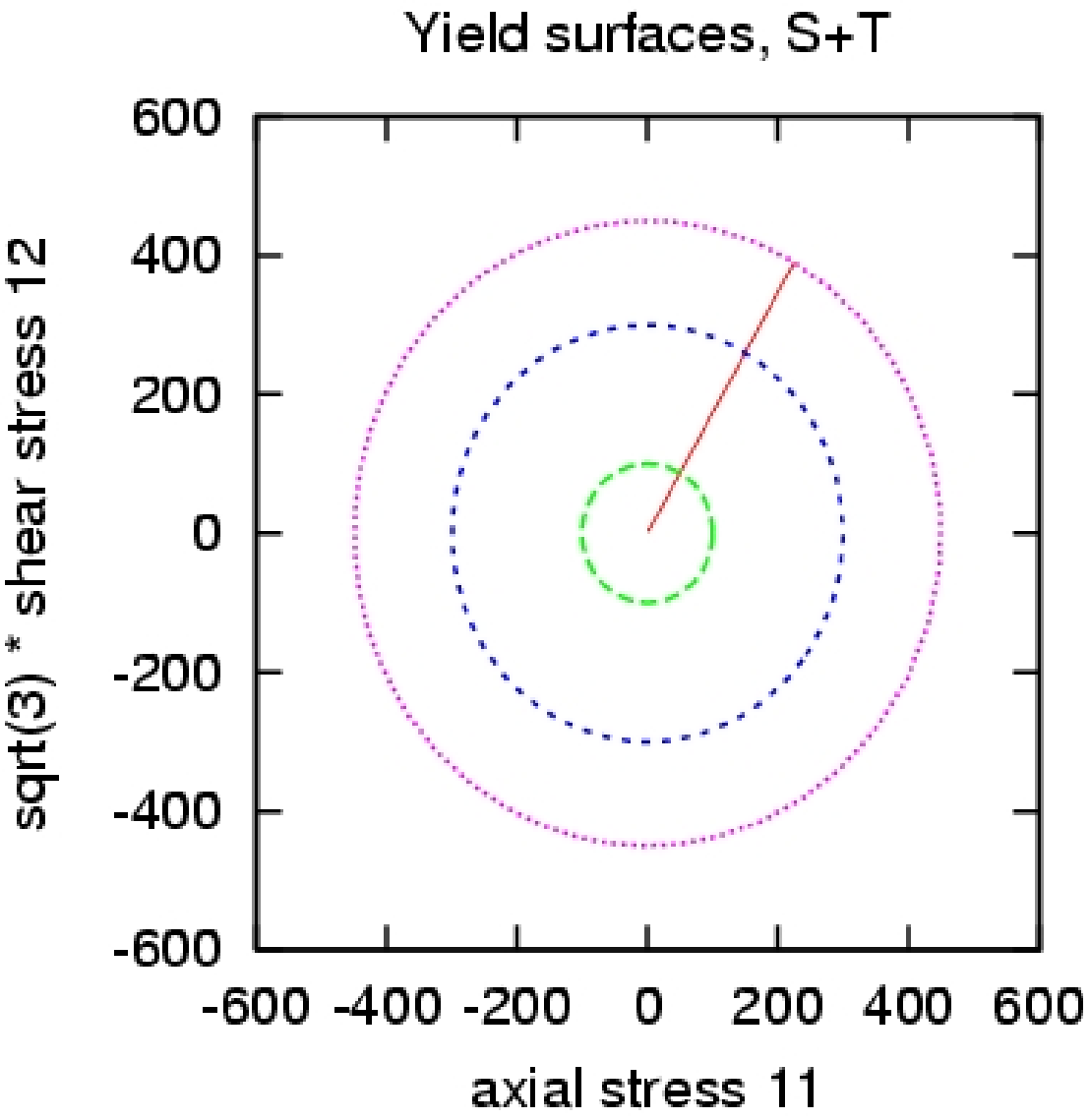

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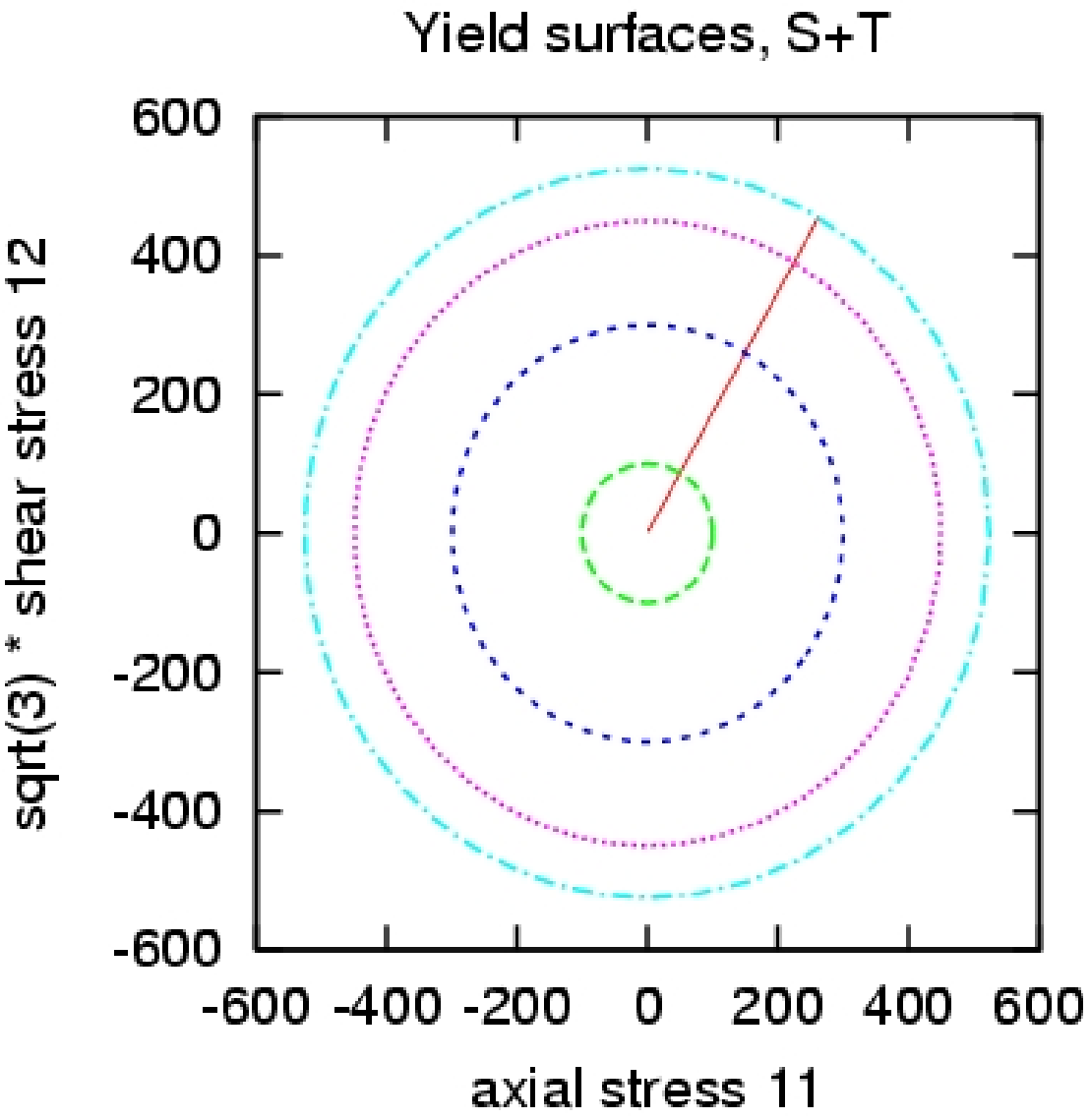

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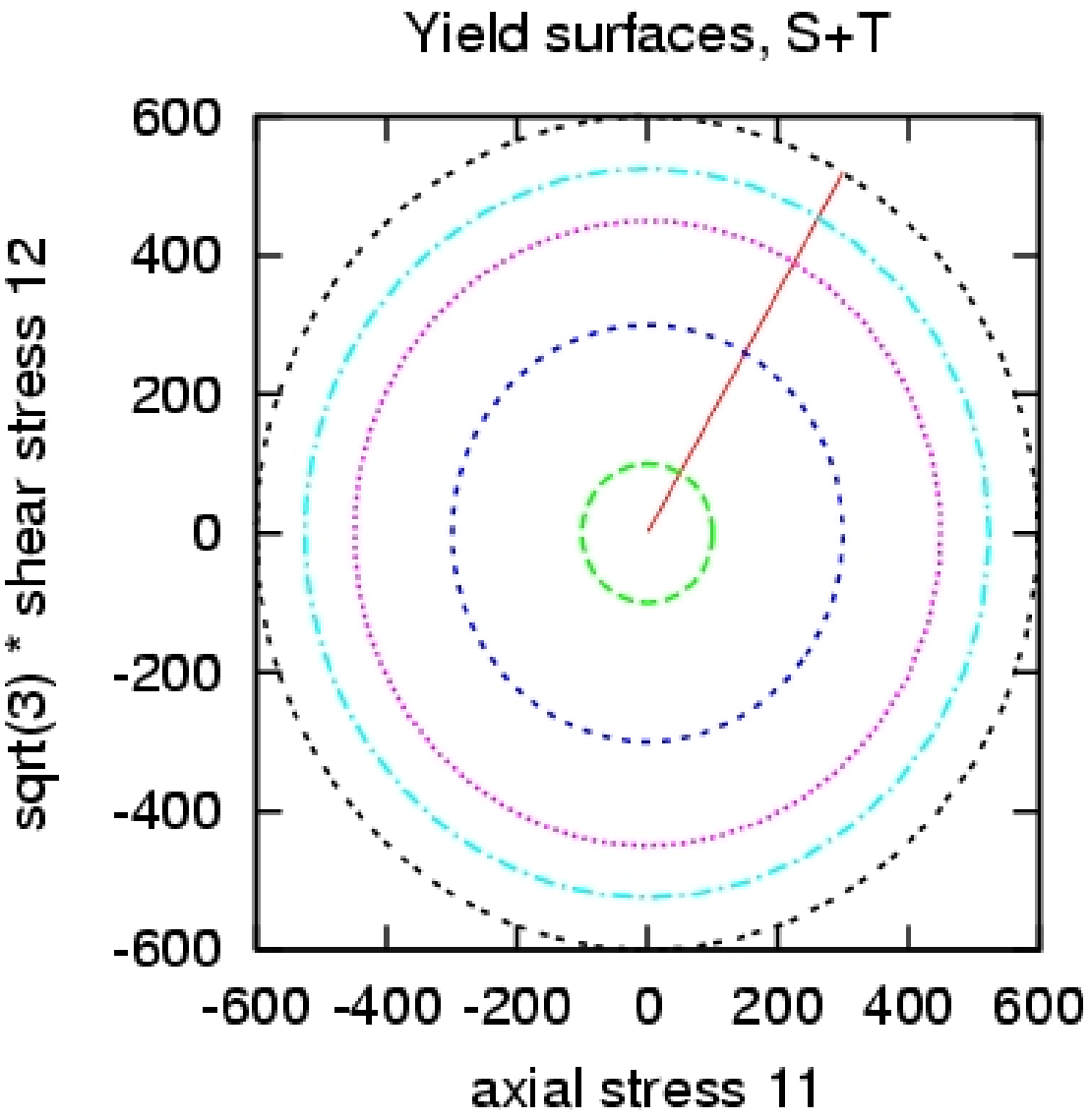

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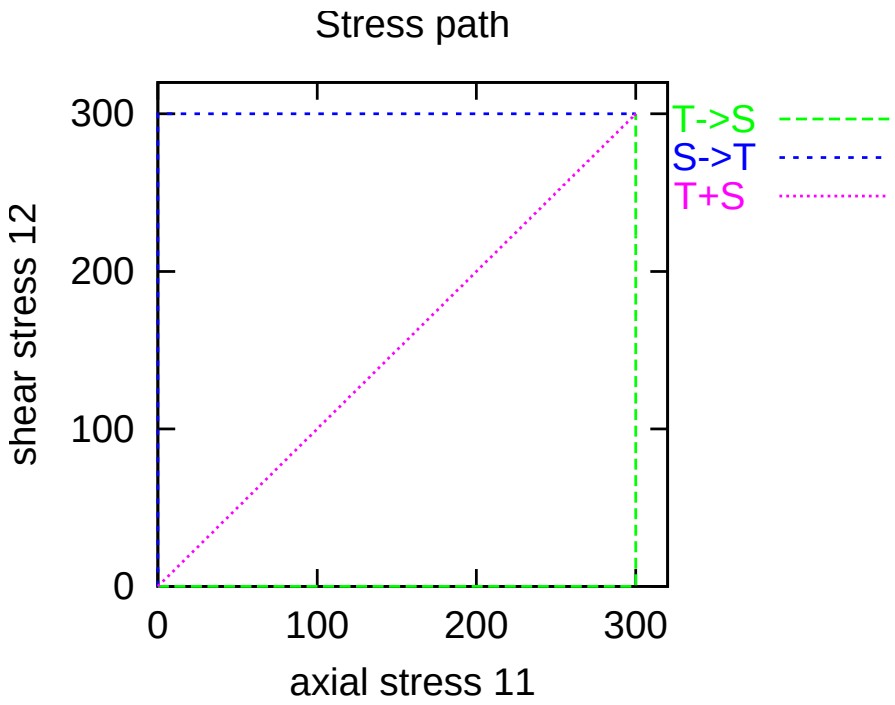
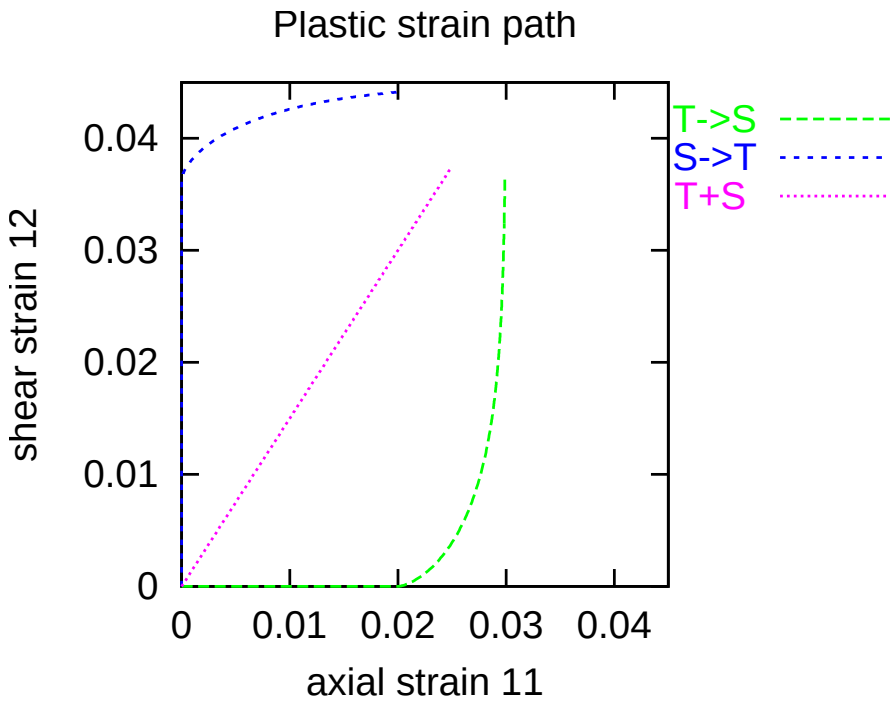
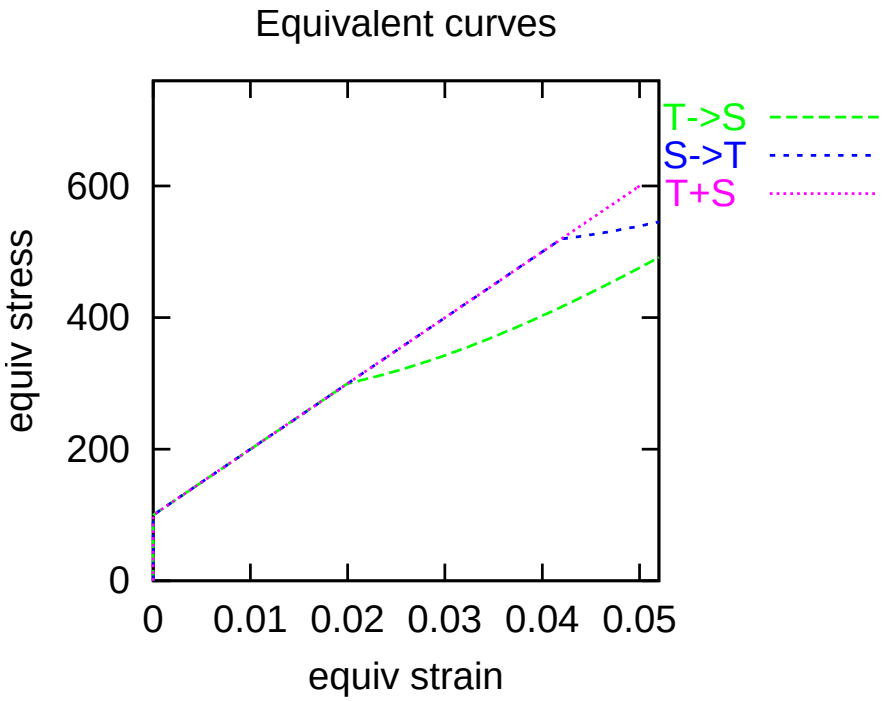
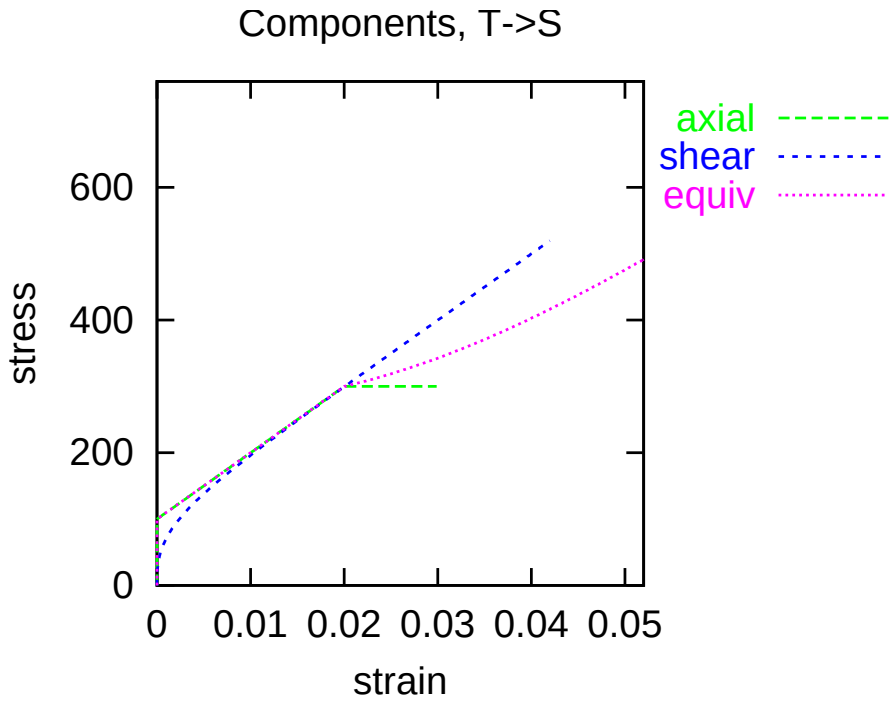




T-S Stress and plastic strain paths





T-S Yield surface evolution

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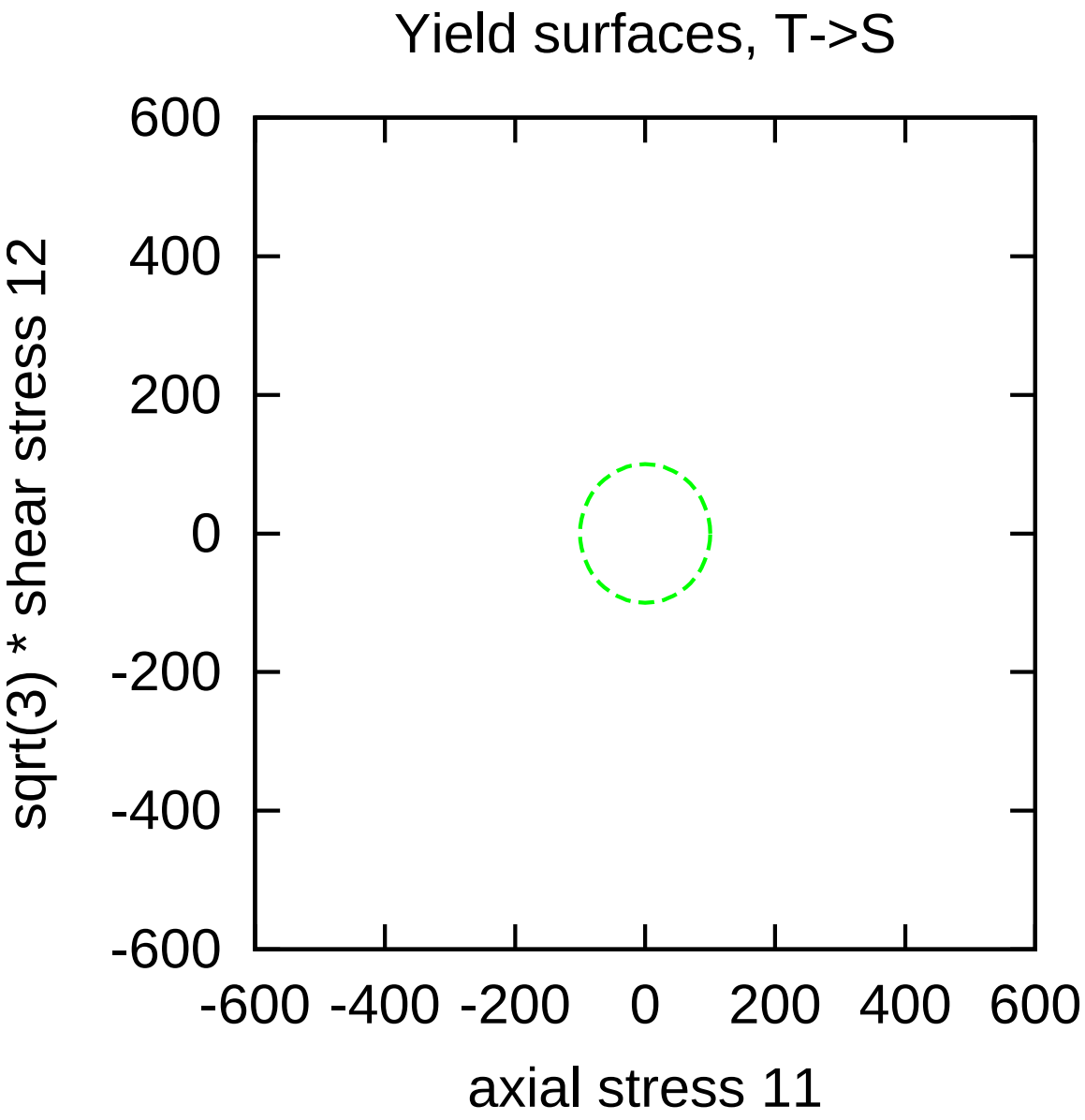

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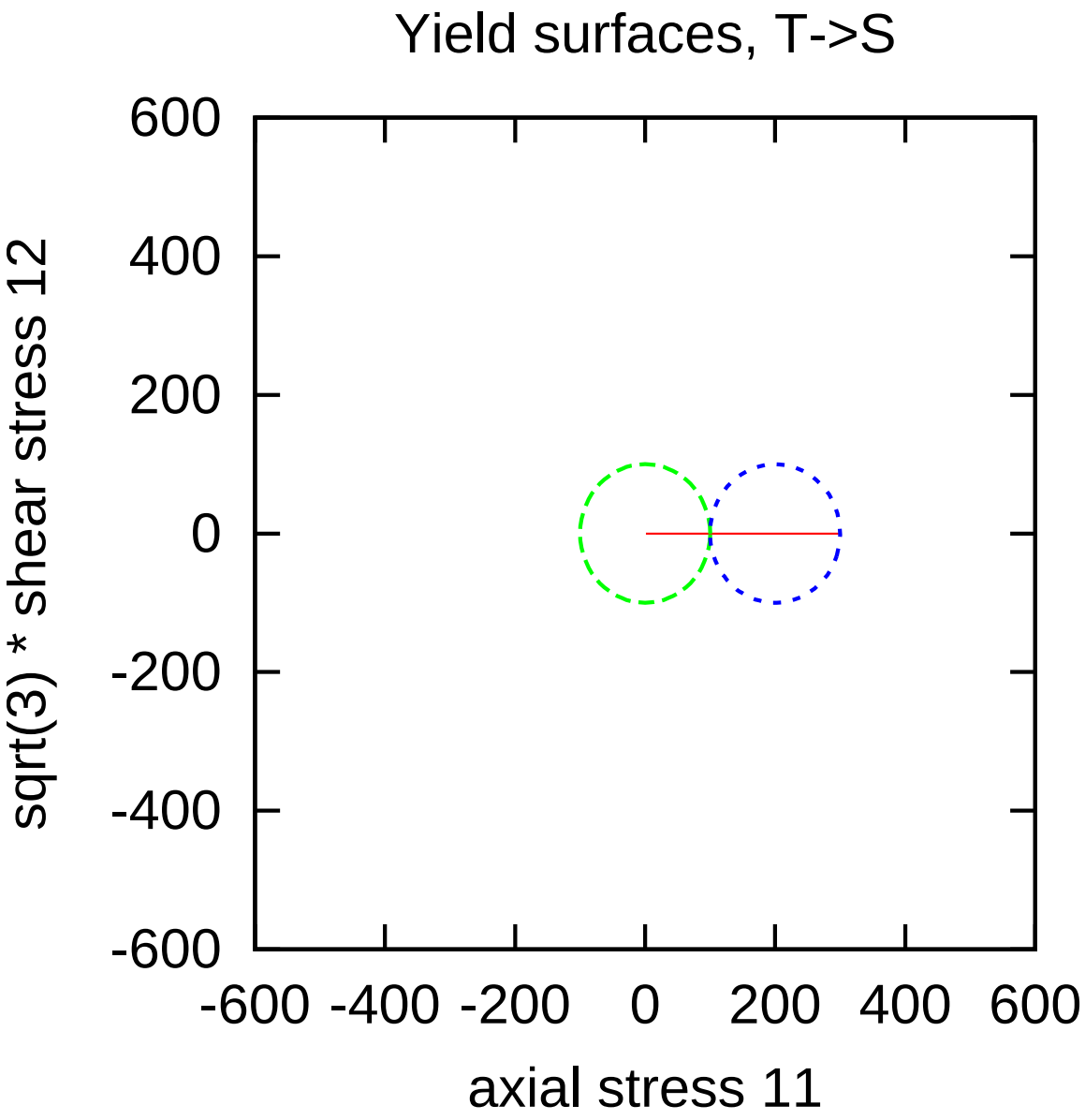

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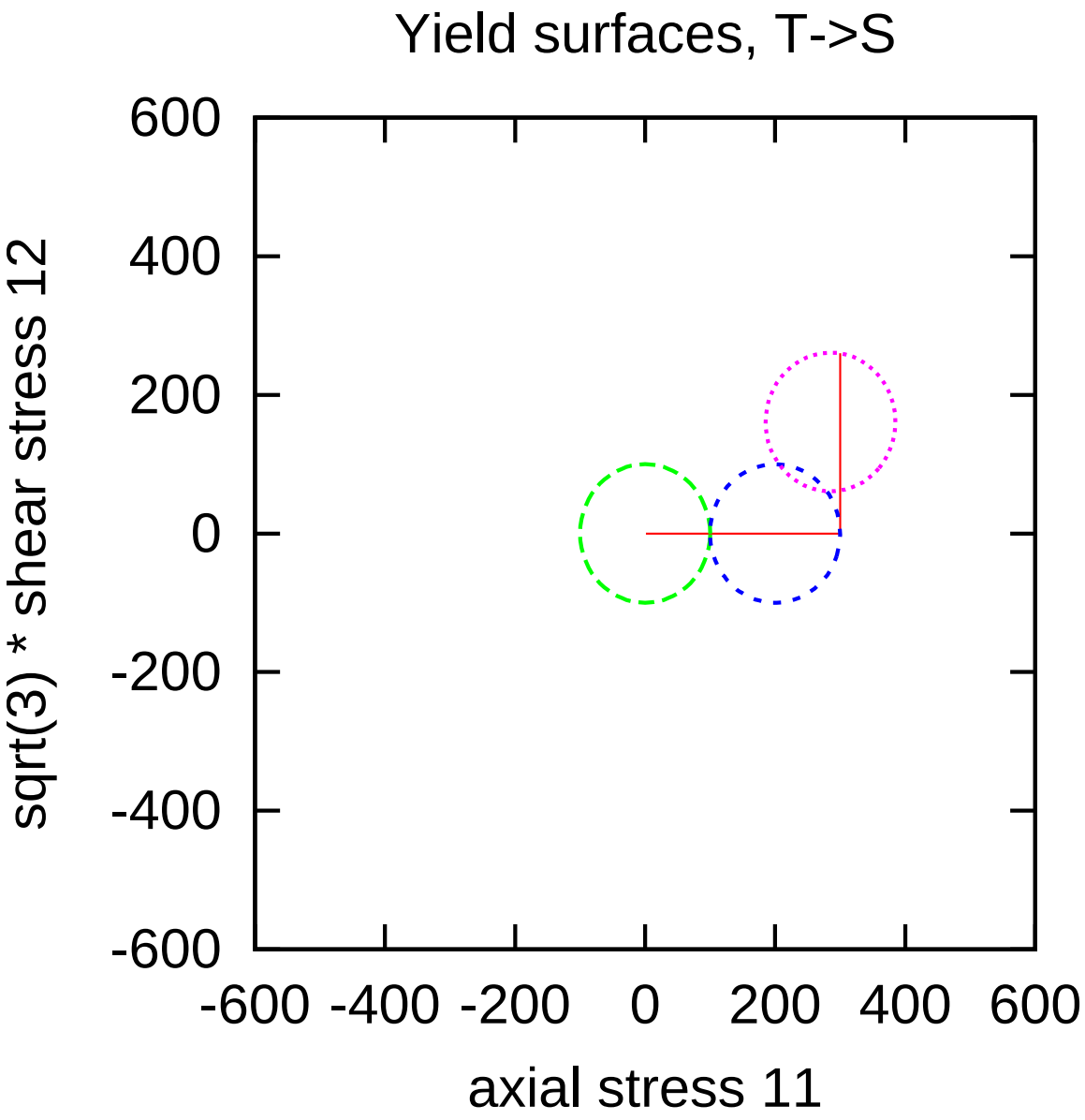

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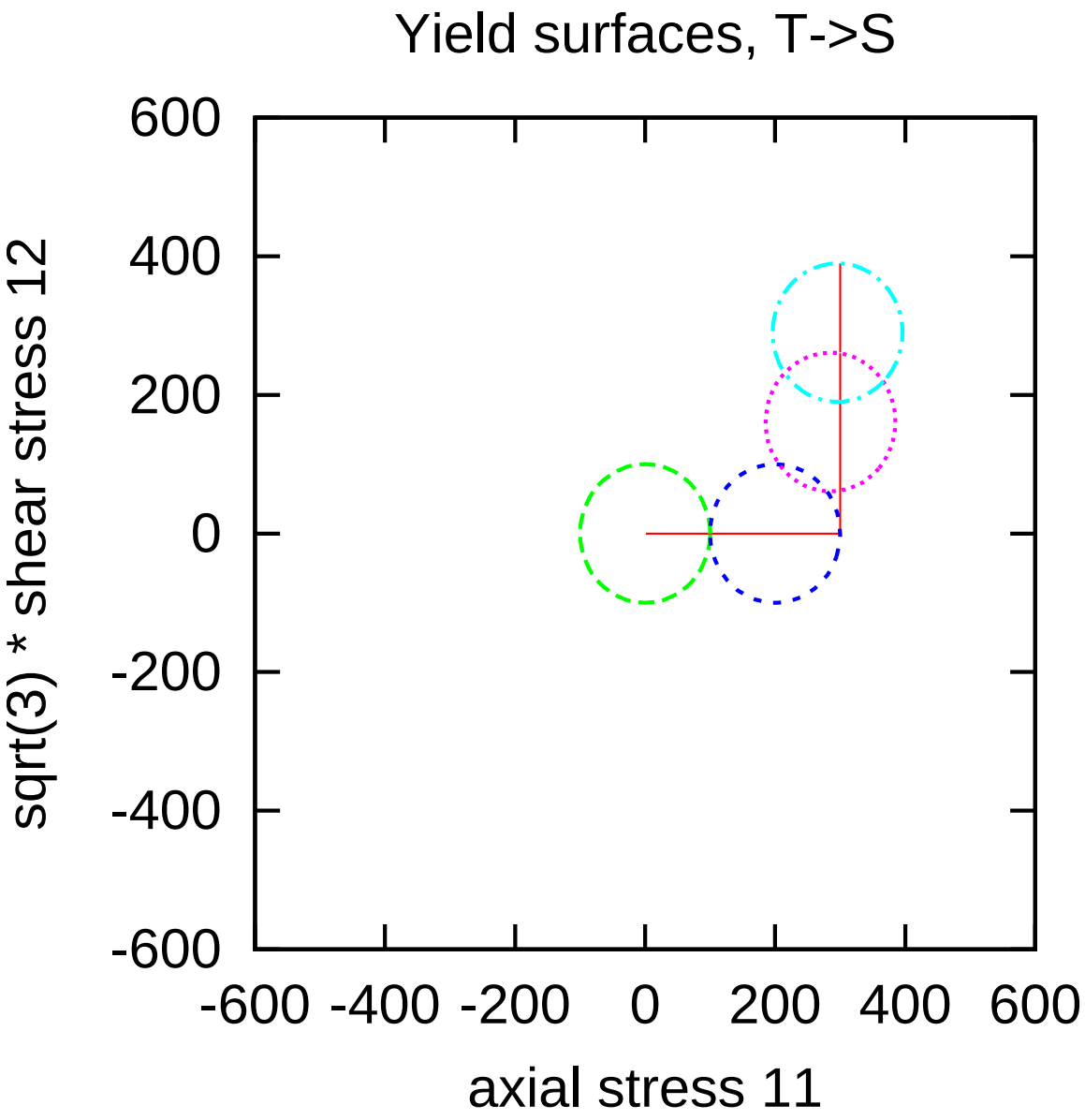

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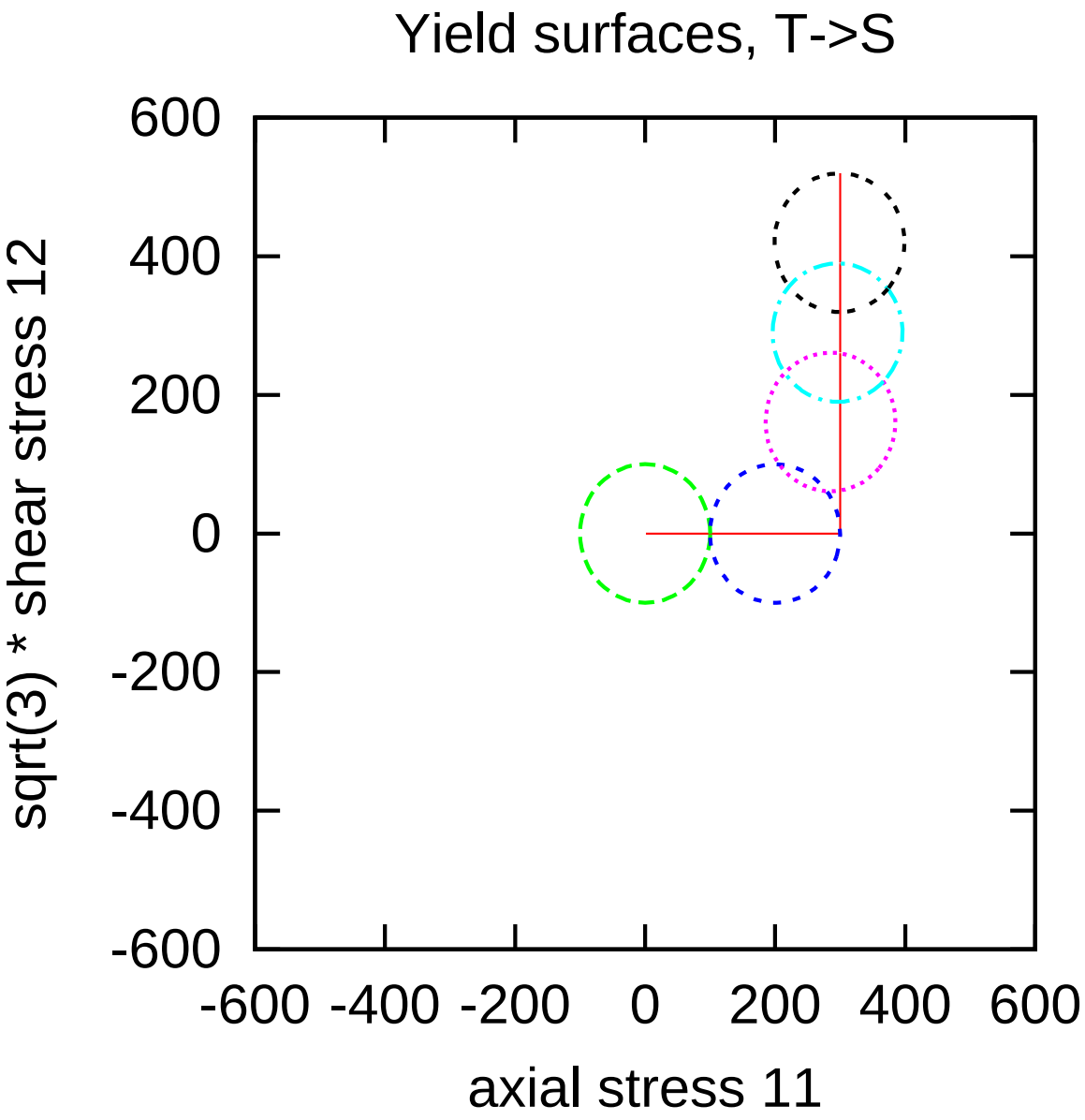

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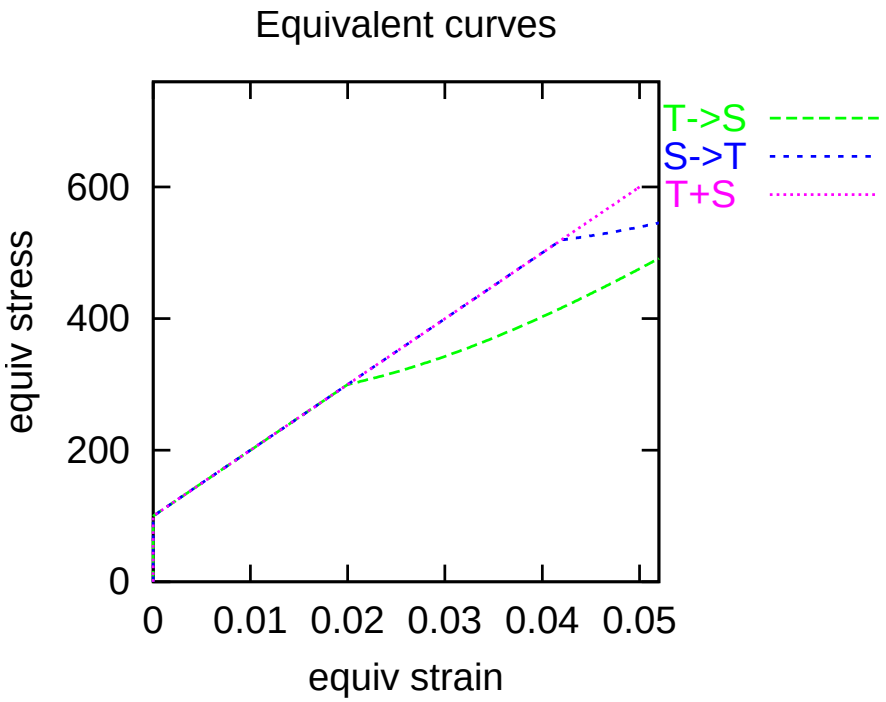

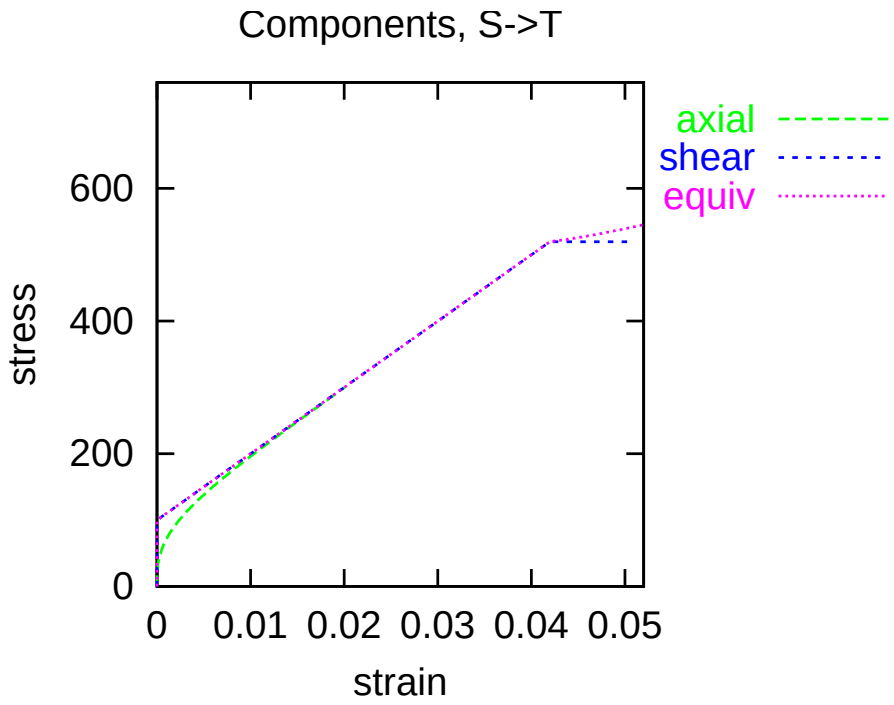

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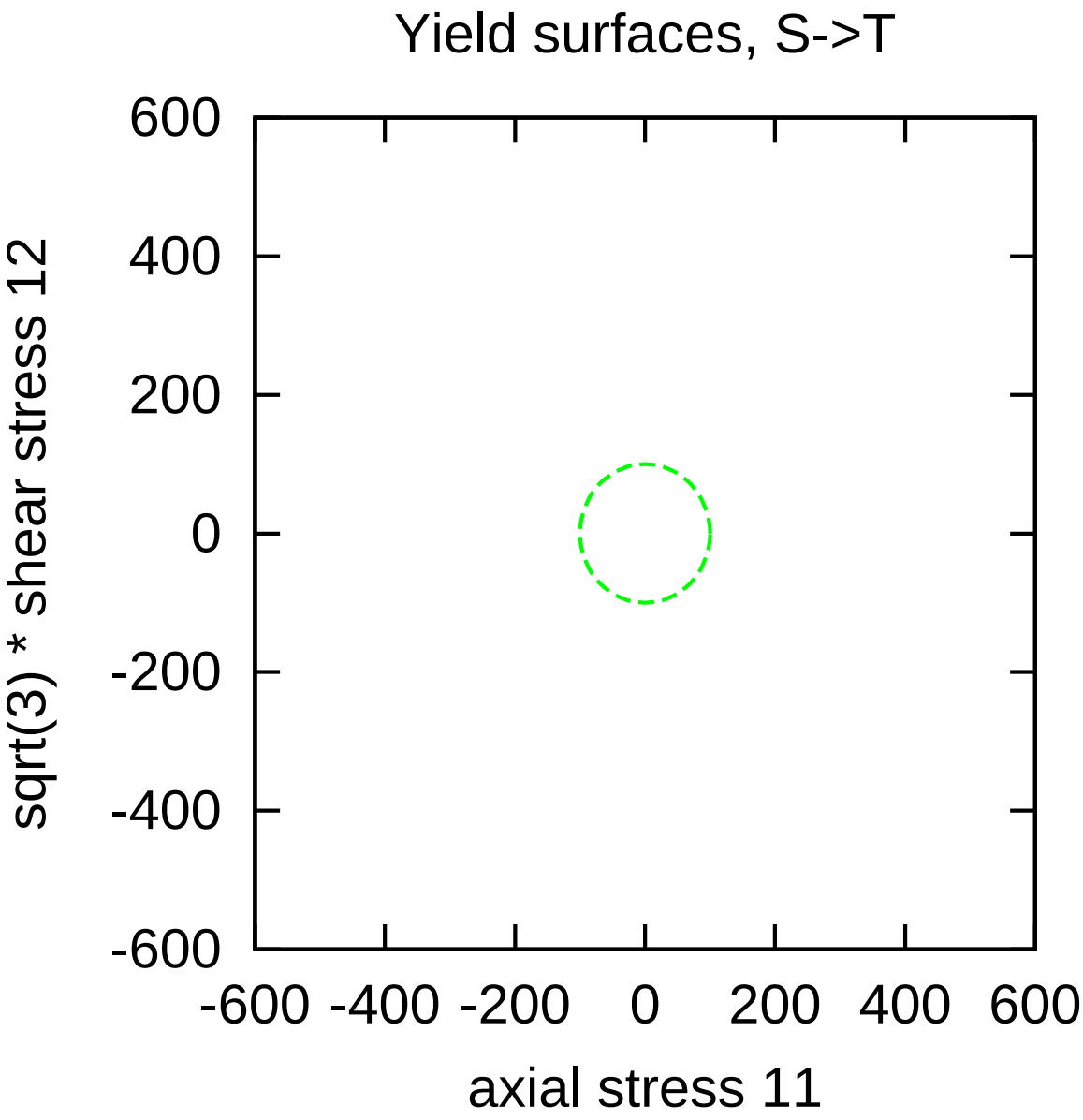

S-T Yield surface evolution

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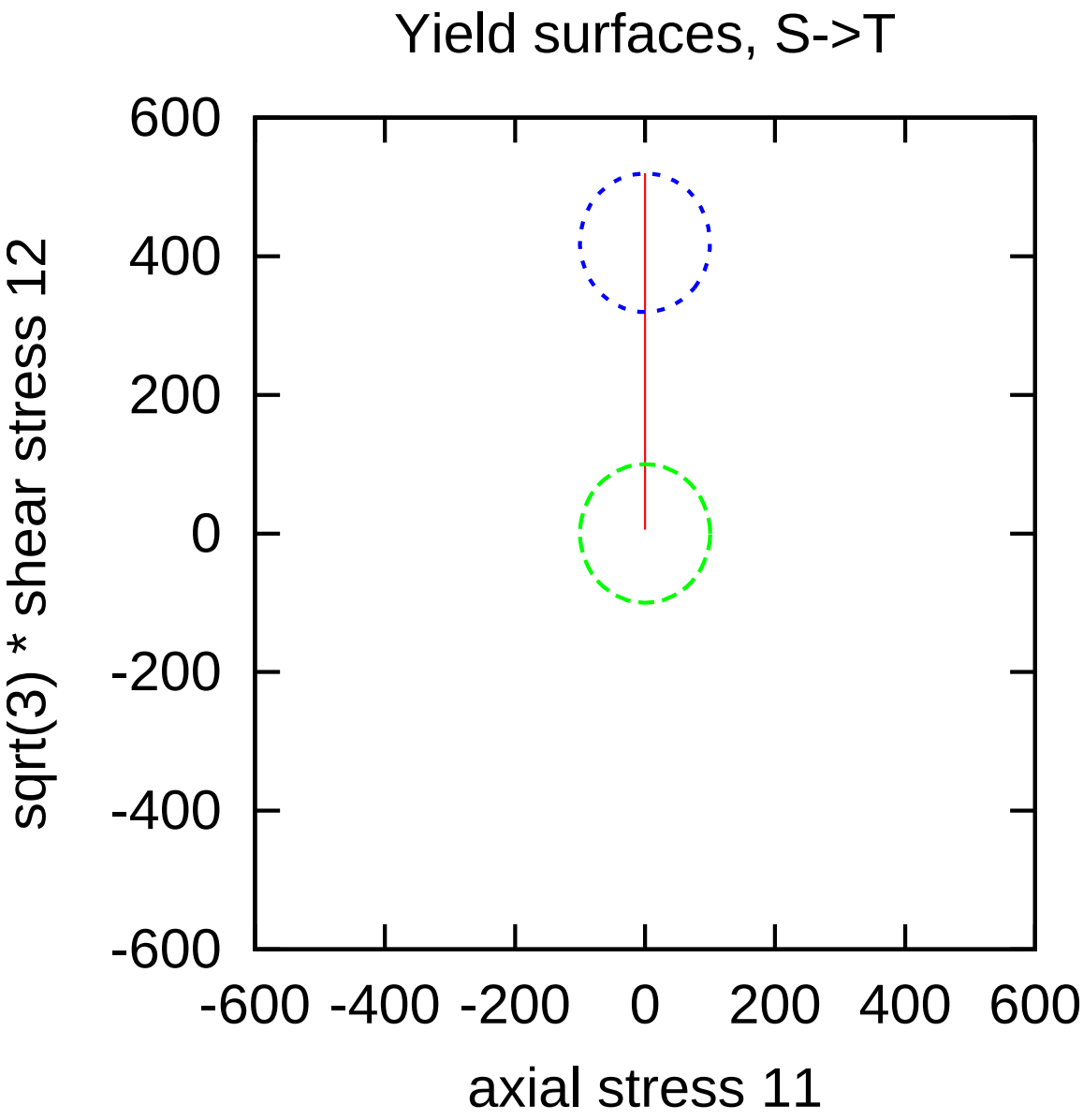

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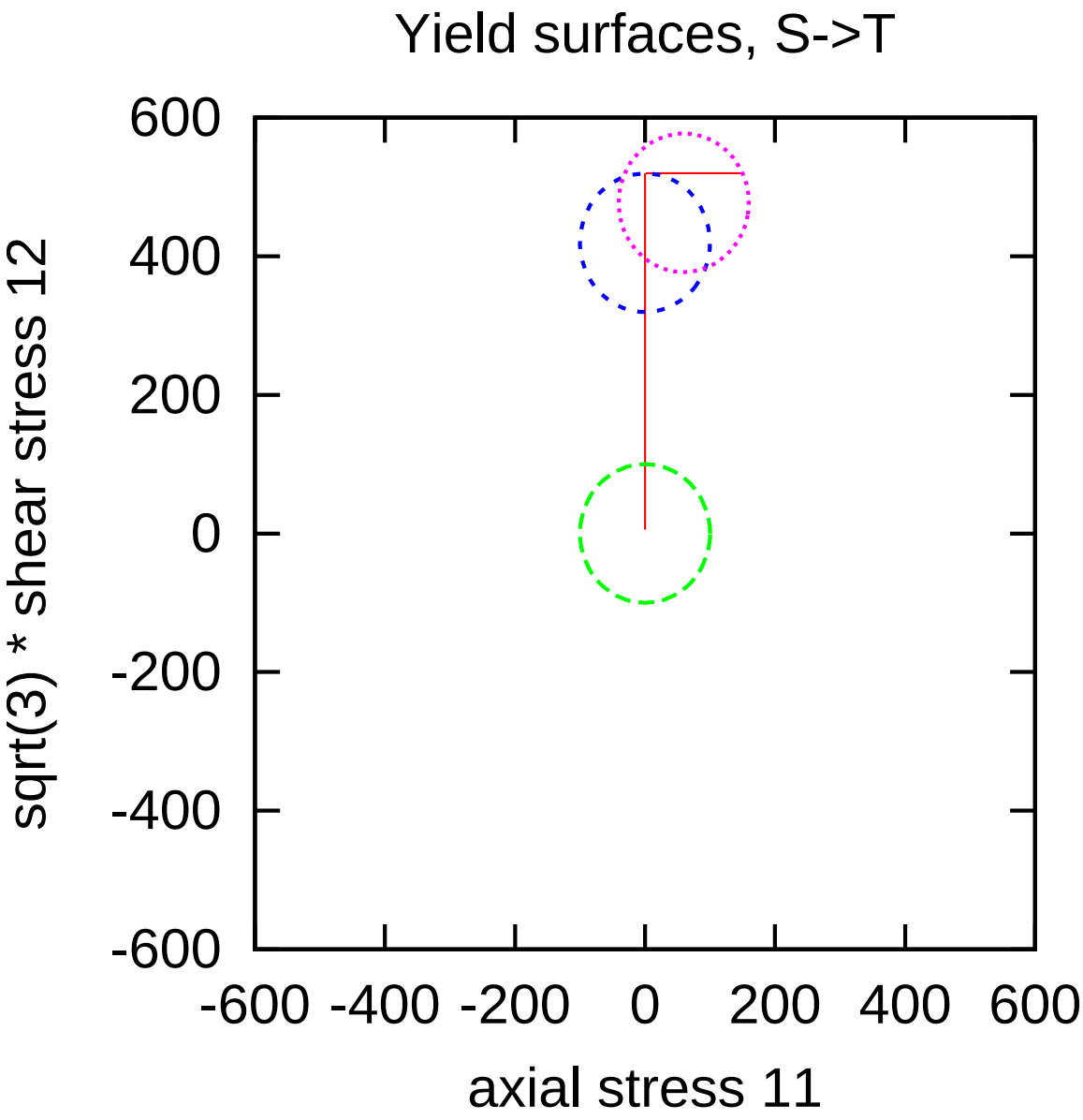

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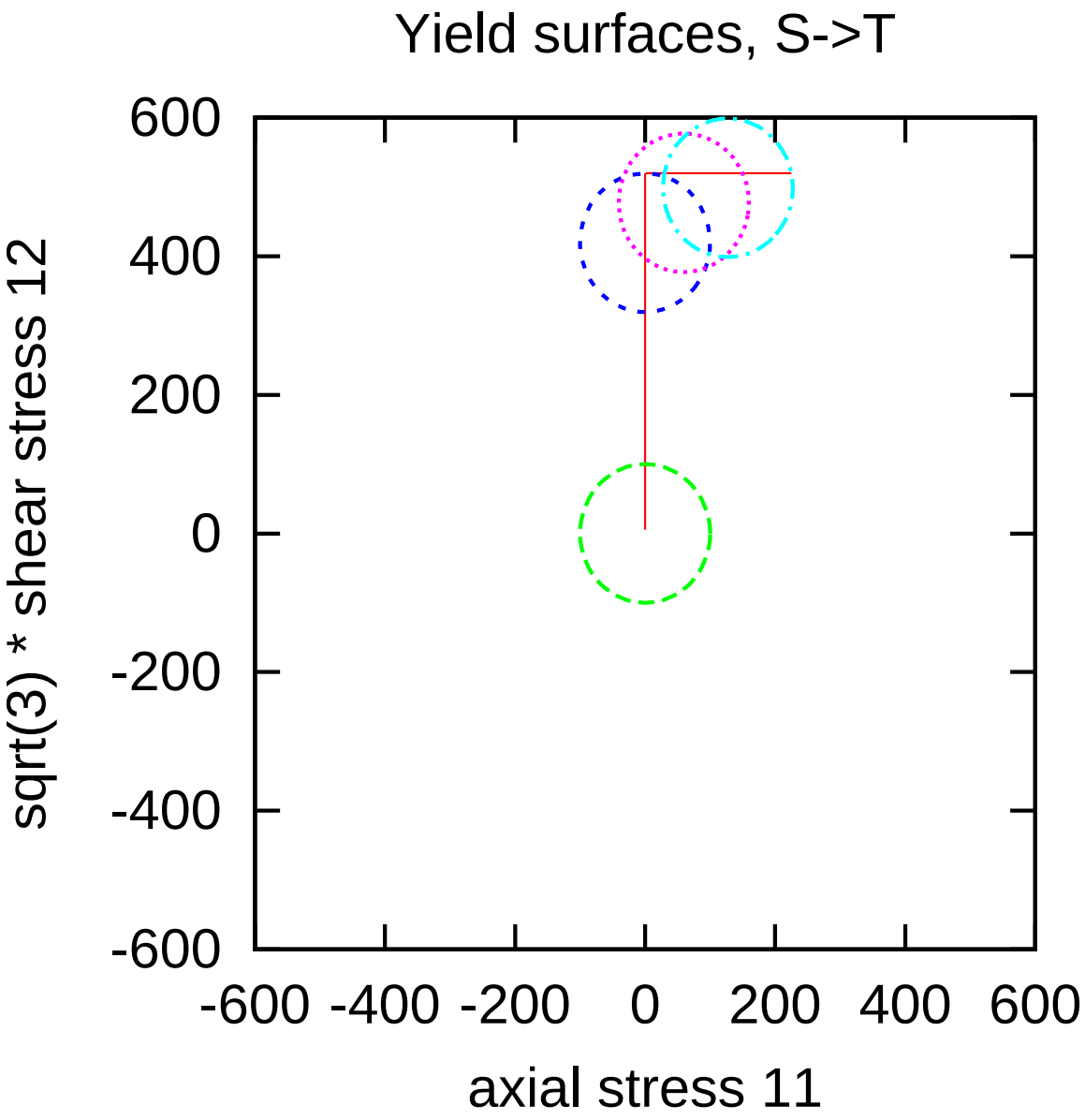

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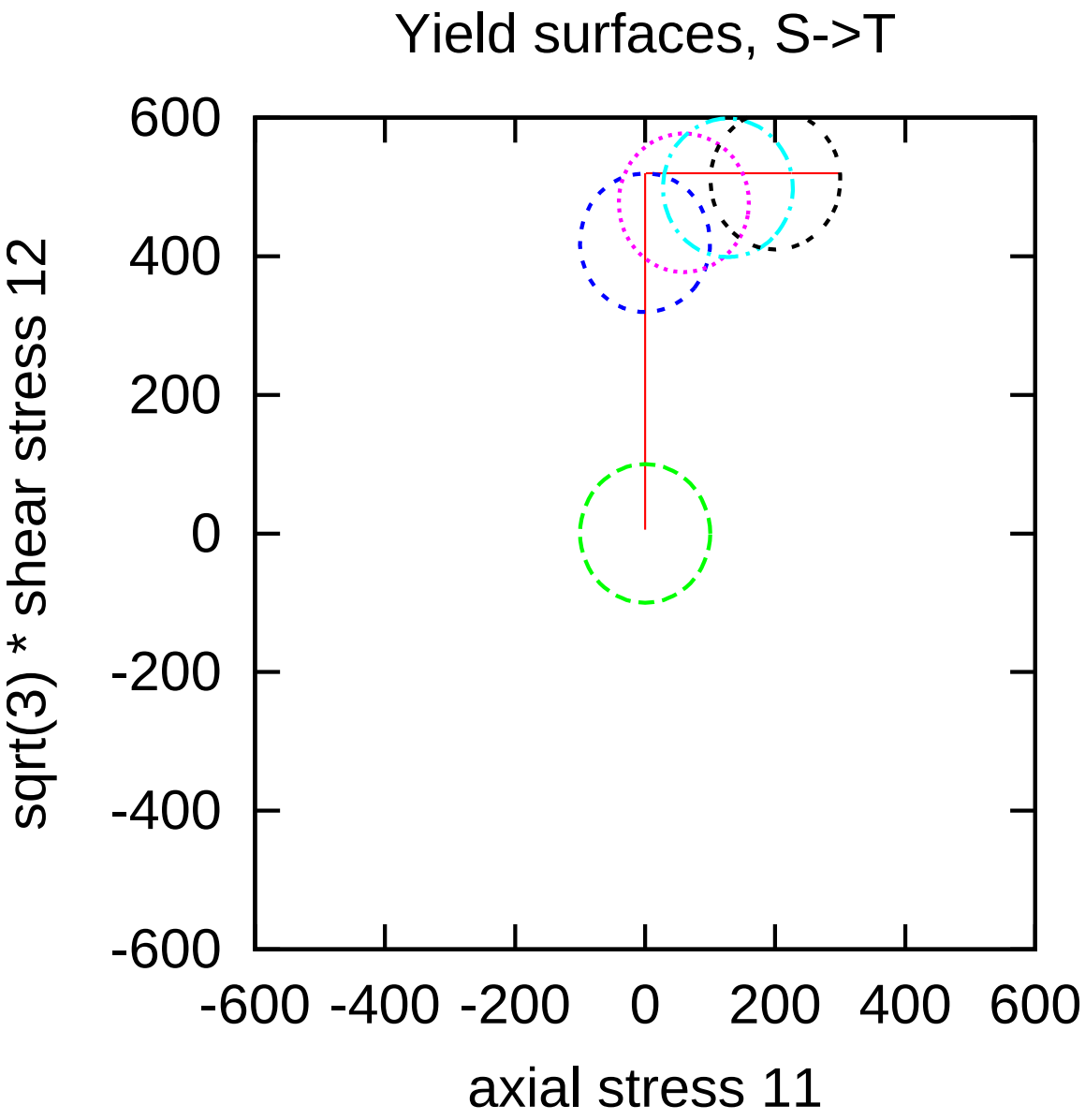
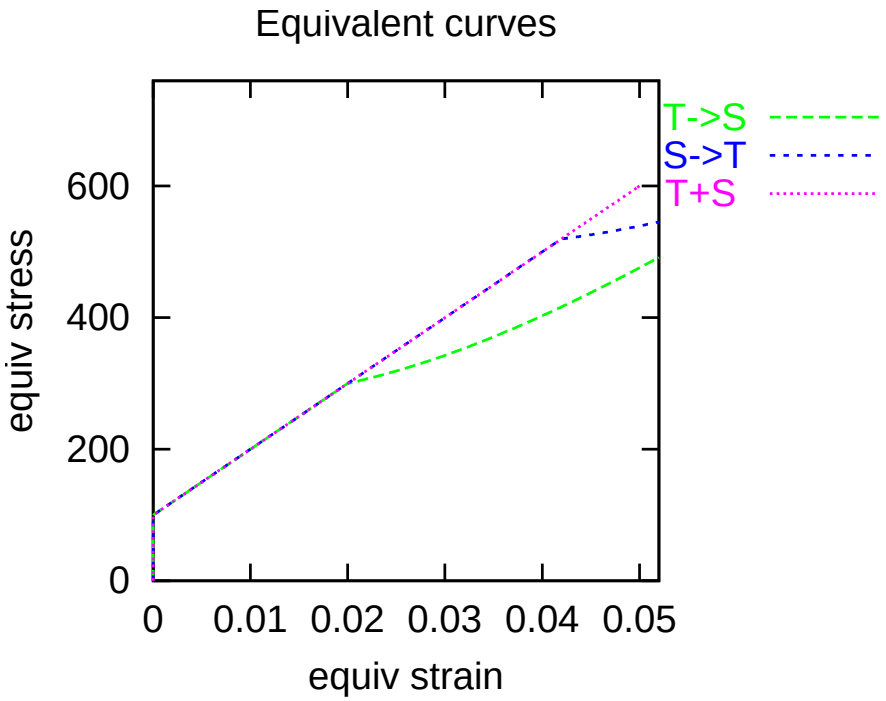
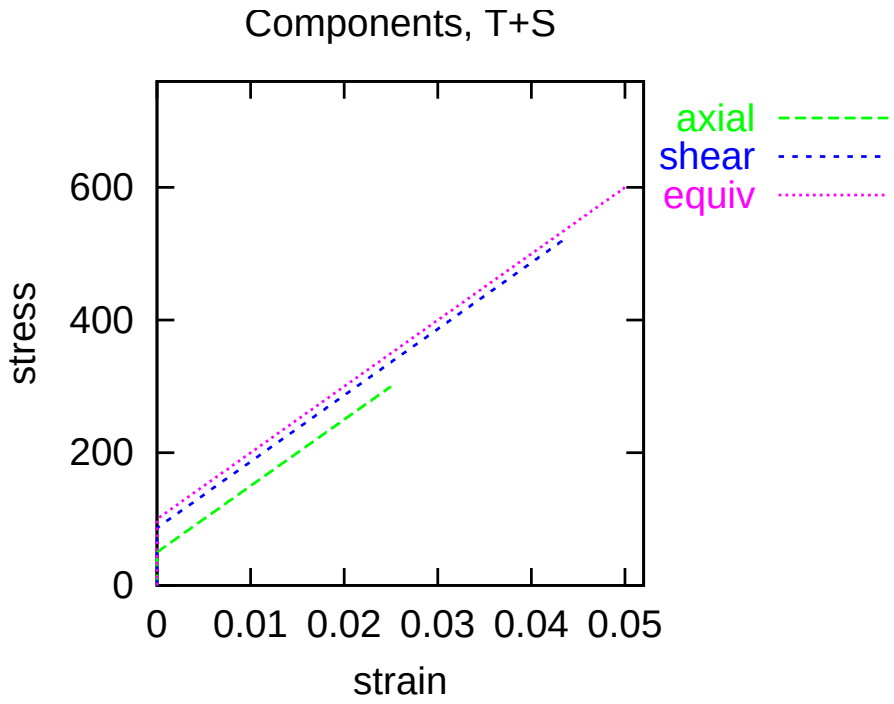


T+S Stress and plastic strain paths





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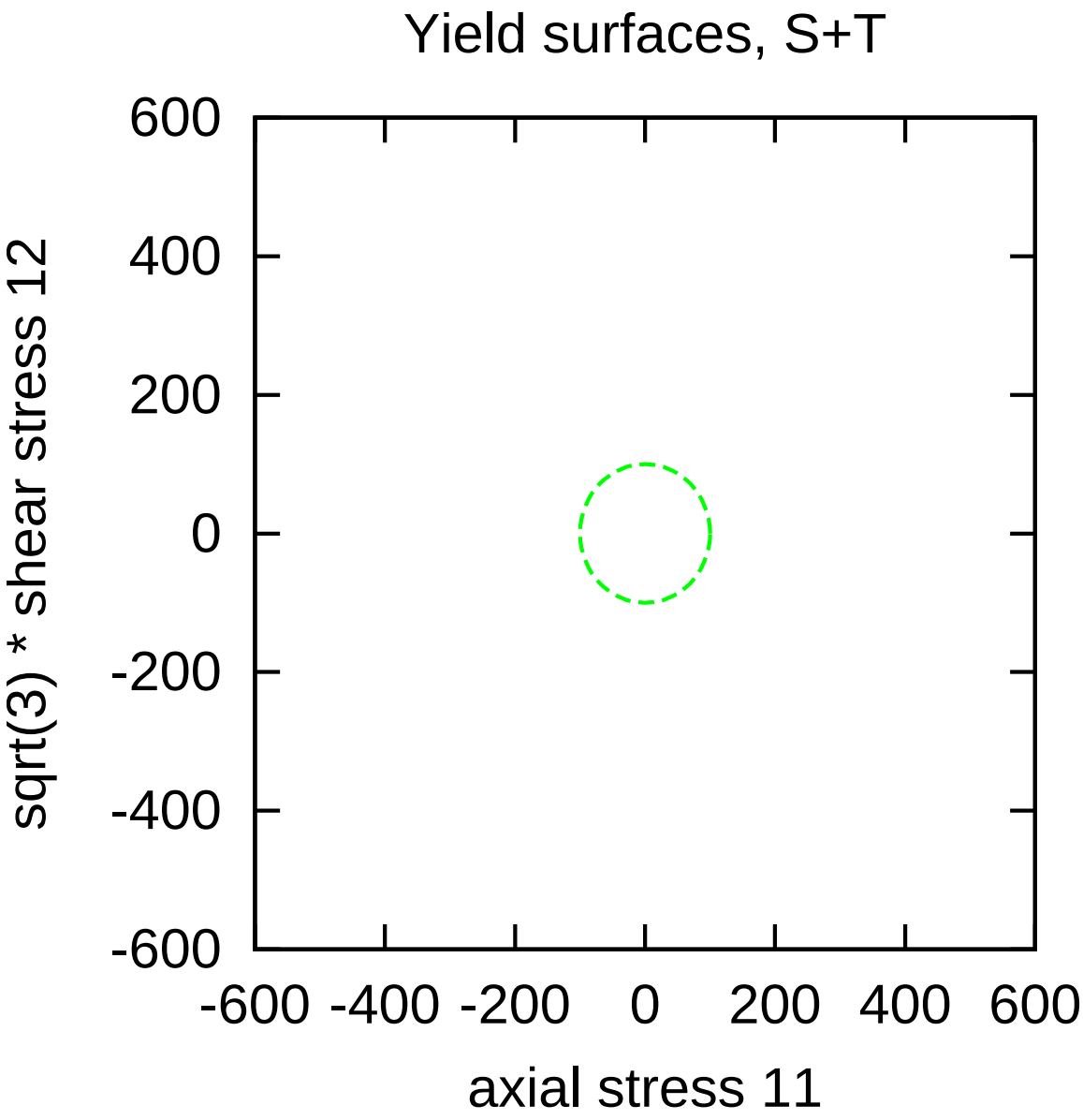

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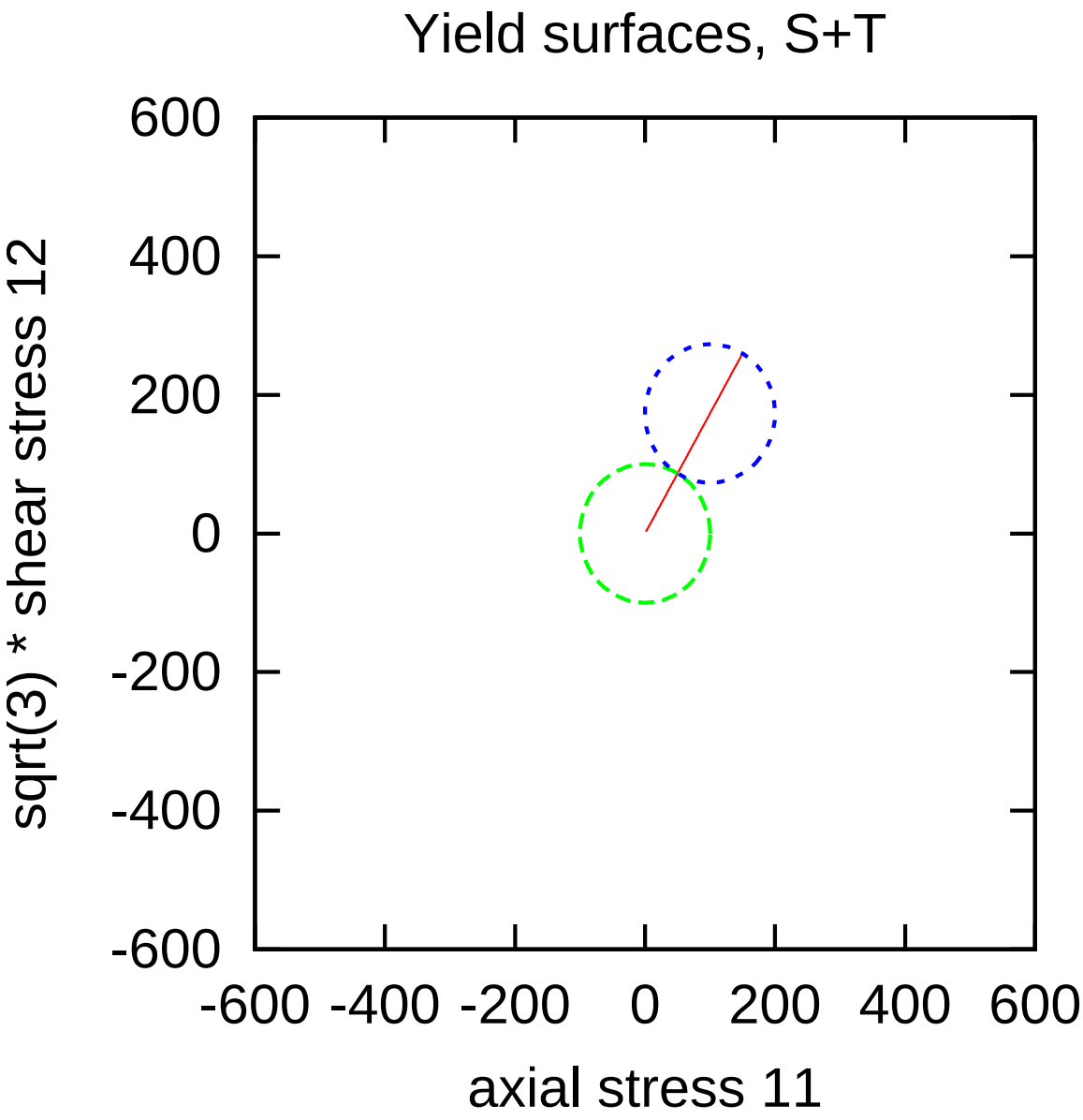

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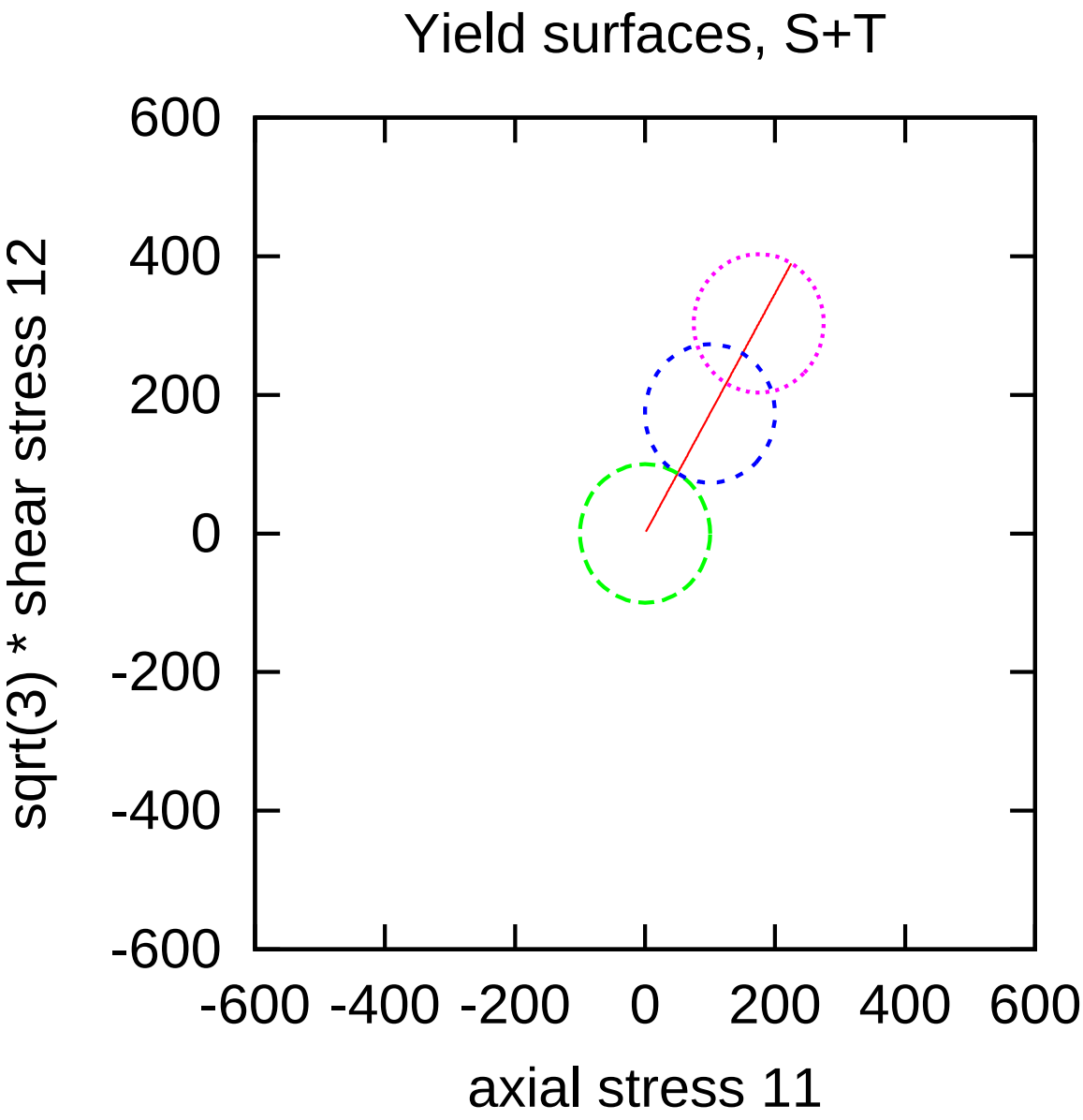

T+S Yield surface evolution

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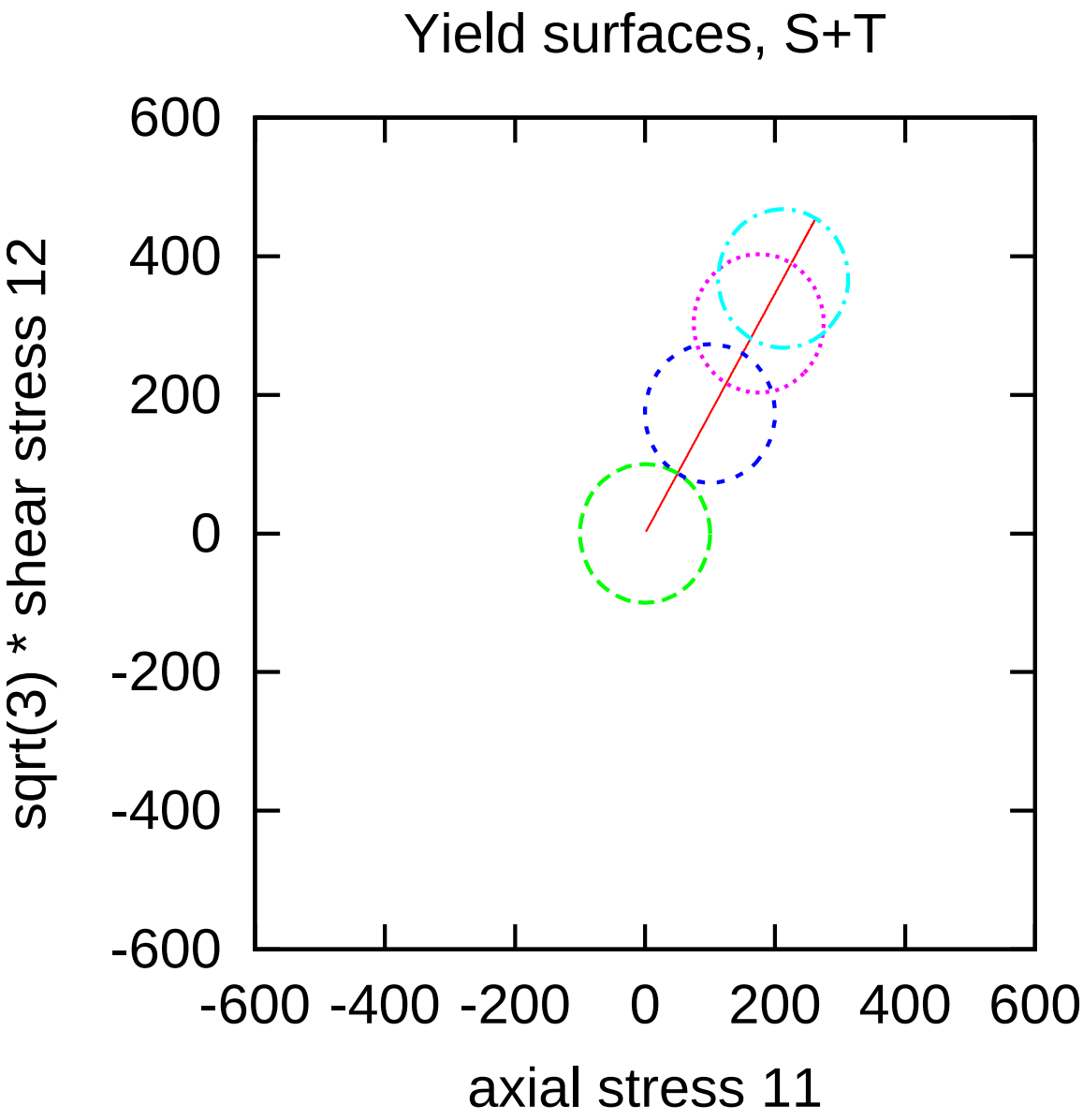

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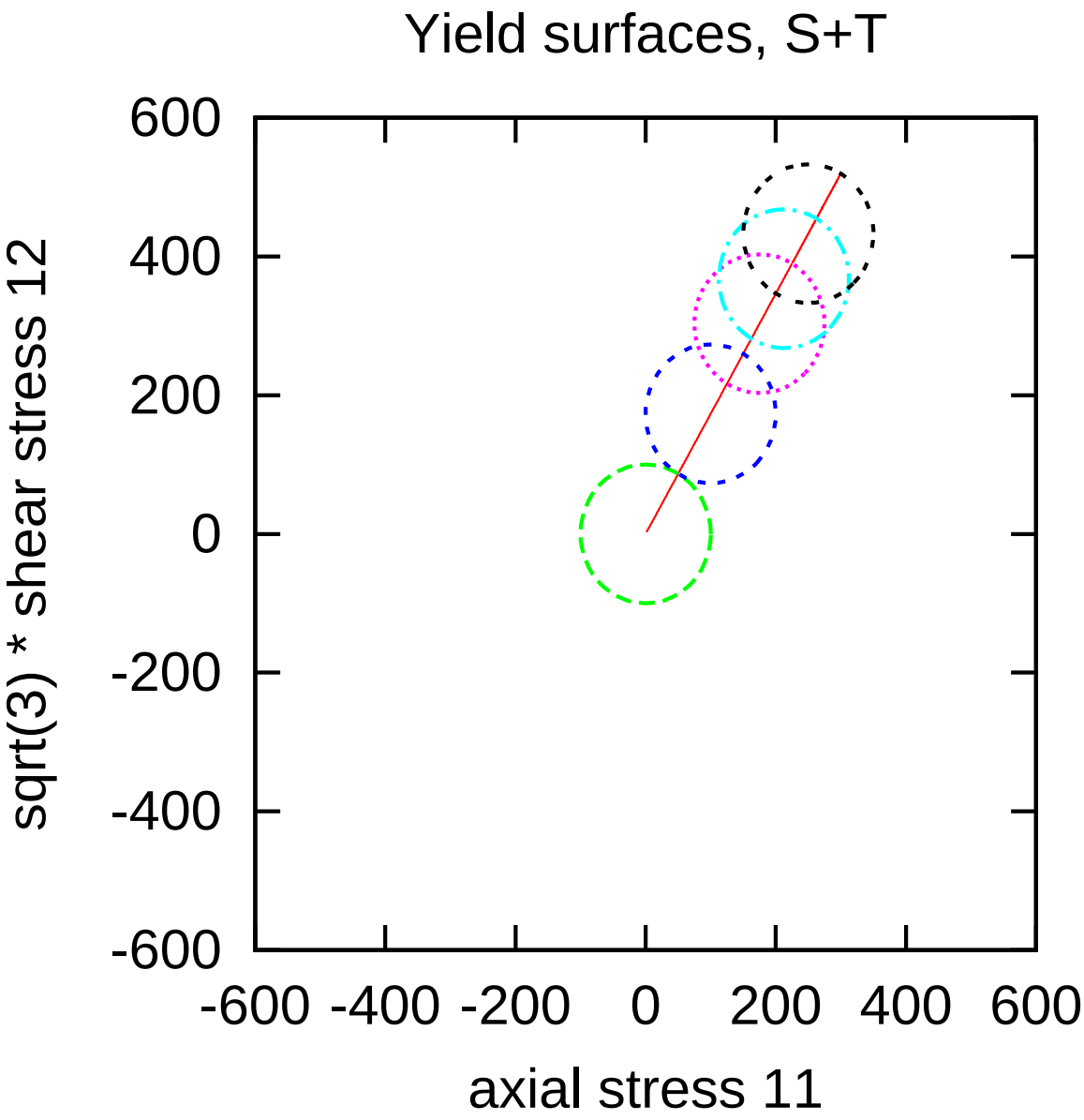

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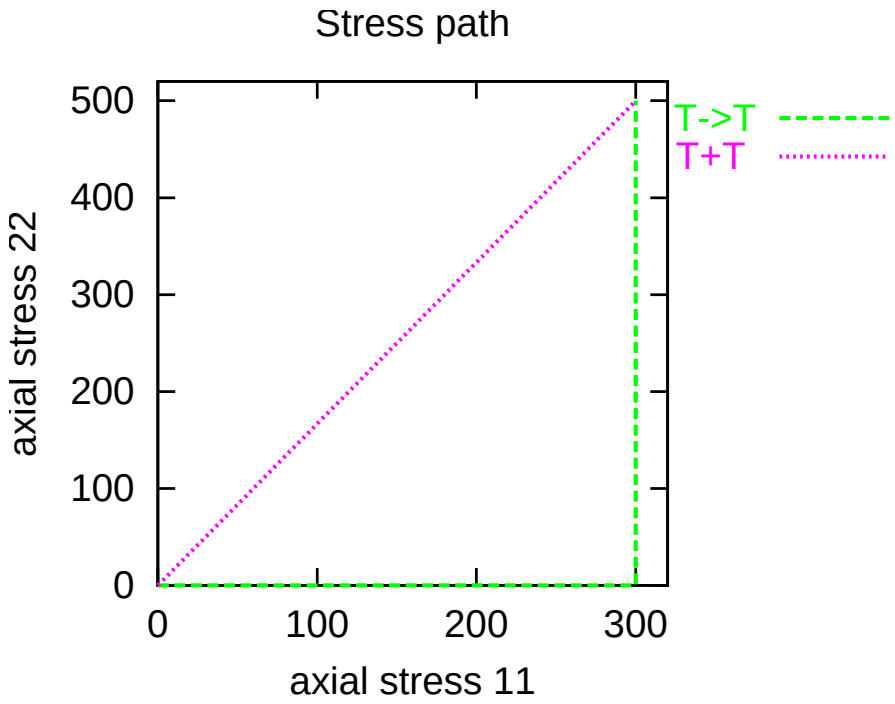
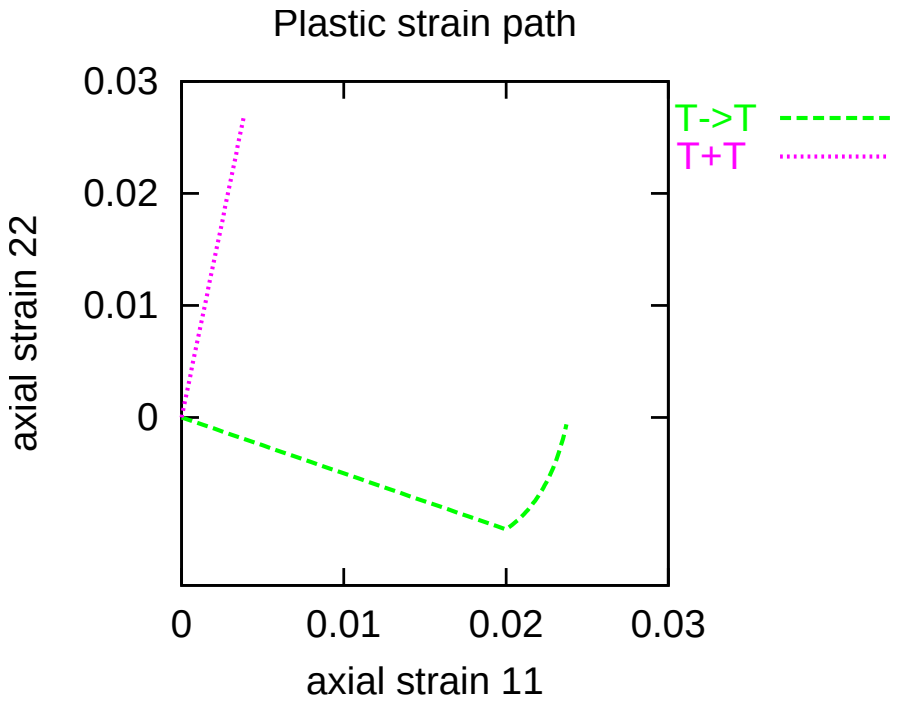
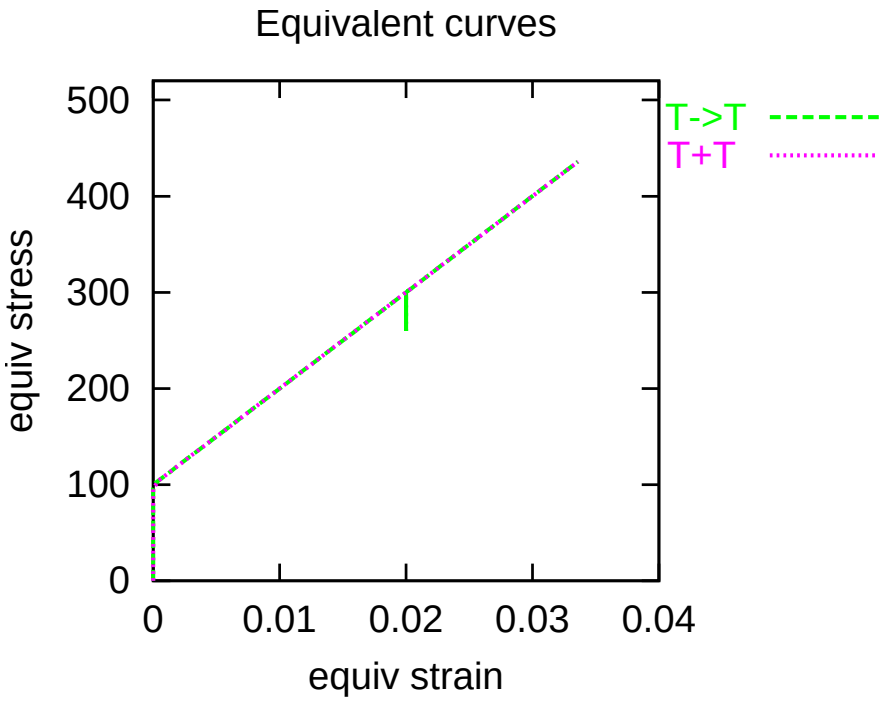
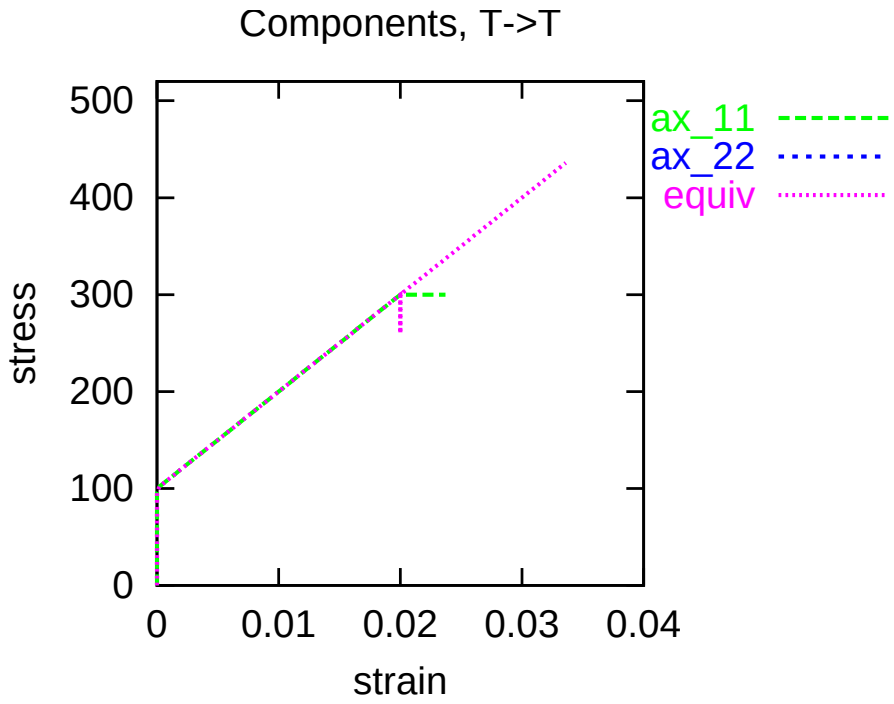




T-T Stress and plastic strain paths





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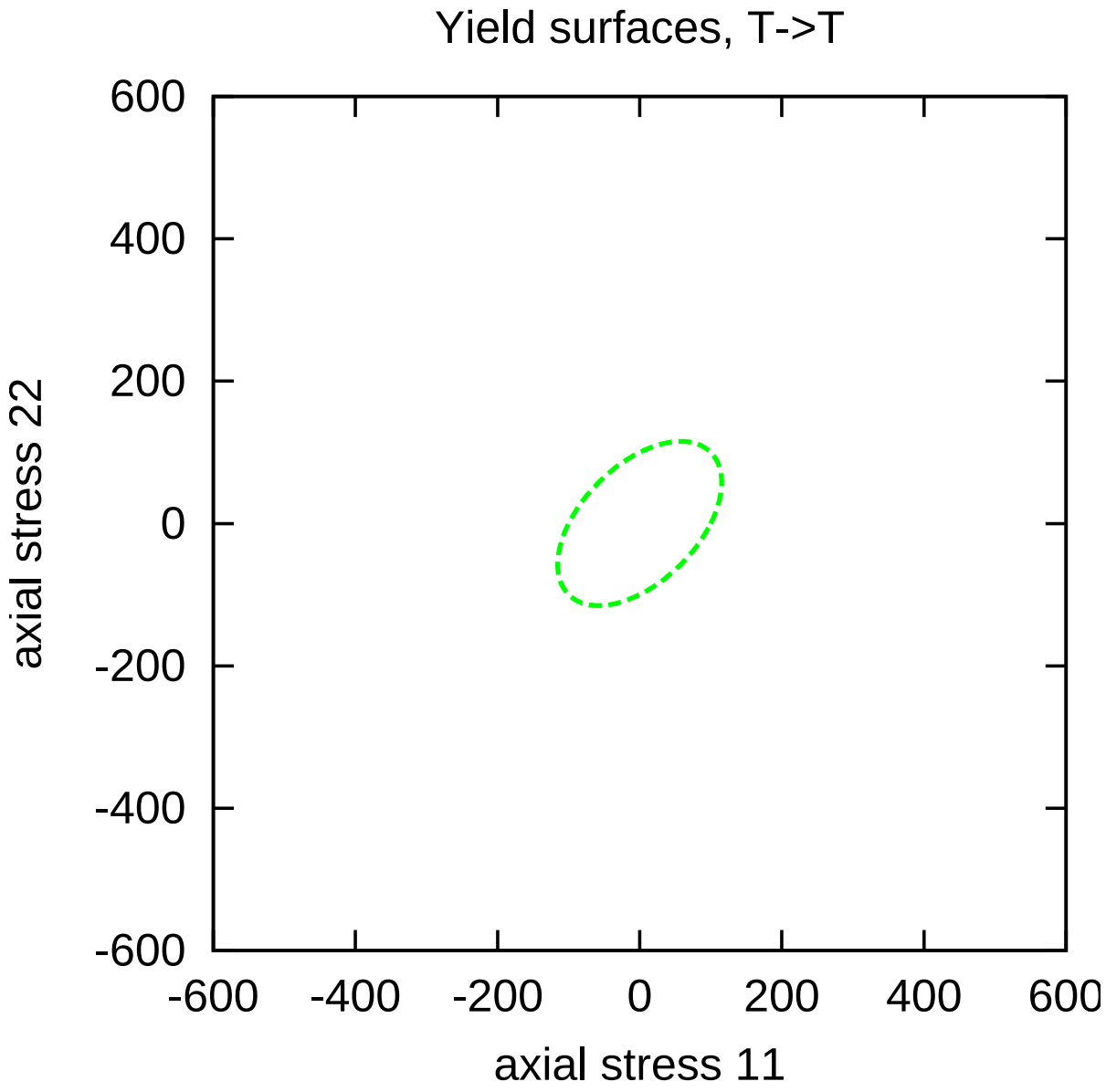

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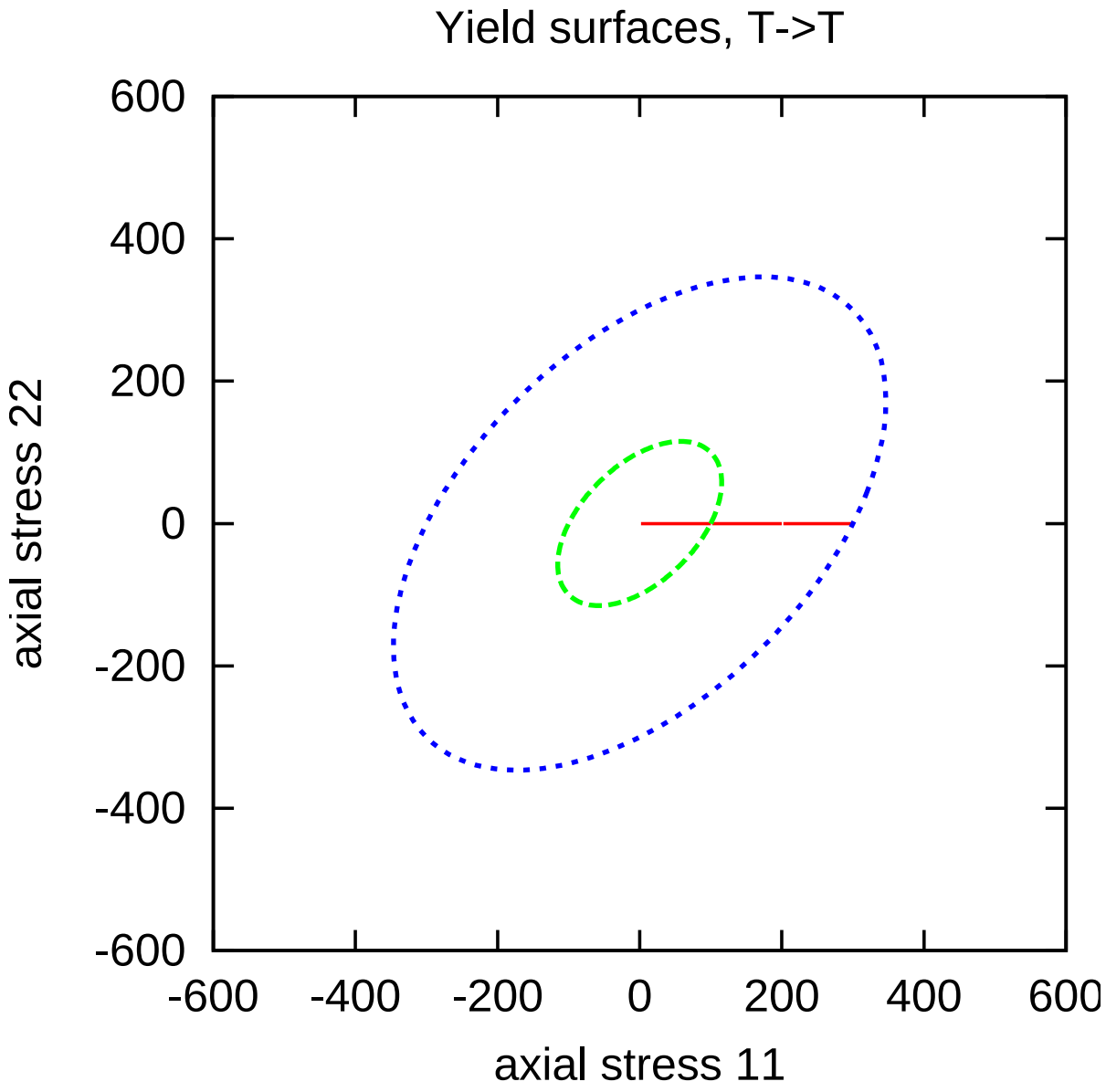

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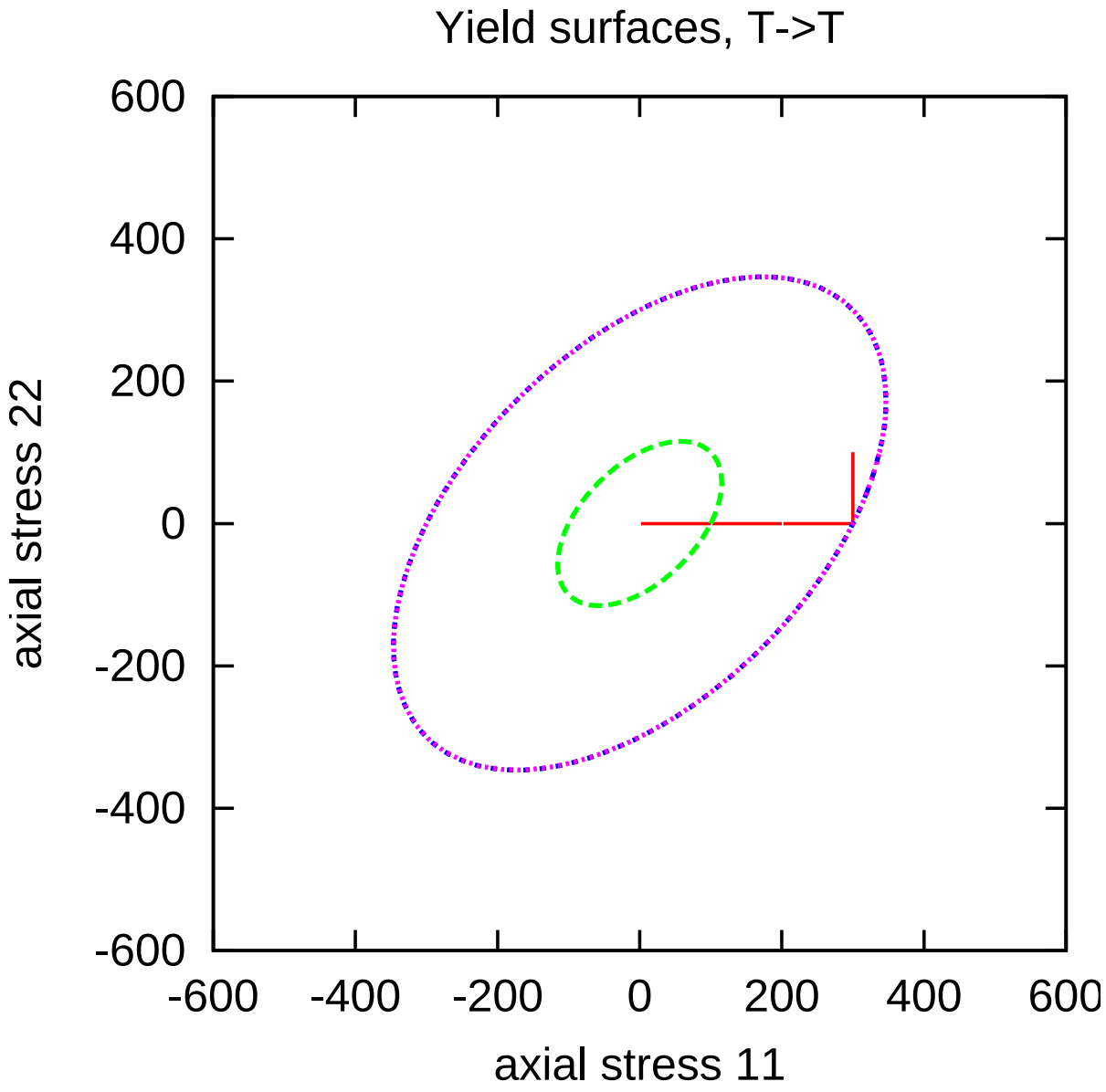

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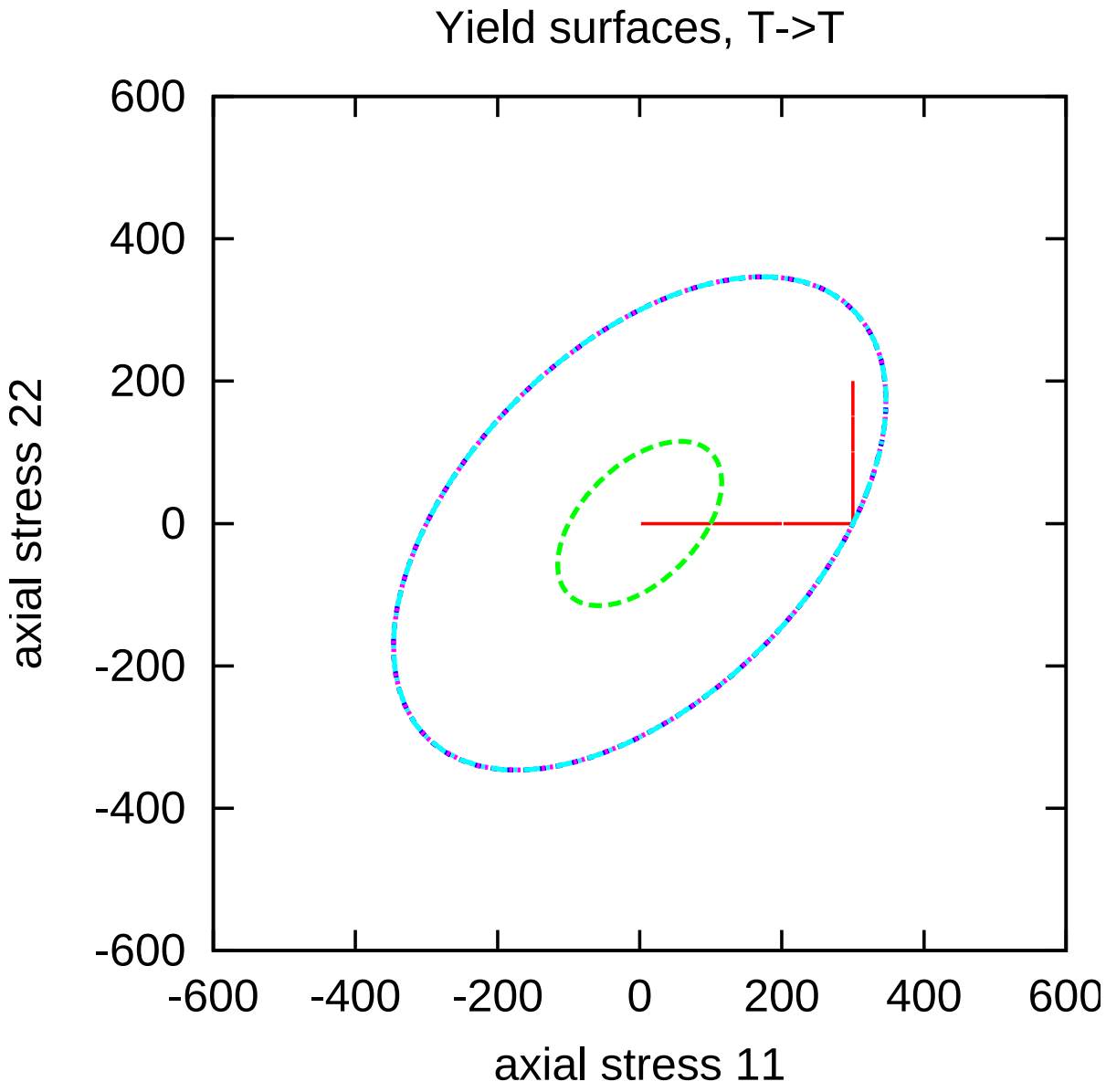

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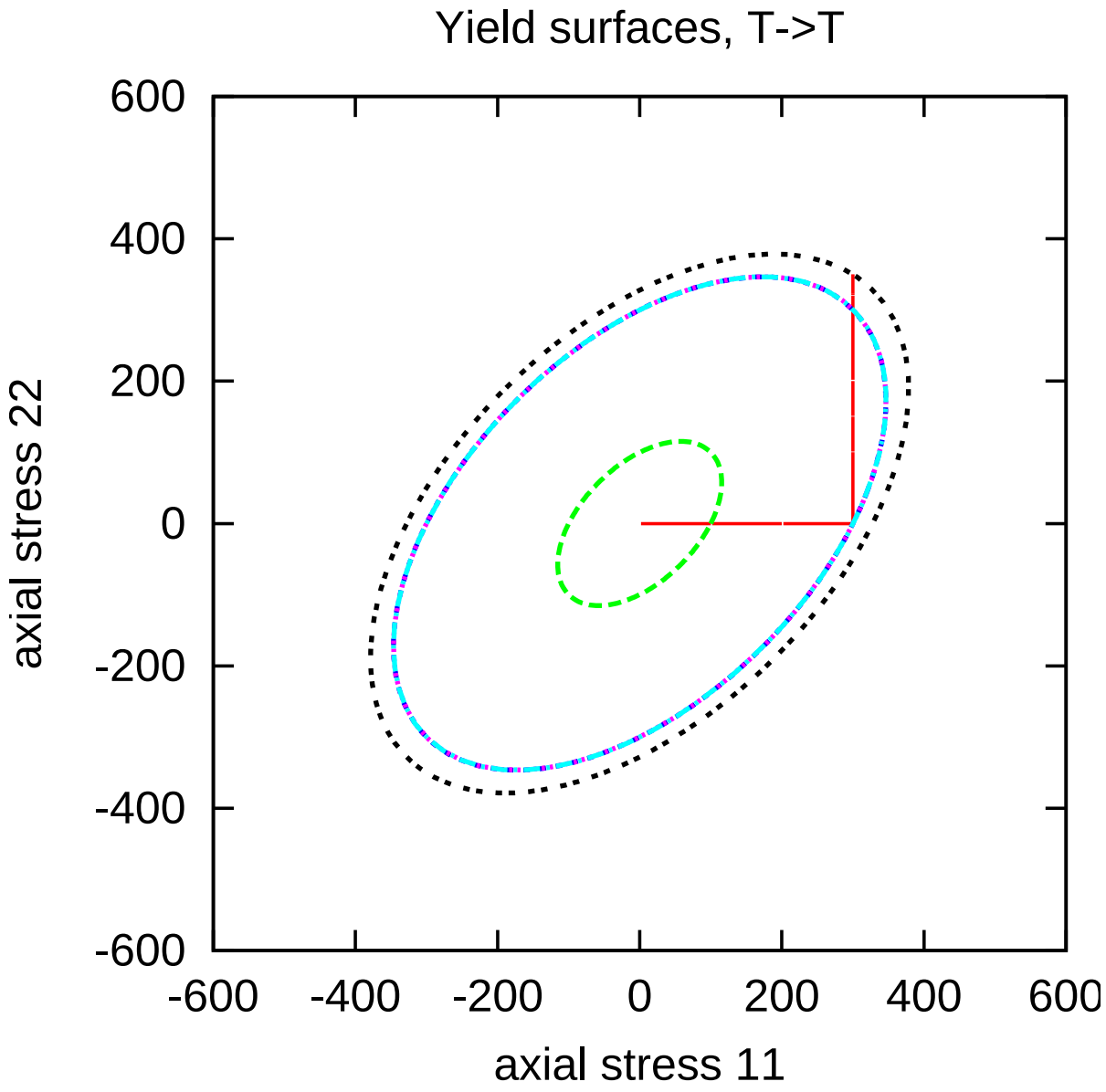
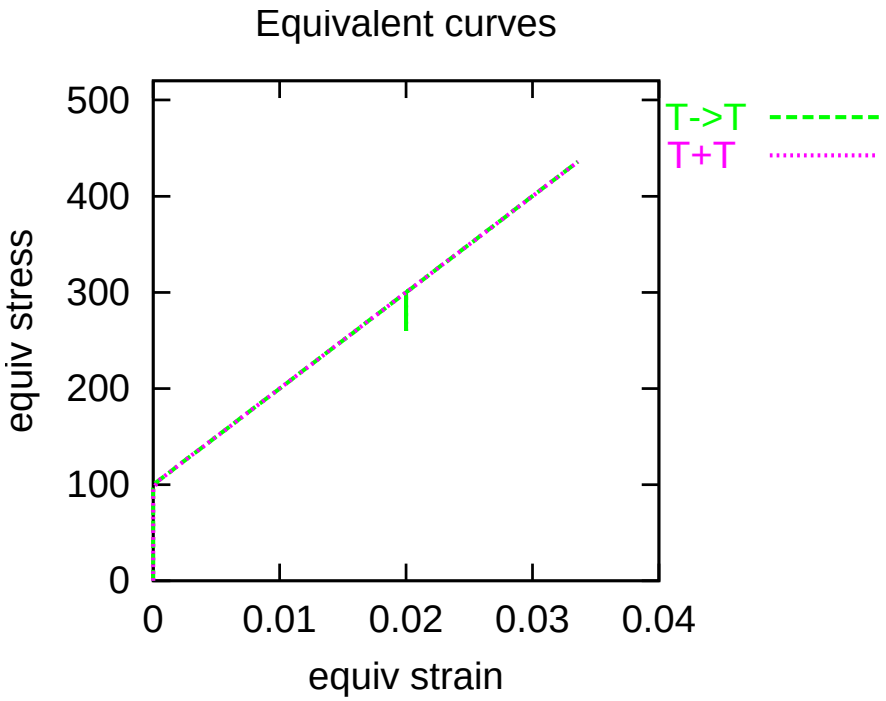
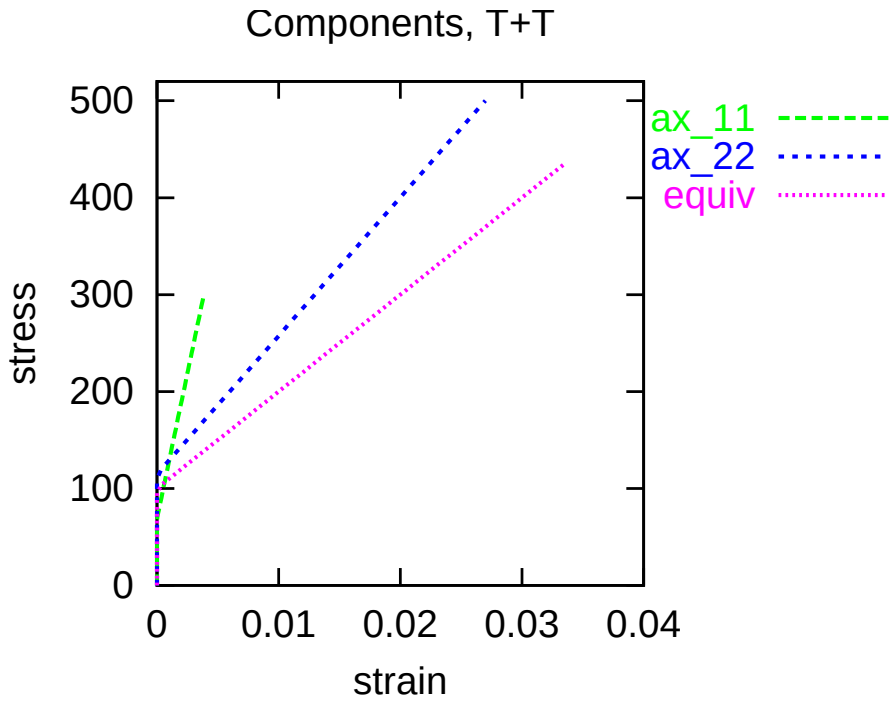


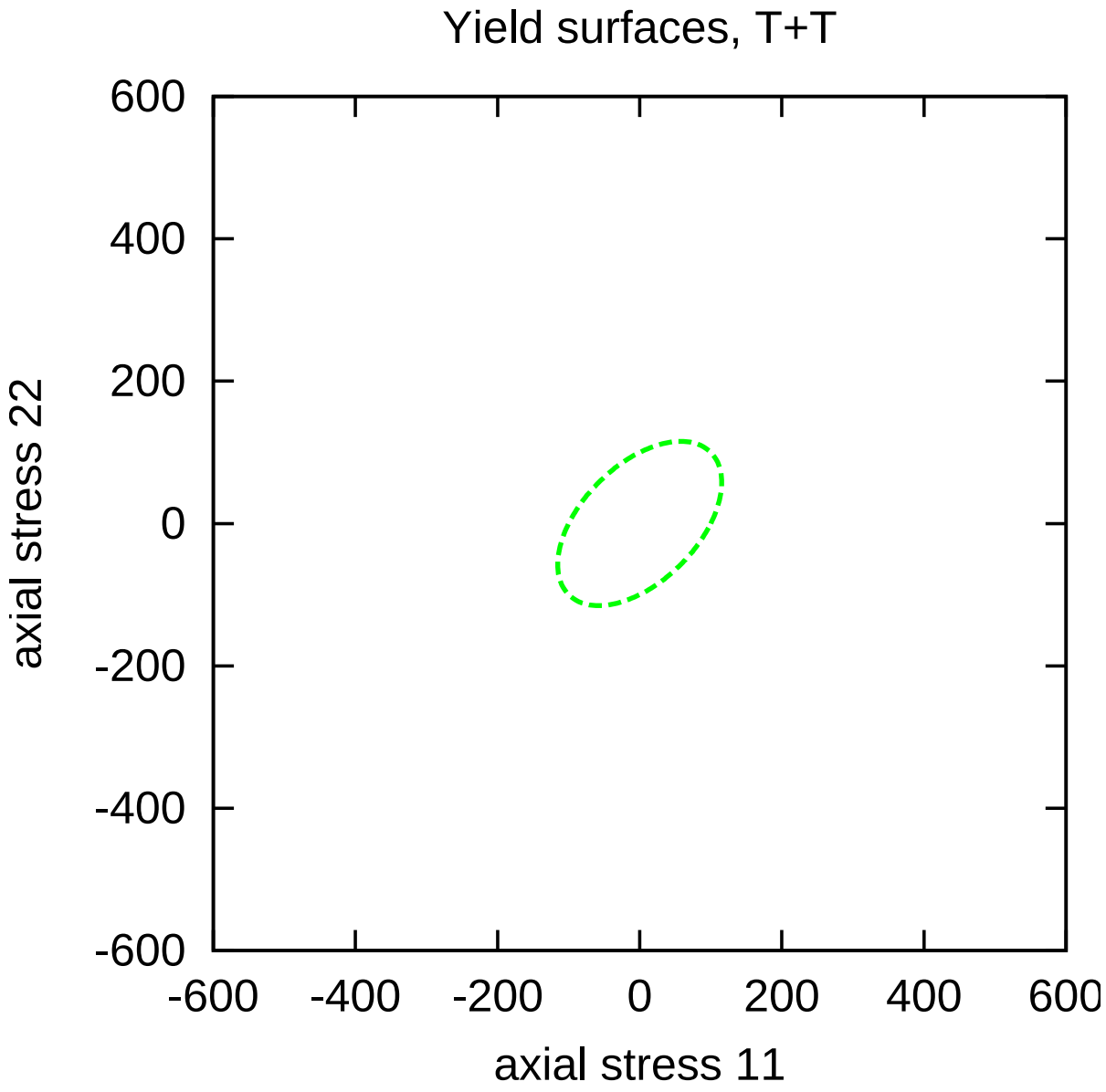
T+T Stress and plastic strain paths





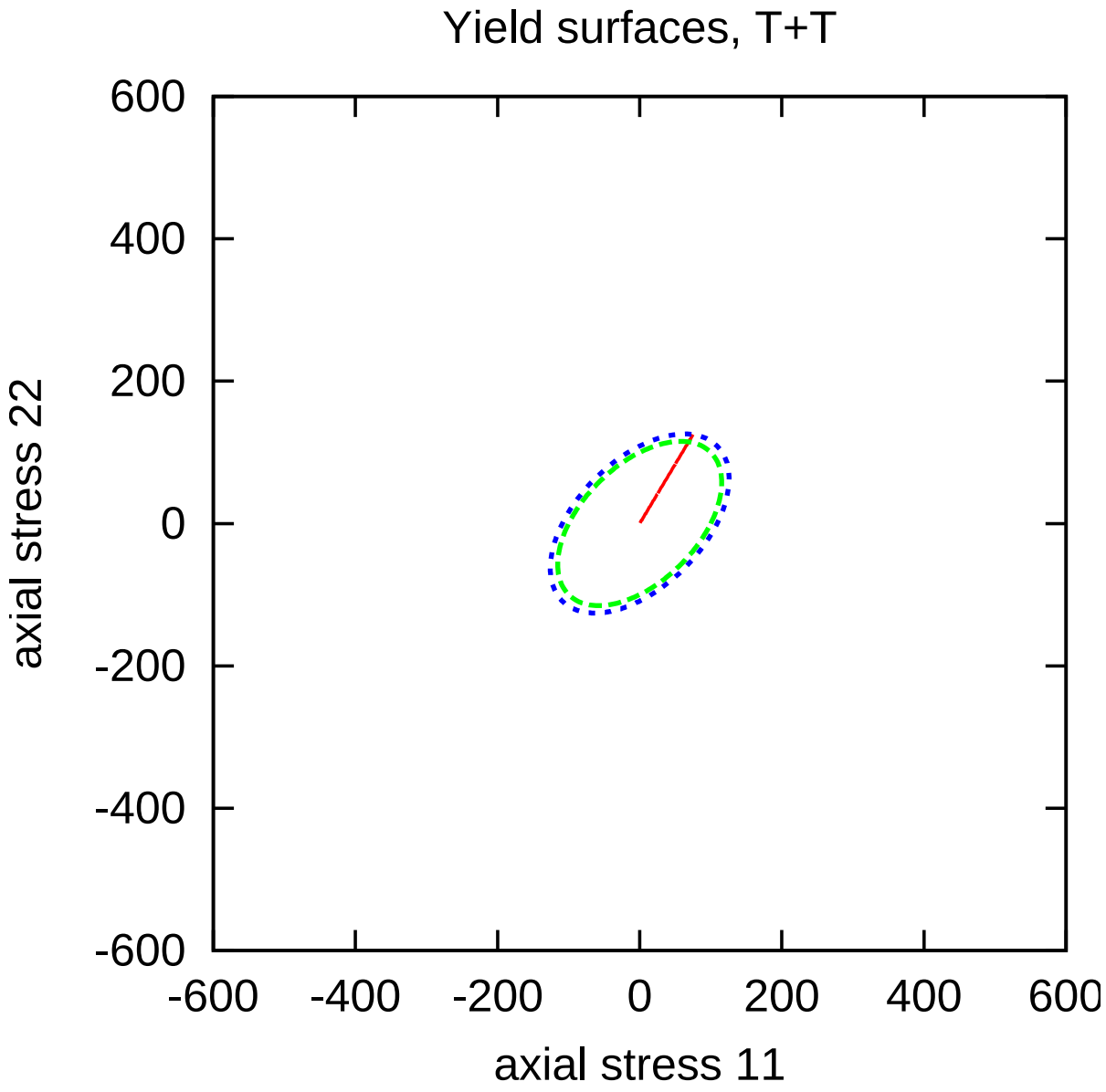
T+T Yield surface evolution

summary



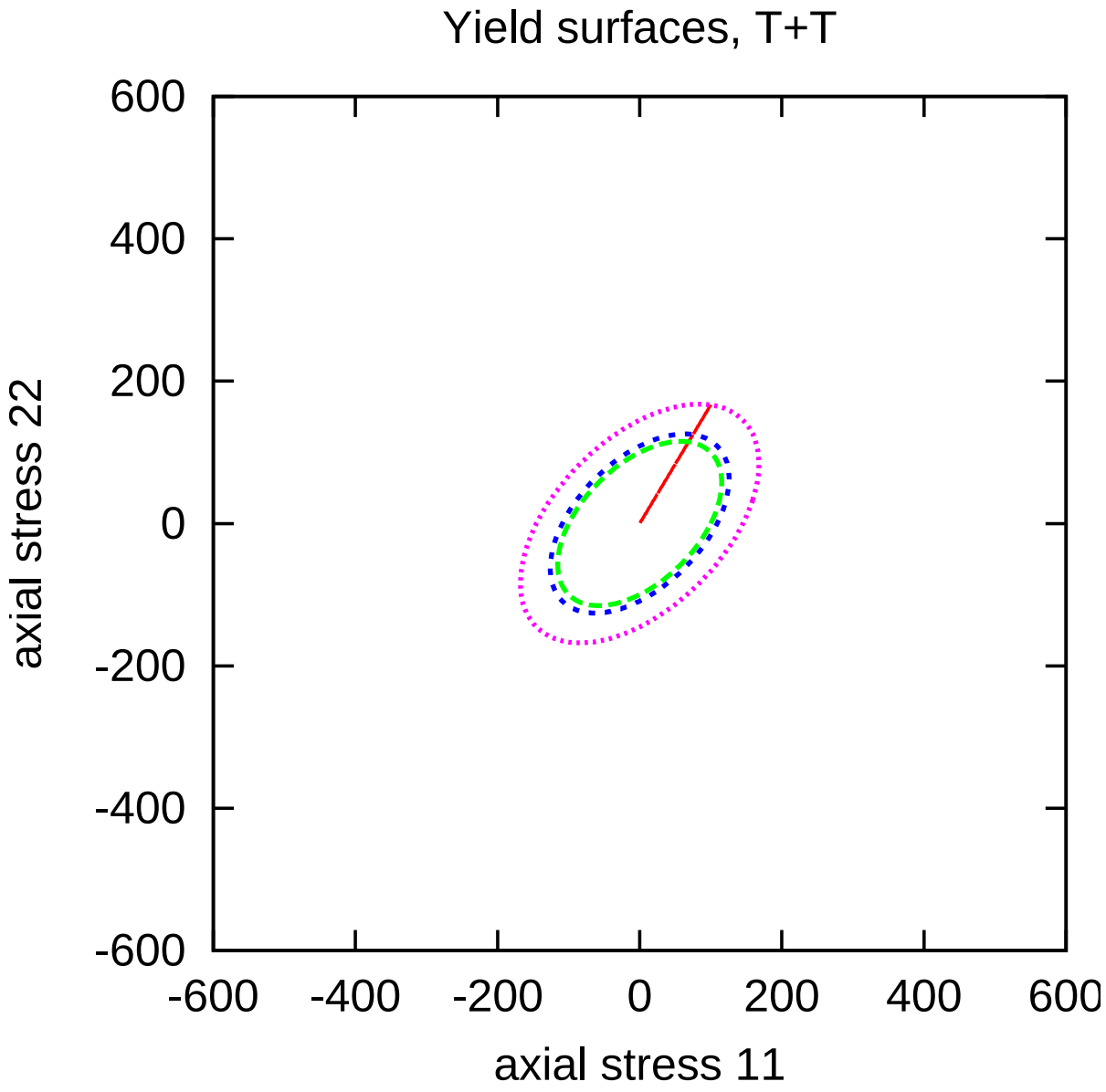
T+T Yield surface evolution

summary



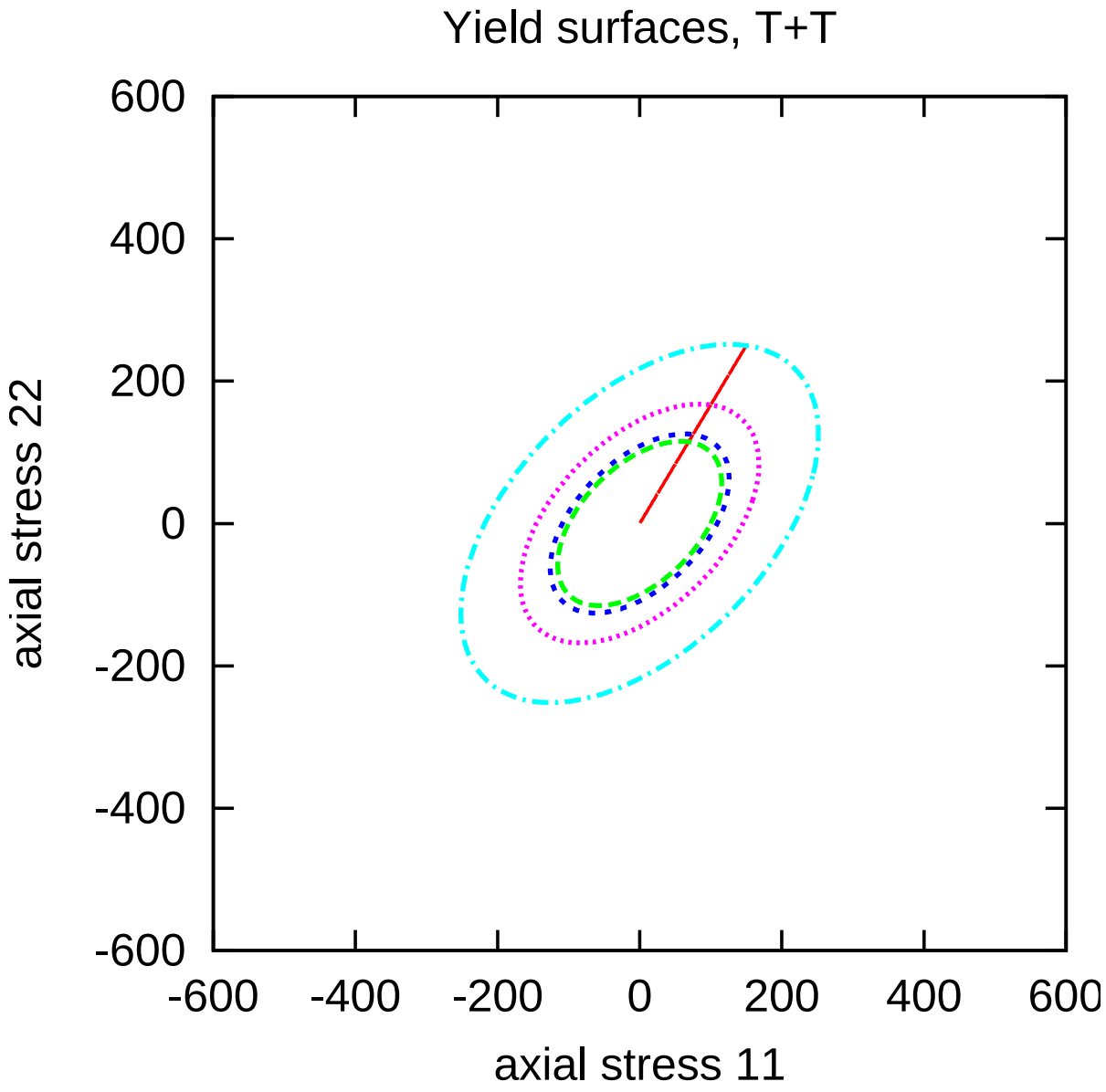
T+T Yield surface evolution

summary



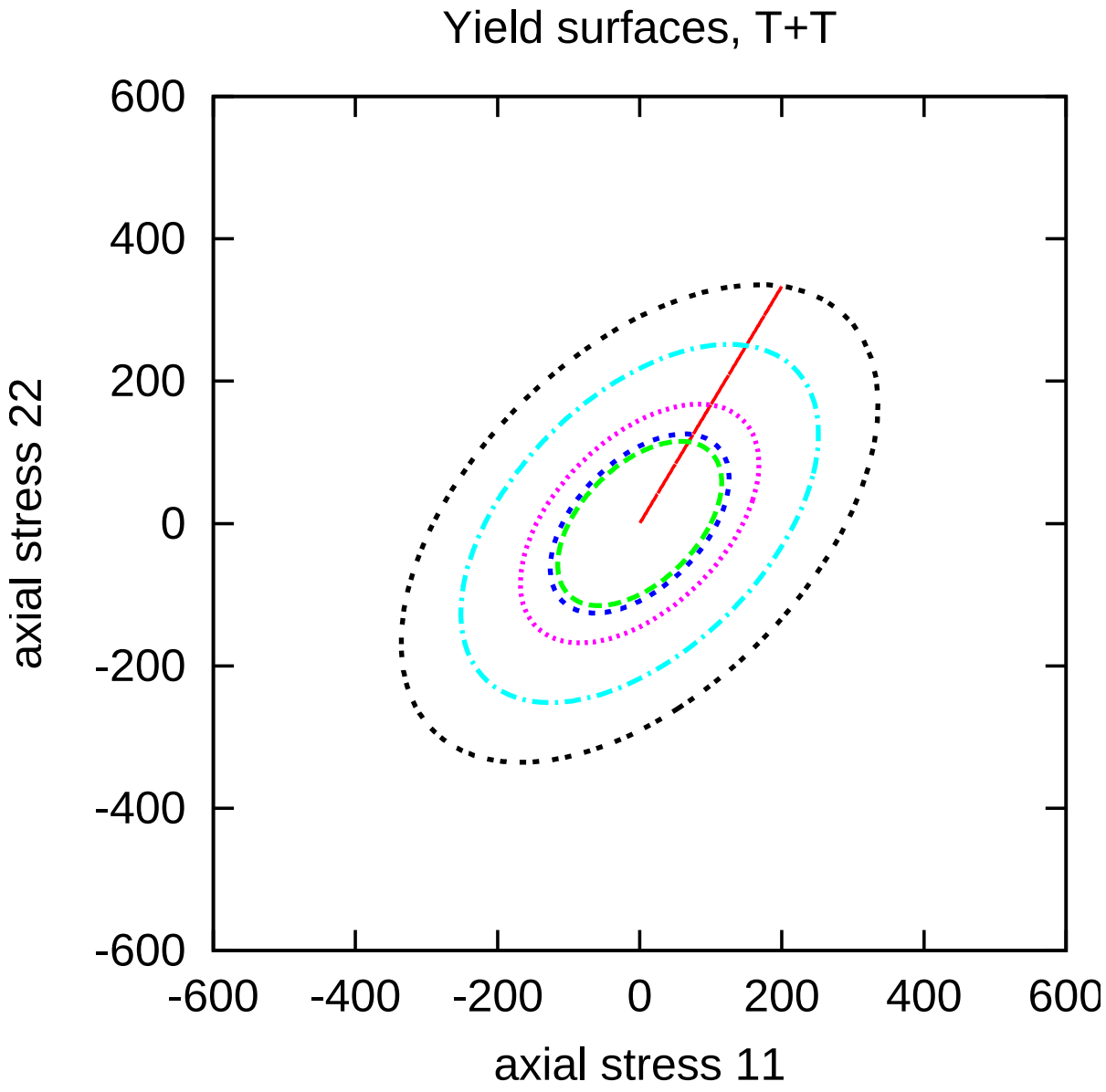
T+T Yield surface evolution

summary



T+T Yield surface evolution

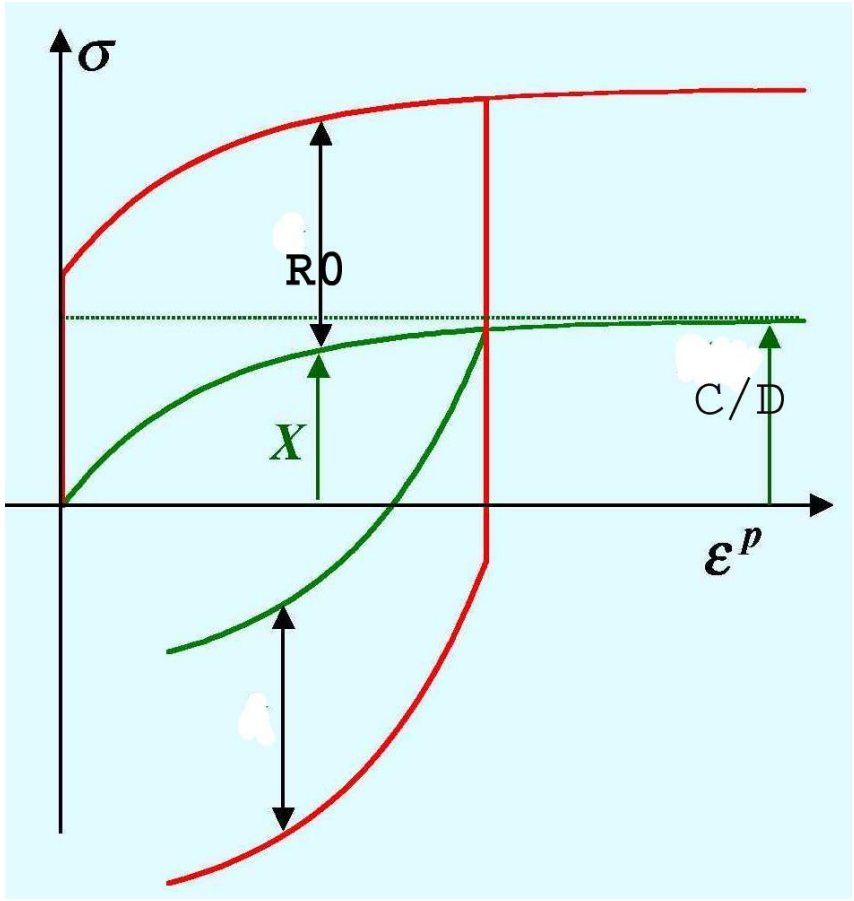
summary



Various unified models and their identification

- **Isotropic/kinematic hardening in non-pro loading**
- ***The most common effects in real world material***
  - ⋆ ***Cyclic hardening curve***
  - ⋆ **Plastic effects: criterion, hardening rules**
  - ⋆ **Viscous effects**
- **Case study: identification on a GS cast iron**

Modeling of the first cycle



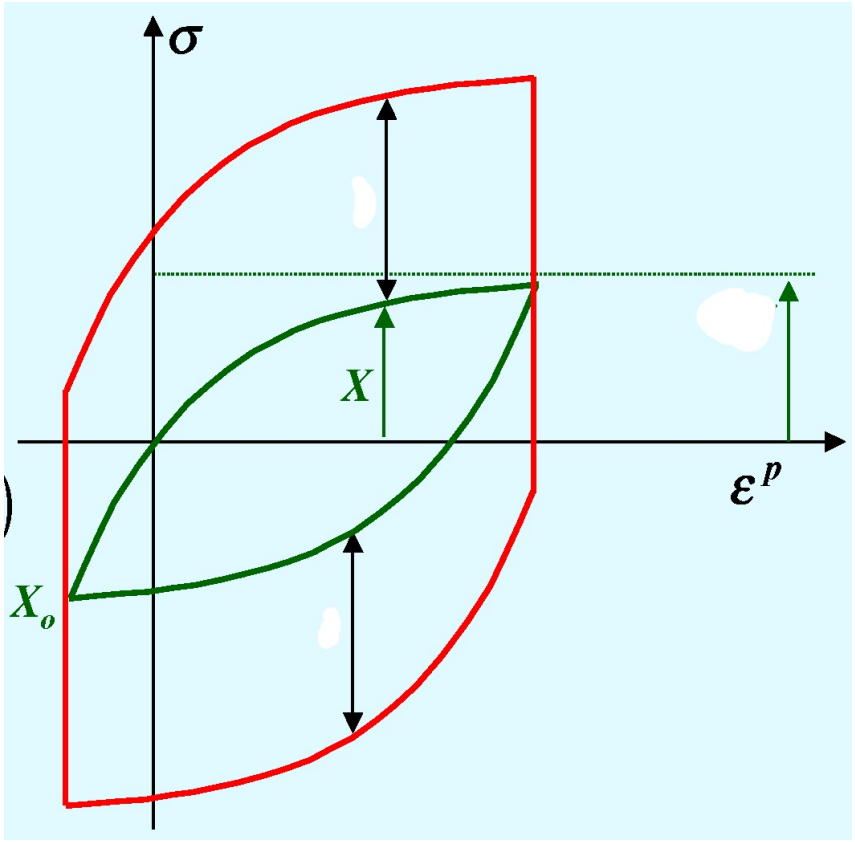
- Initial yield $R_0 = \sigma_y$
- Kinematic variable evolution:

$$dX = Cd\varepsilon^p - DX|d\varepsilon^p|$$

- Asymptotic value of X: C/D
- Initial slope: D
- General expression for the first tension

$$X = \frac{C}{D}\left(1 - \exp\left(D\varepsilon^p\right)\right)$$

Modeling of the hysteresis loops



- General expression ($\eta = +1/-1$, tension/compression branch)

$$X = \eta\frac{C}{D} + \left(X_0 - \eta\frac{C}{D}\right)\exp\left(D(\varepsilon^p - \varepsilon_0^p)\right)$$

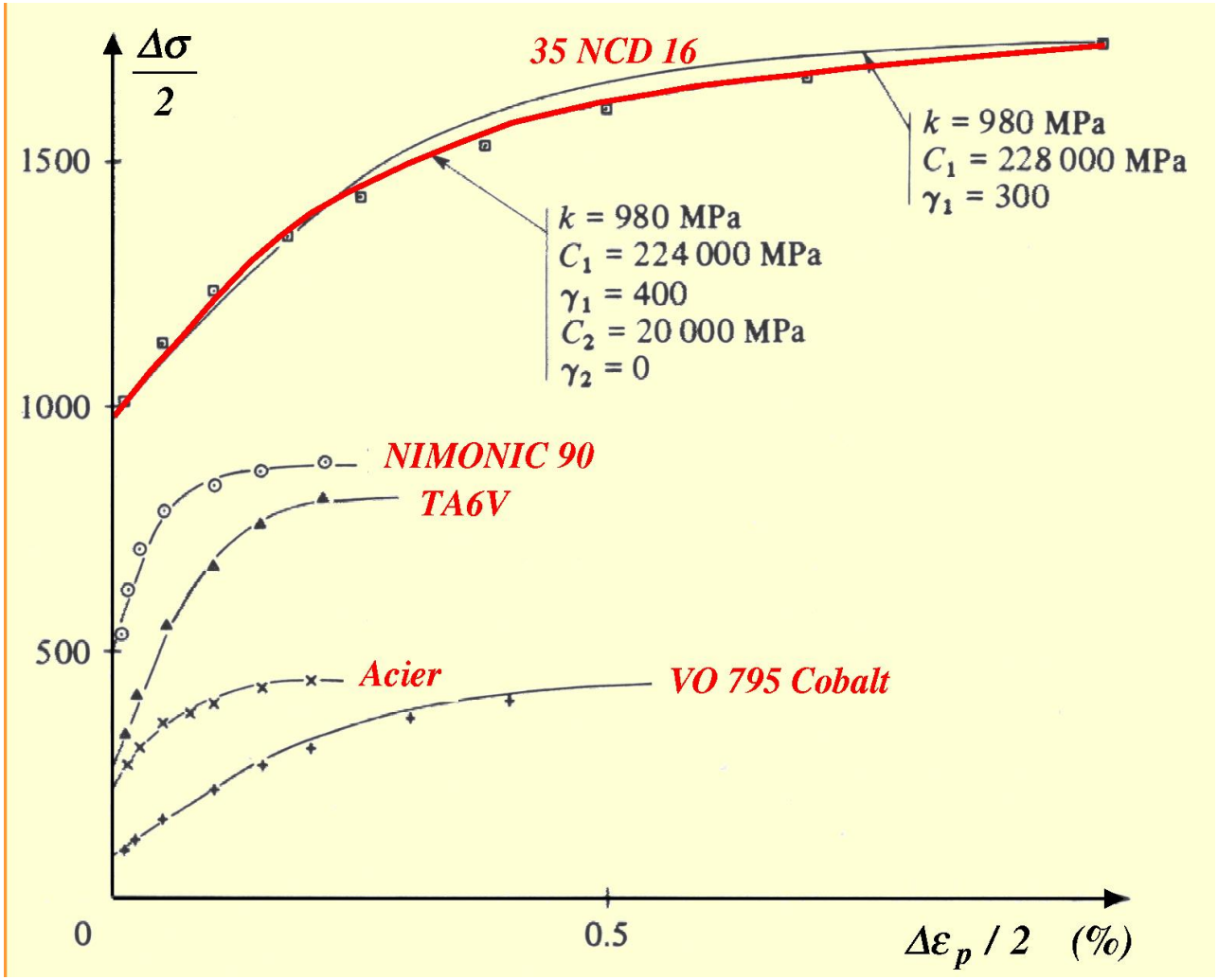
- Initial value in each loop:

$$|X_0| = \frac{C}{D}\tanh\left(D\frac{\Delta\varepsilon^p}{2}\right)$$

- Cyclic hardening curve:

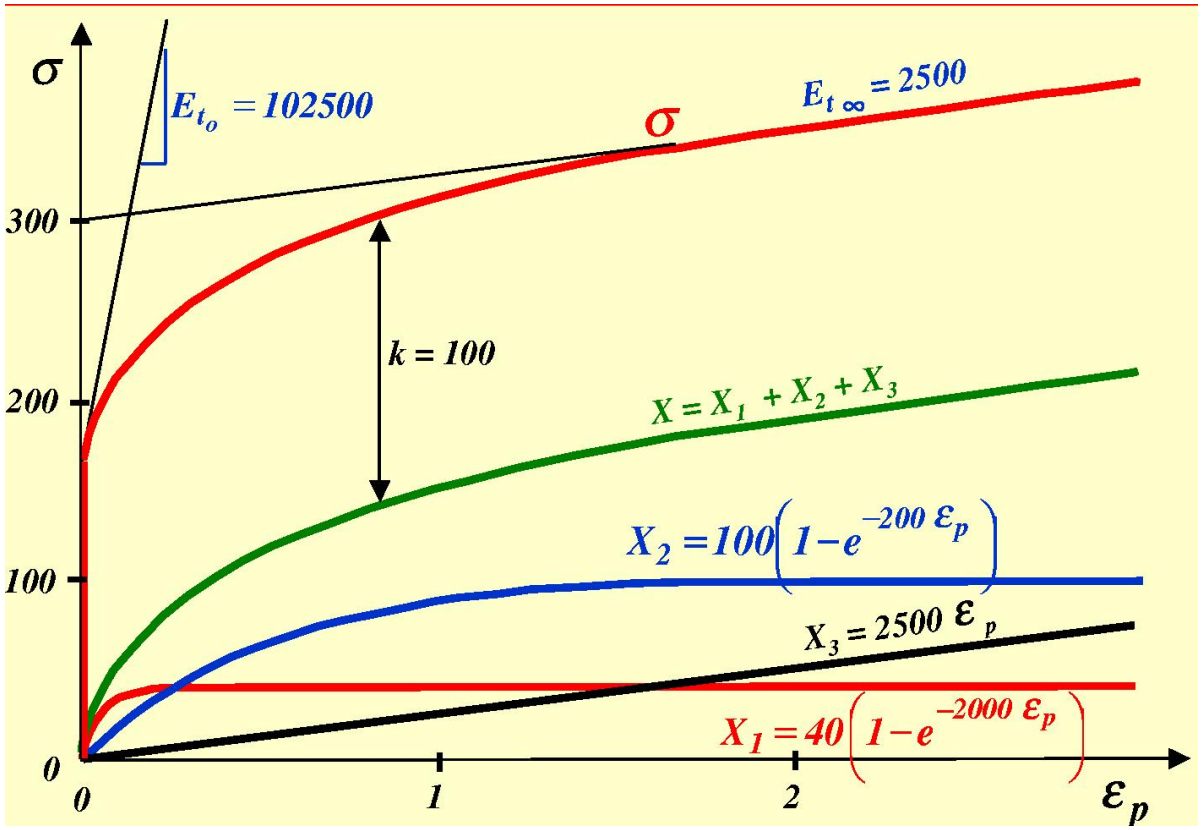
$$\frac{\Delta\sigma}{2} = \sigma_y + \frac{C}{D}\tanh\left(D\frac{\Delta\varepsilon^p}{2}\right)$$

Cyclic hardening curves for several materials



Obtained by plotting the top of the hysteresis loops

Multikinematic hardening



- Introduce several kinematic variables, $\underset{\sim}{X}_i$

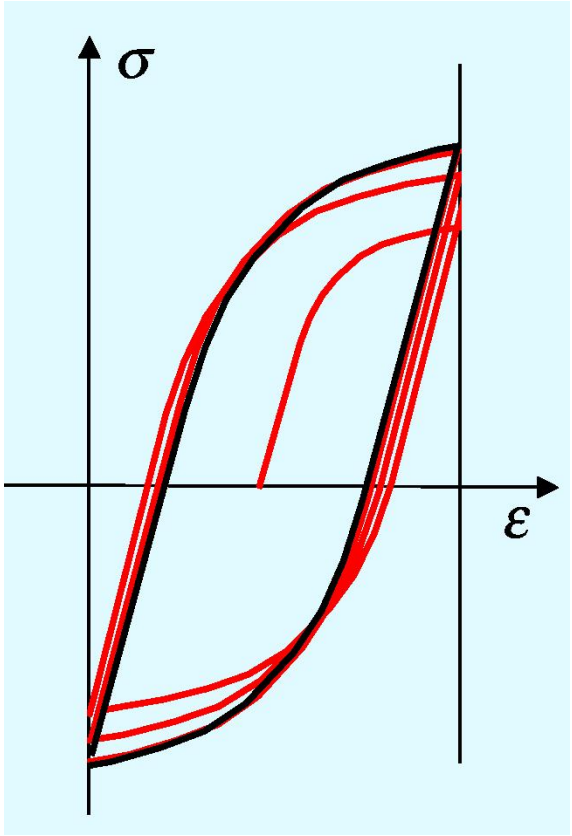
$$\dot{\underset{\sim}{X}}_i = \frac{2}{3} C_i \dot{\underset{\sim}{\varepsilon}}^p - D_i \underset{\sim}{X}_i \dot{p}$$

- Consider the total kinematic hardening as:

$$\underset{\sim}{X} = \sum_i \underset{\sim}{X}_i$$

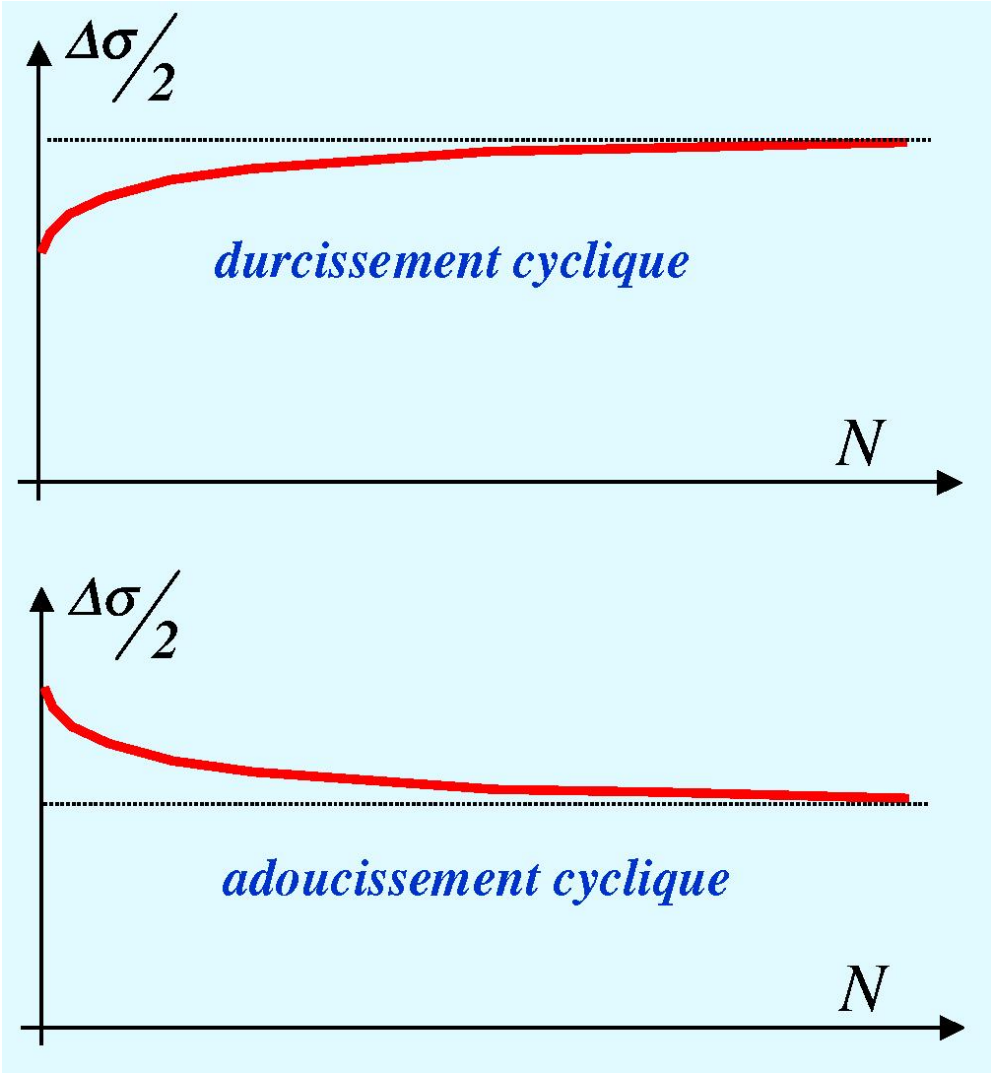
Development for given values or D_i (including 0, for linear kinematic hardening)

Simulation of cyclic hardening/cyclic softening



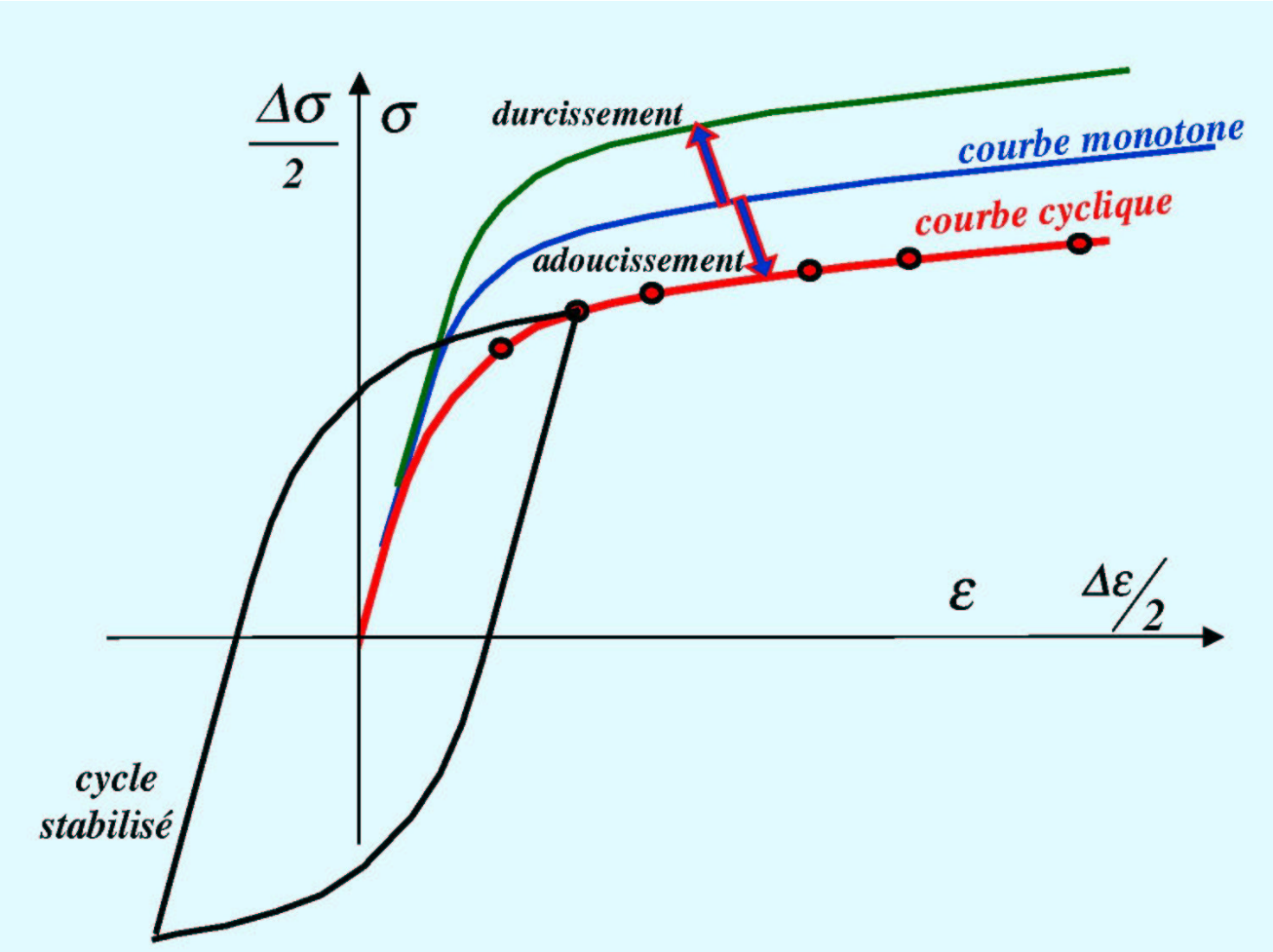
$$R = \sigma_y + Q\,(1 - \exp(-bp))$$

Hardening, $Q > 0$



Softening, $Q < 0$

Comparison between monotonic and cyclic curves



Rapid evolution for kinematic hardening: $100 < D < 5000$

Slow variation for isotropic hardening: $1 < b < 50$

Various unified models and their identification

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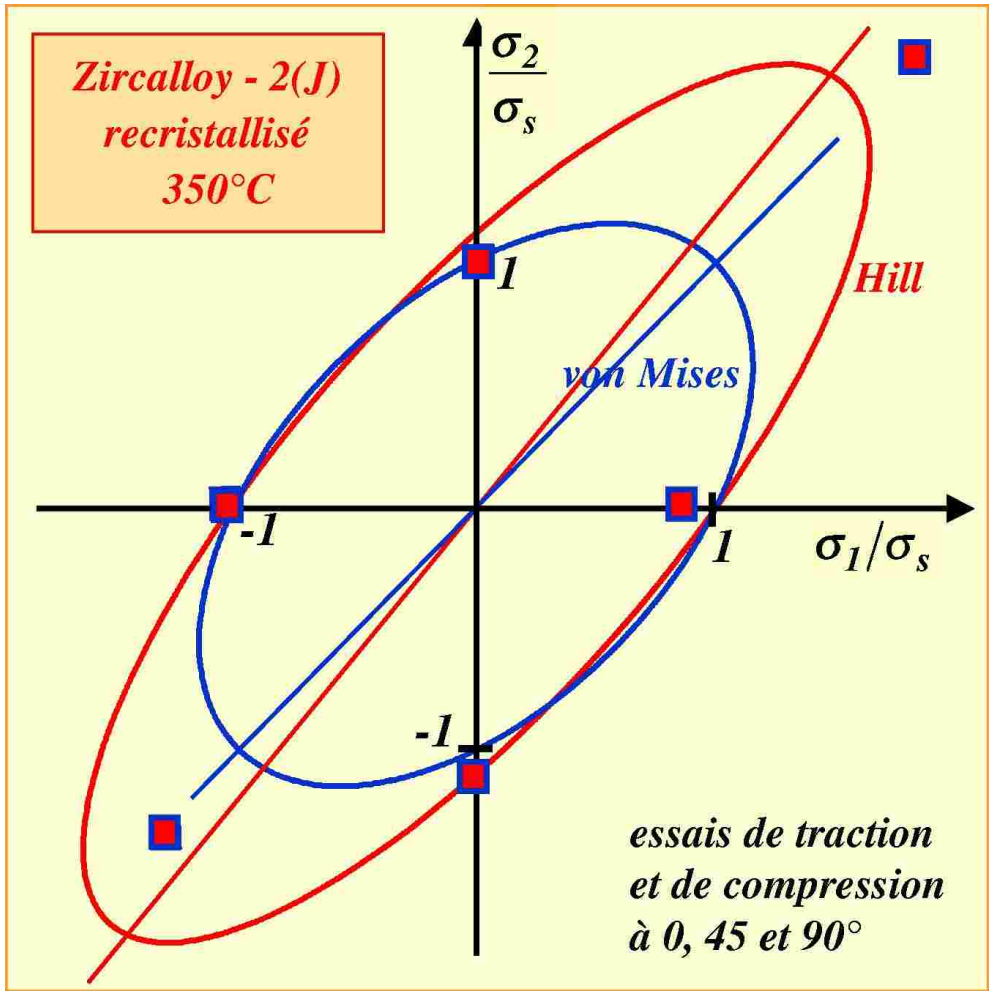
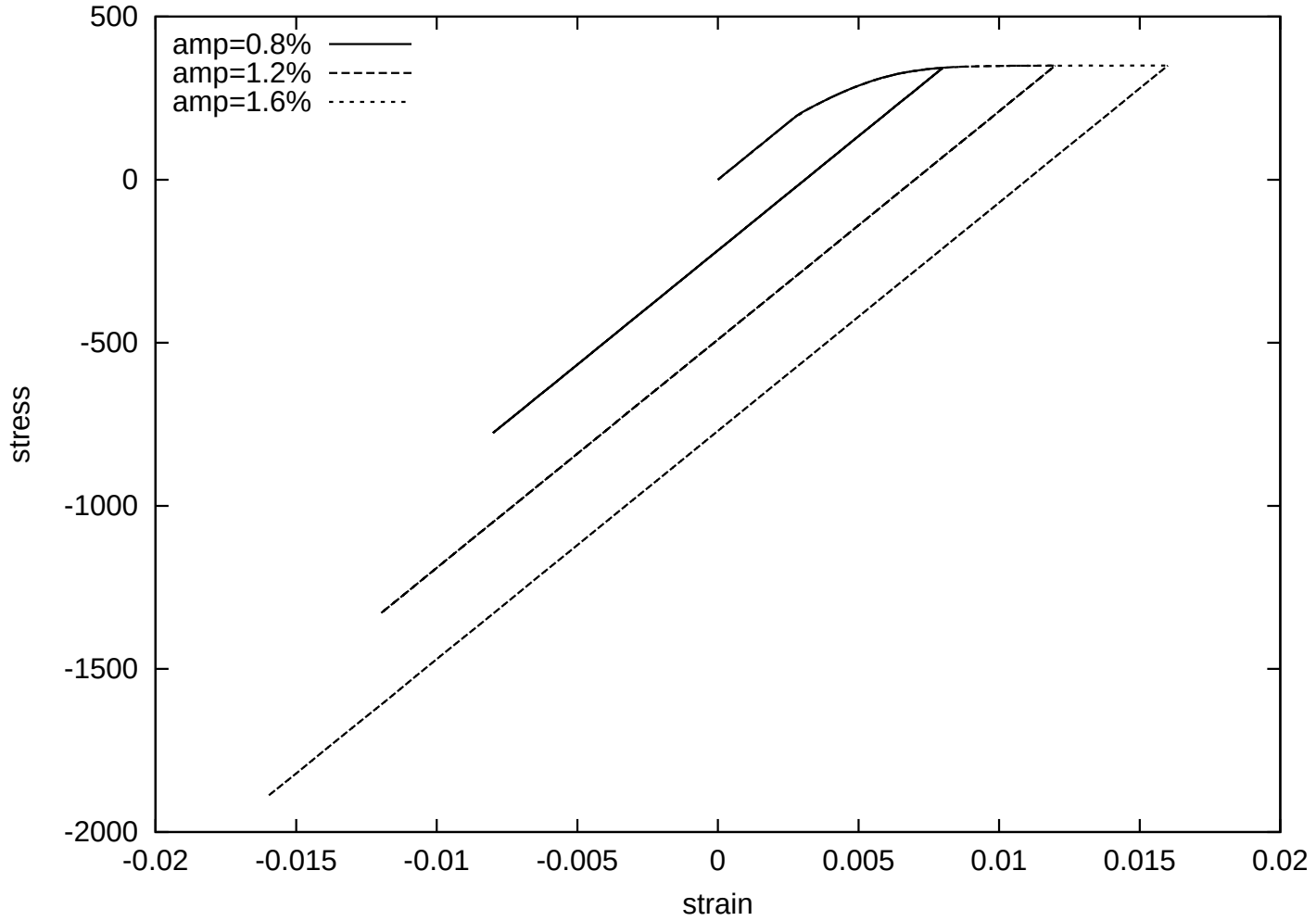
Hill's criterion



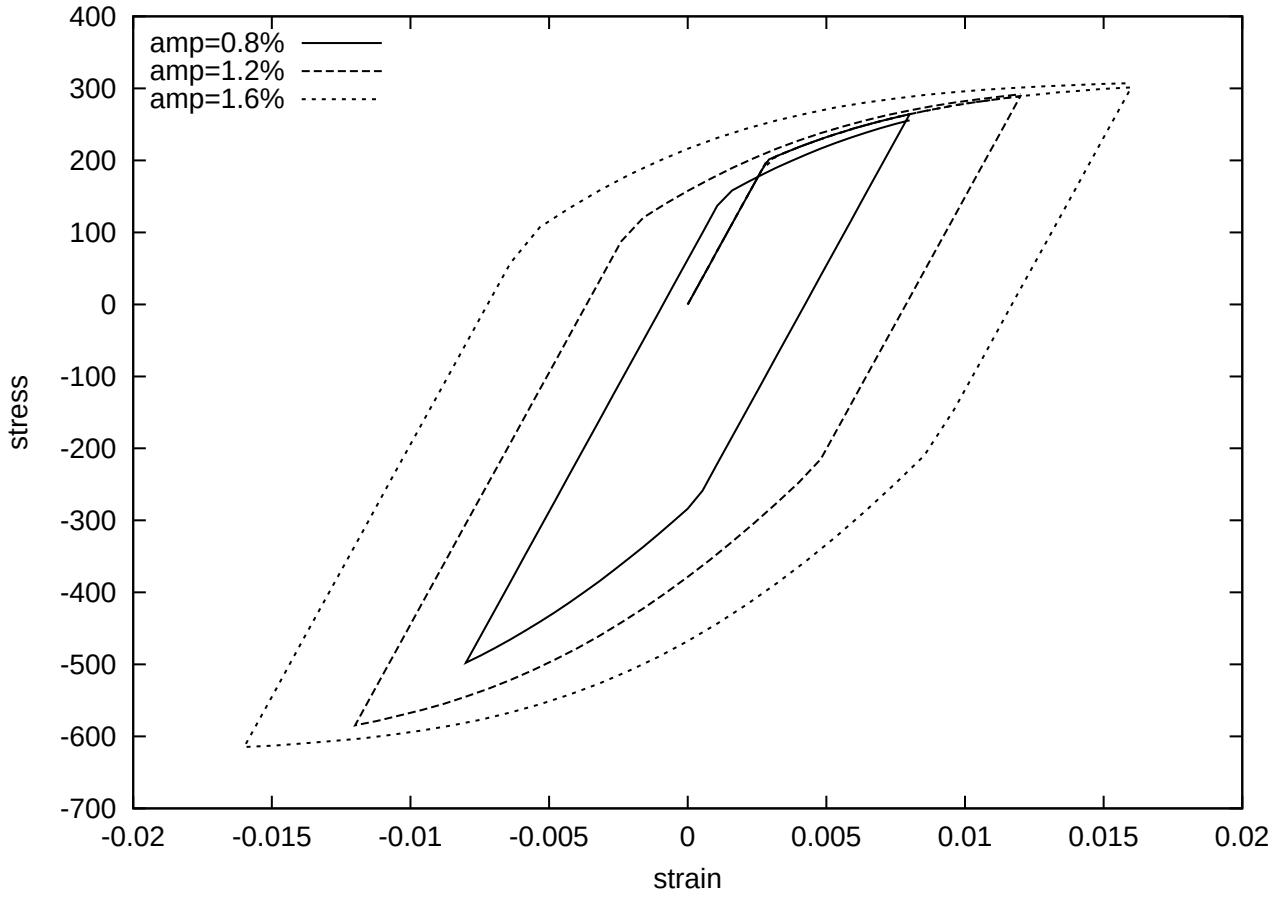
Hill criterion for biaxial tension

Yield in tension only



If $\mathrm{Tr}\,\sigma > 0$, *Von Mises criterion; else elastic behavior*

Unsymmetrical flow

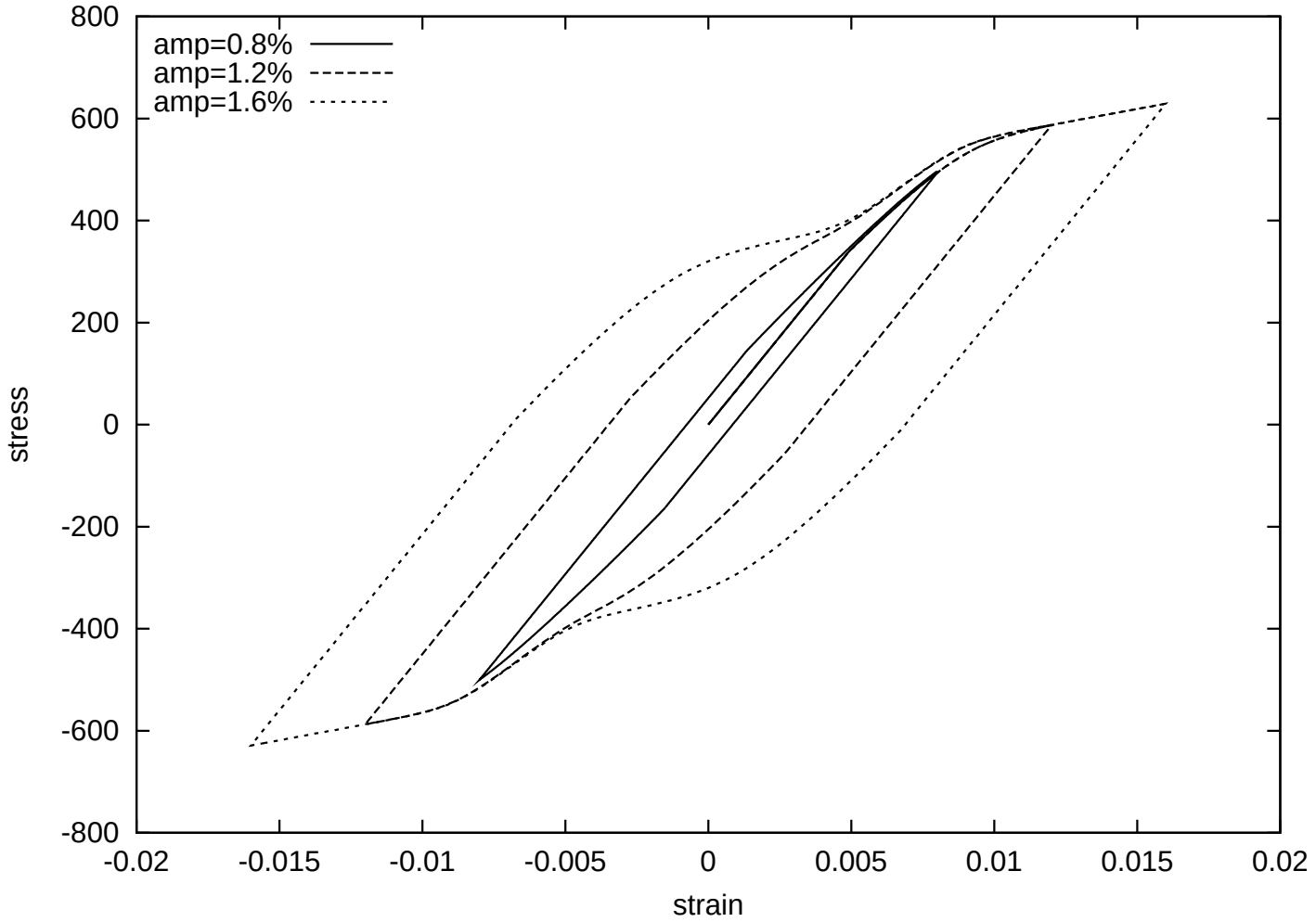


Non associated plasticity: von Mises type flow direction; Drucker-Prager type flow intensity

$$f(\underset{\sim}{\boldsymbol{\sigma}}, \underset{\sim}{\boldsymbol{X}}) = (1-\alpha) J(\underset{\sim}{\boldsymbol{\sigma}} - \underset{\sim}{\boldsymbol{X}}) + \alpha \mathrm{Tr}\,(\sigma - \underset{\sim}{\boldsymbol{X}}) - R_O$$

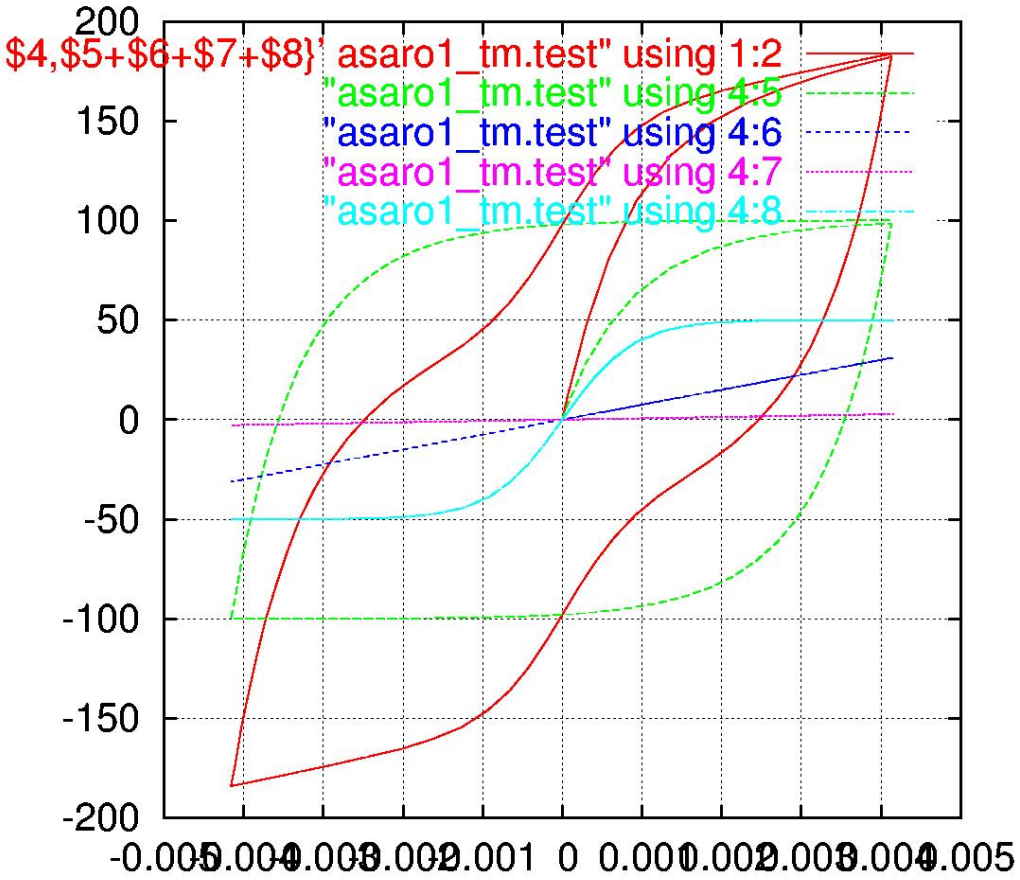
$$\sigma_y^t = R_0 \qquad \sigma_y^c = (1-2\alpha)\sigma_y^c$$

Non-linear-but-without-hysteresis kinematic hardening (1)



S-shape *(type III – Asaro model) for aluminium alloys*

Ingredients of the S-shape kinematic model

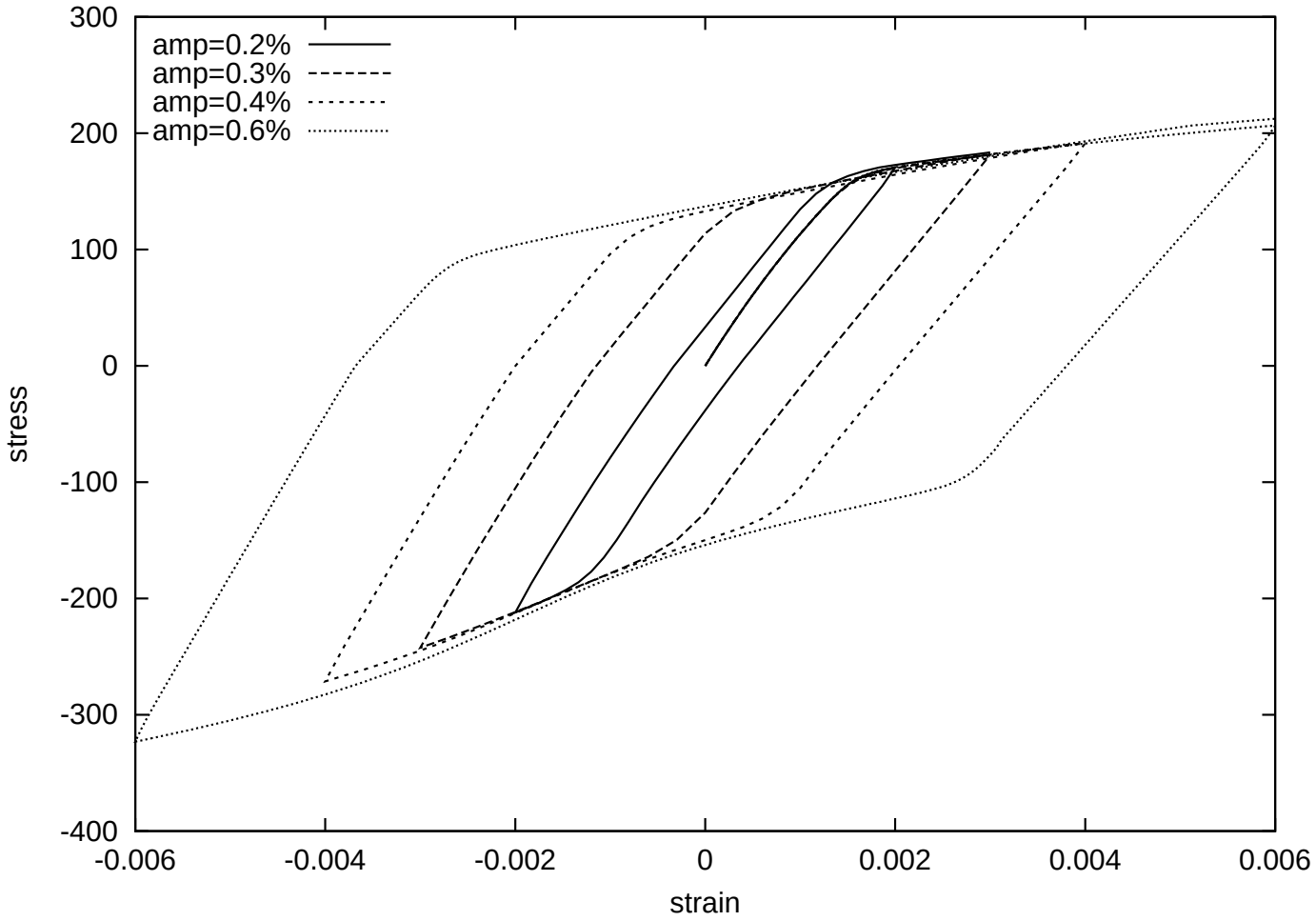


$$\underset{\sim}{\boldsymbol{X}}_i = C\frac{\tanh(D\,\alpha_{eq})}{D\,\alpha_{eq}}\underset{\sim}{\boldsymbol{\alpha}}_i$$

$$\alpha_{eq} = \left(1.5\underset{\sim}{\boldsymbol{\alpha}} : \underset{\sim}{\boldsymbol{\alpha}}\right)^{1/2}$$

$$\underset{\sim}{\boldsymbol{\alpha}} = \underset{\sim}{\boldsymbol{\varepsilon}}^p$$

Cast iron (1)



Experiments by Hjelm (JEMT, 1994), Hjelm *et al* (JEMT, 1995)

Ingredients of the cast iron model

- Modified criterion :

$$f_t = \left(J^2 + (R_c - R_t)\mathrm{Tr}\,\underset{\sim}{\boldsymbol{\sigma}}\right)^{1/2} - (R_t R_c)^{1/2}$$
$$f_c = J - R_c$$

- Unsymmetric kinematic variable :

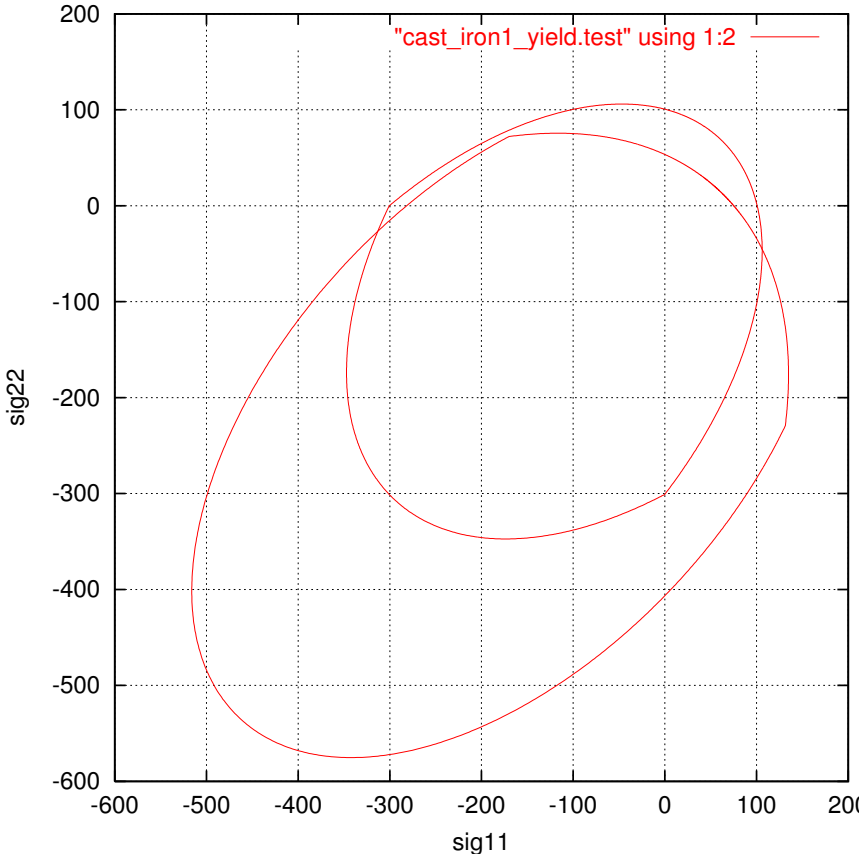
$$\dot{\underset{\sim}{\boldsymbol{\alpha}}} = \dot{\lambda}(\underset{\sim}{\boldsymbol{n}} - \frac{3D}{2C}\underset{\sim}{\boldsymbol{X}})$$

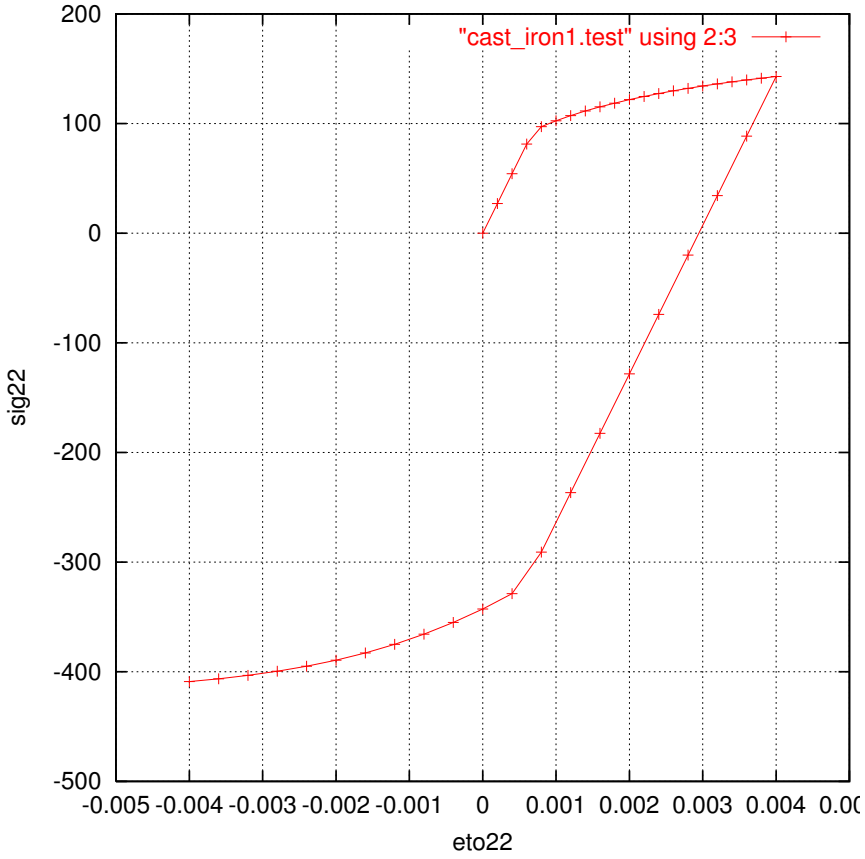
Coefficient set (C, D) is (D_t, C_t) in tension, (D_c, C_c) in compression.

- "Tension" means:

$$tr\sigma > \sigma_O$$

Cast iron: initial and subsequent yield surface





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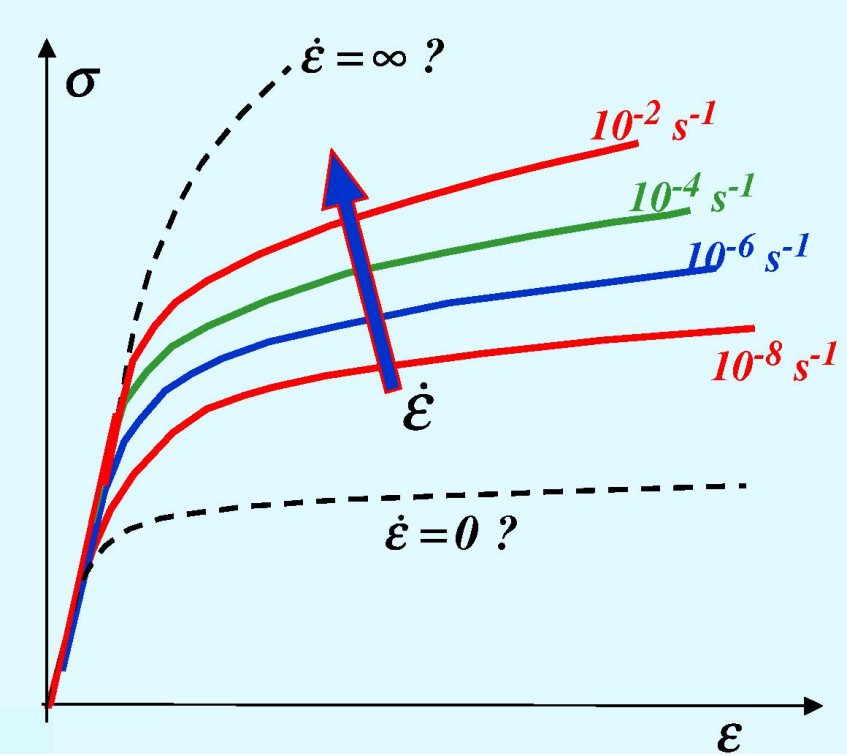
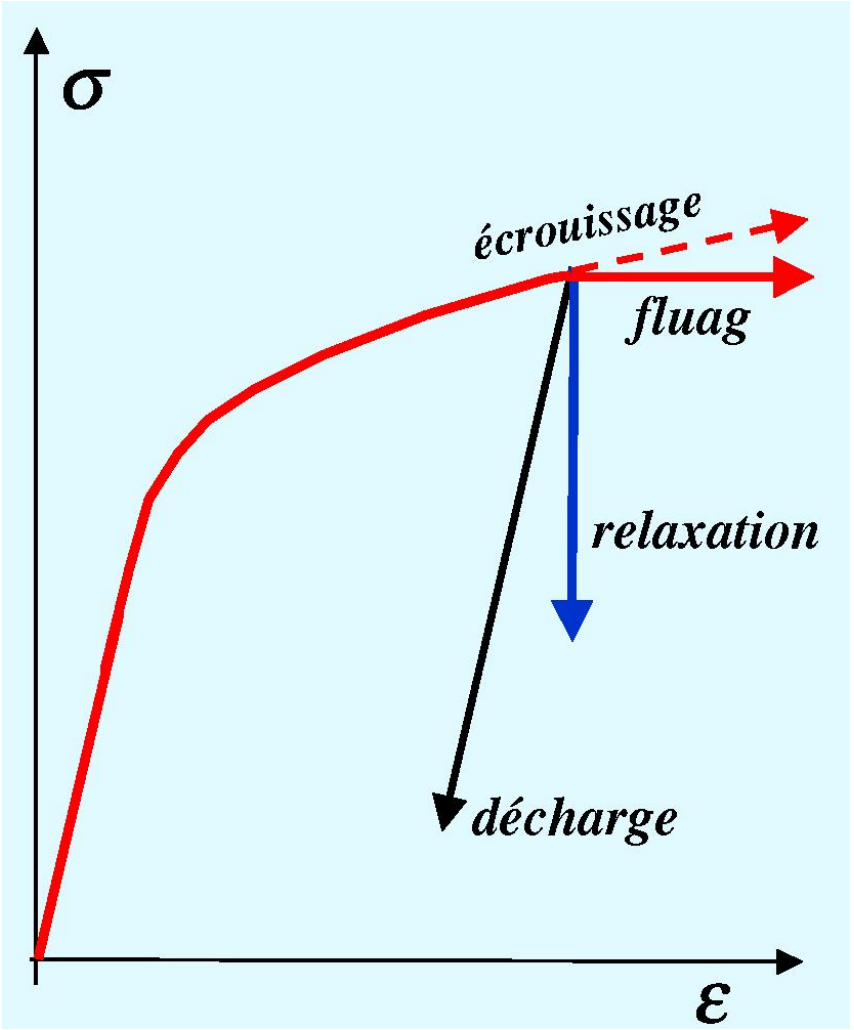
Multikinematic hardening



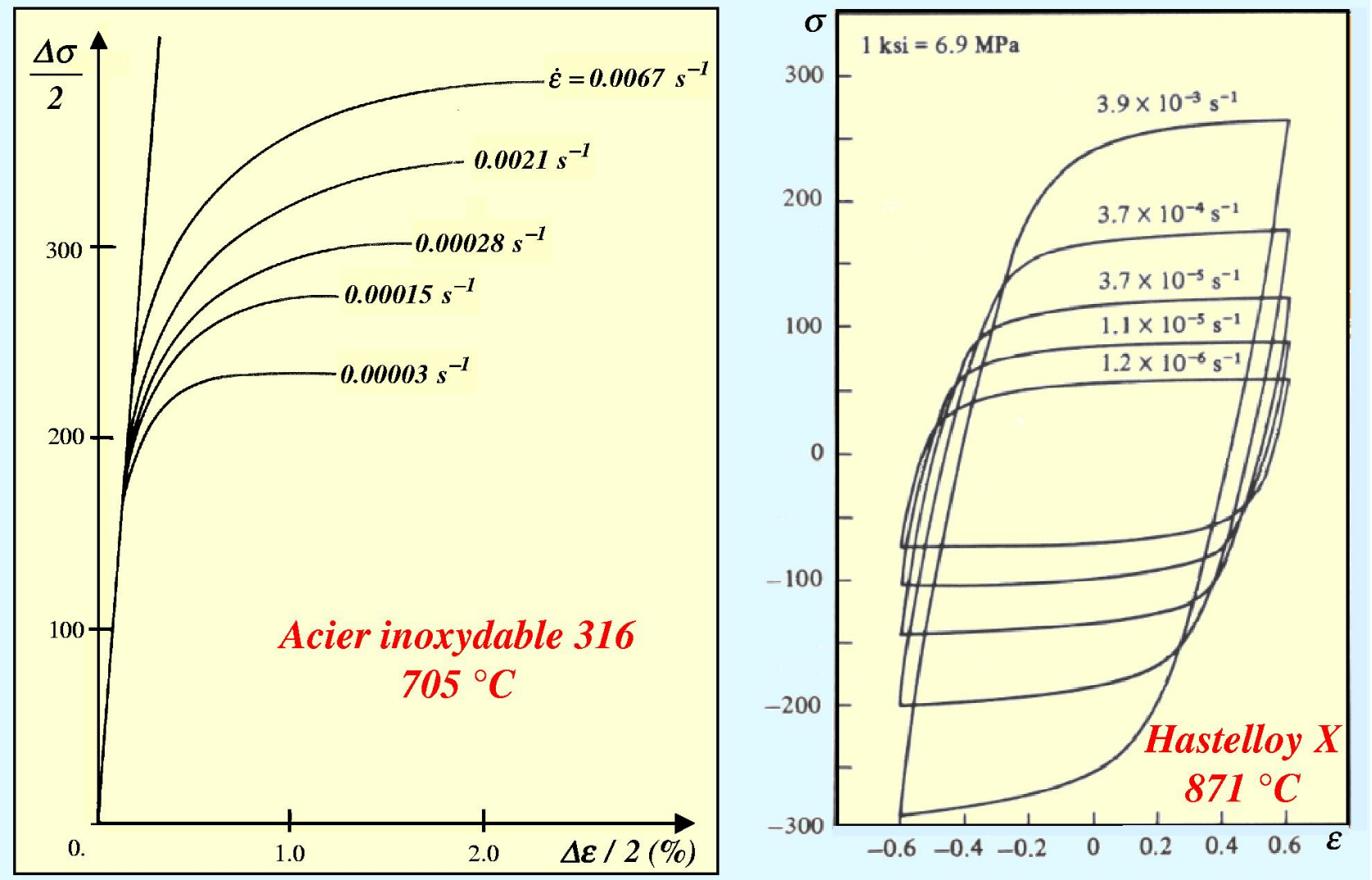
Strain rate effect



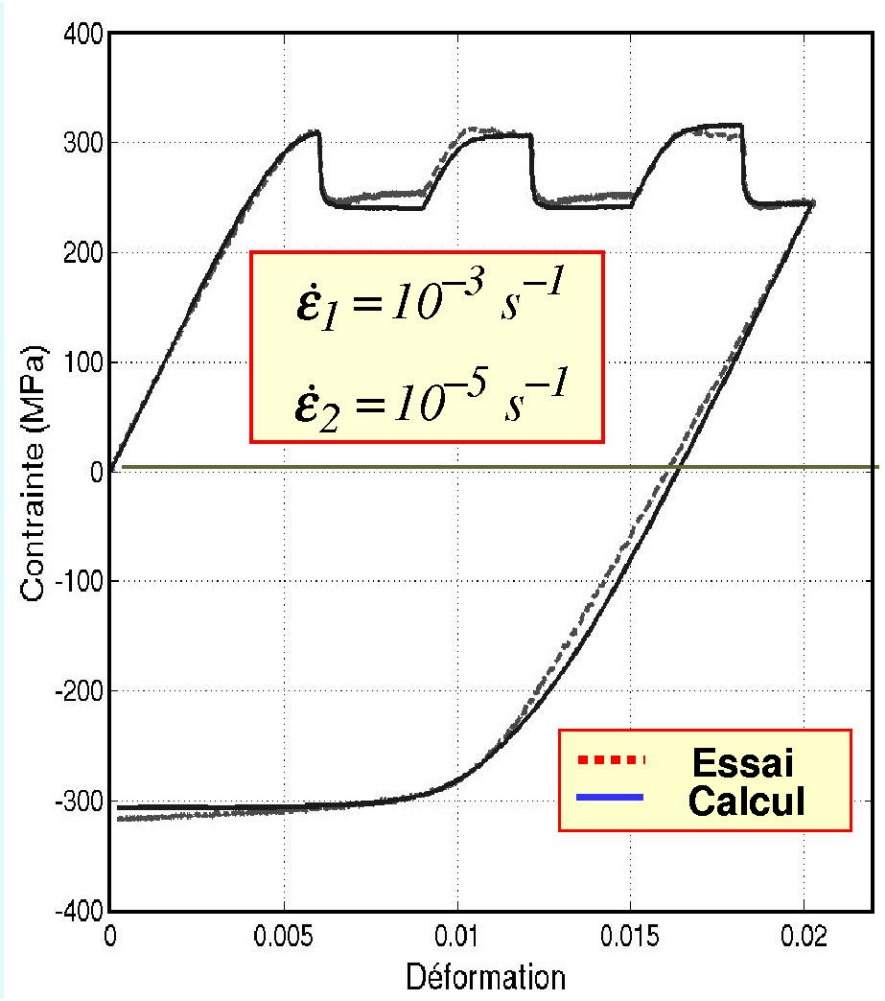
Specific tests for viscous materials

Creep, relaxation

Classical strain rate effect

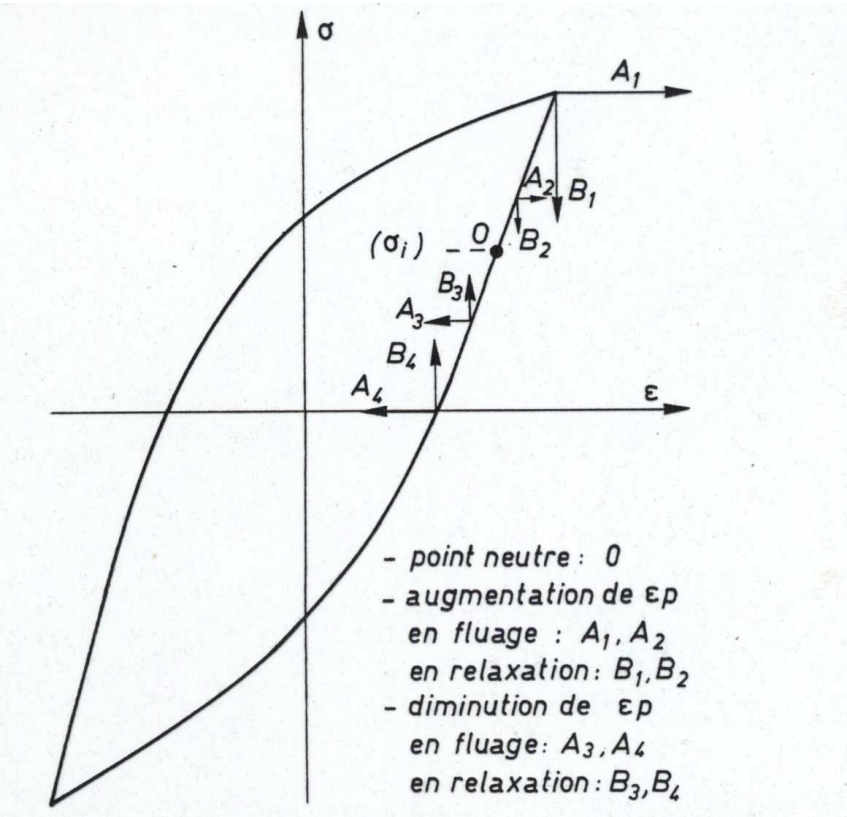


Strain rate jumps on an aluminium alloy

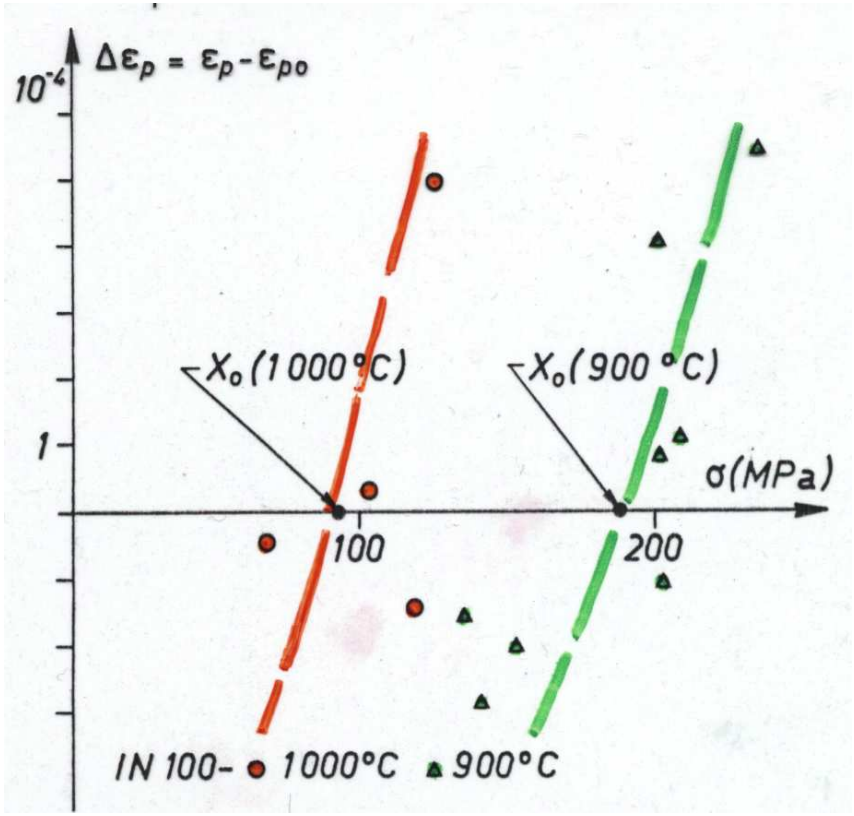


State T7351, 173° C

Experimental evaluation of the internal stress in 1D

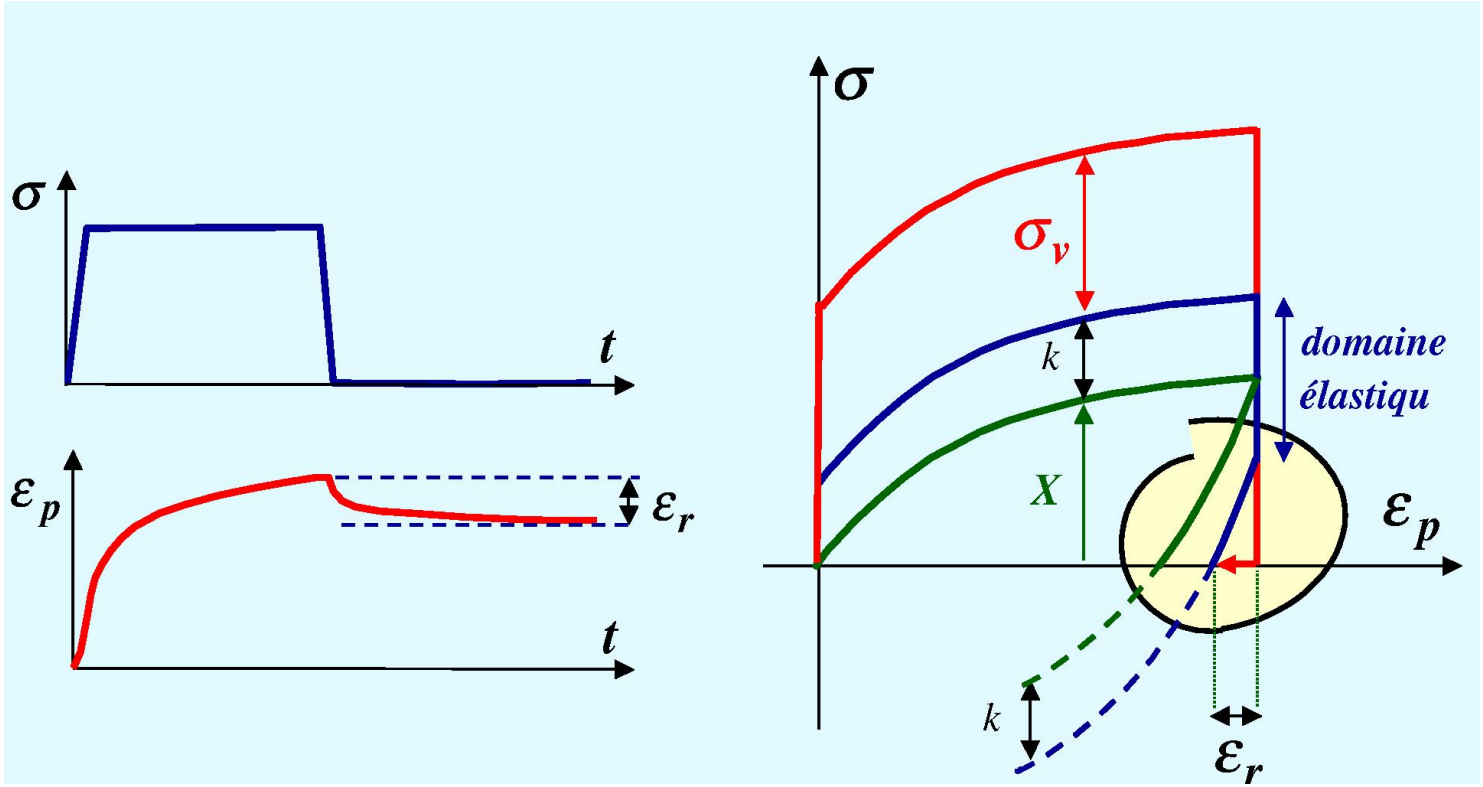


Various loading paths

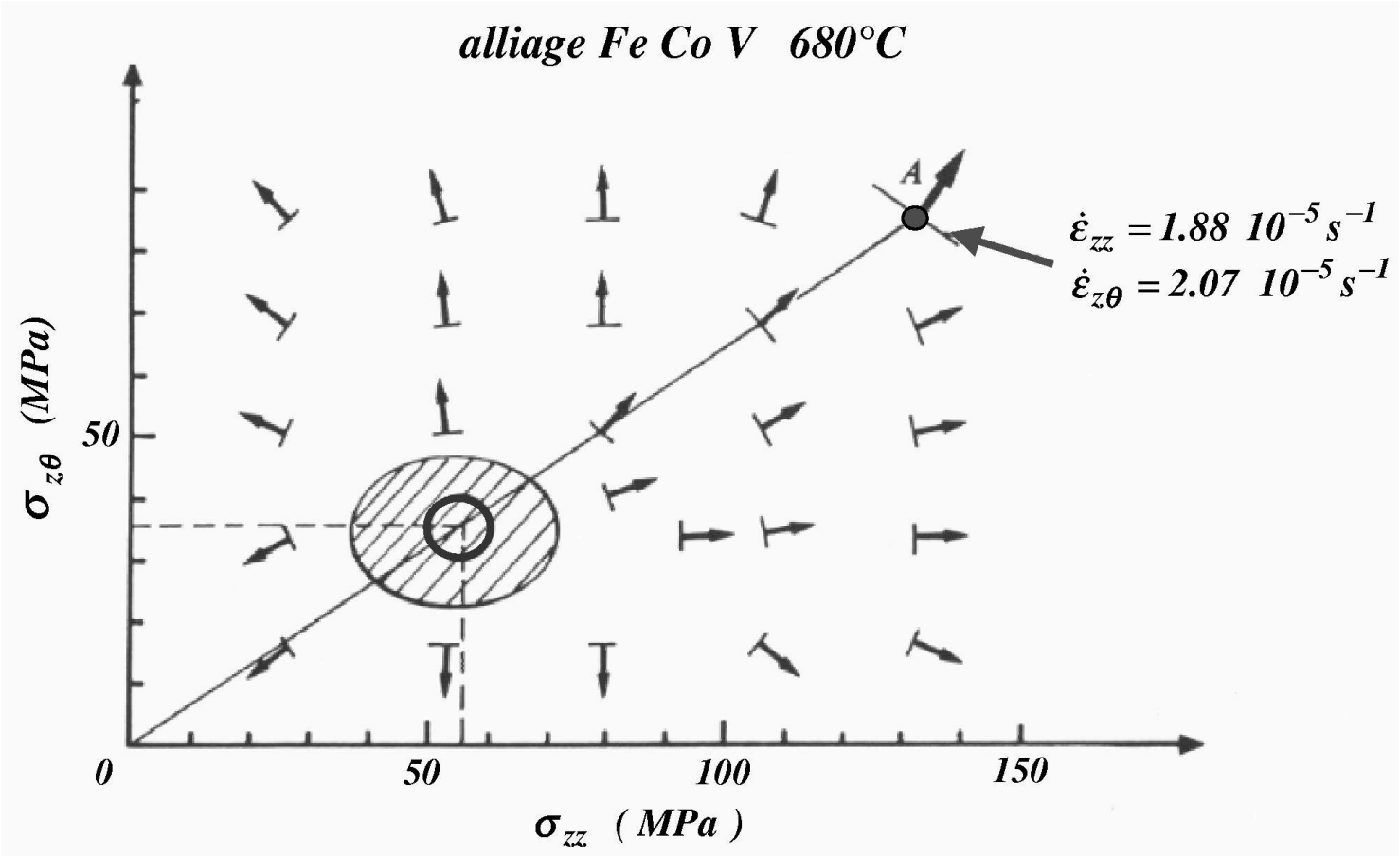


Result on IN100 alloy (creep after cyclic load)

Strain recovery

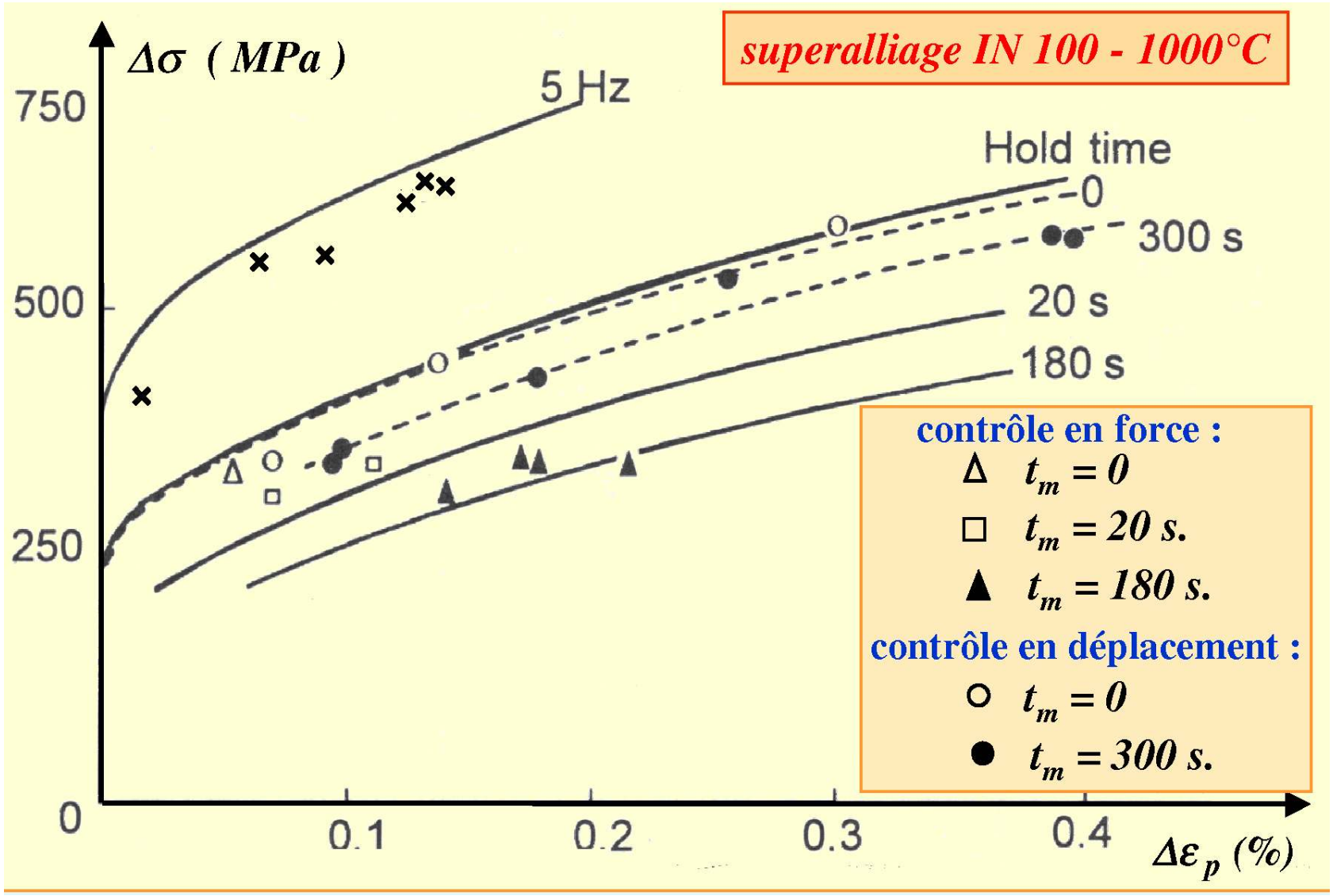


Experimental evaluation of the internal stress in 2D



Test Oytana, University of Besançon

Resulting cyclic hardening curves, IN100 alloy



Stress–strain loops, IN100 alloy

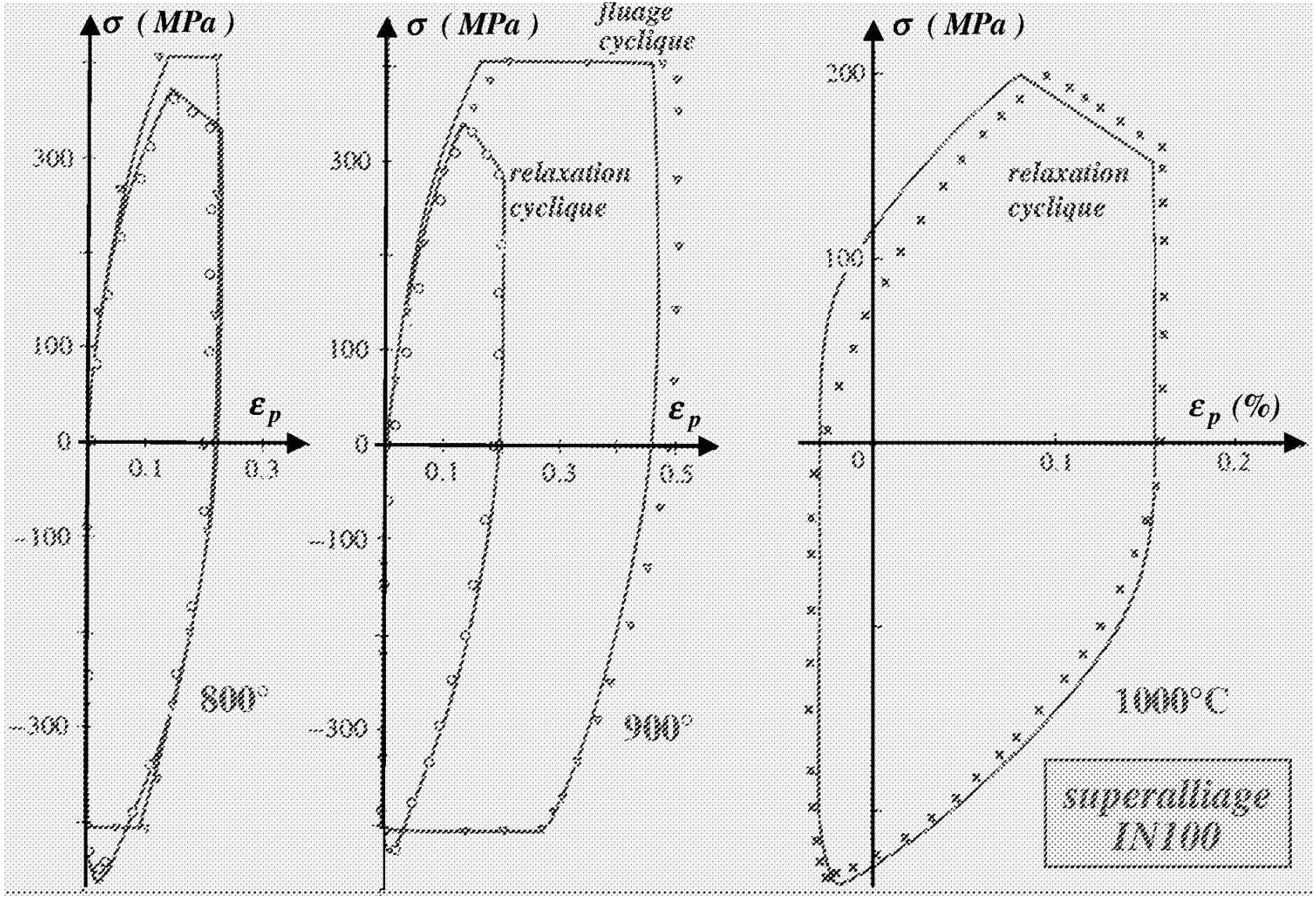
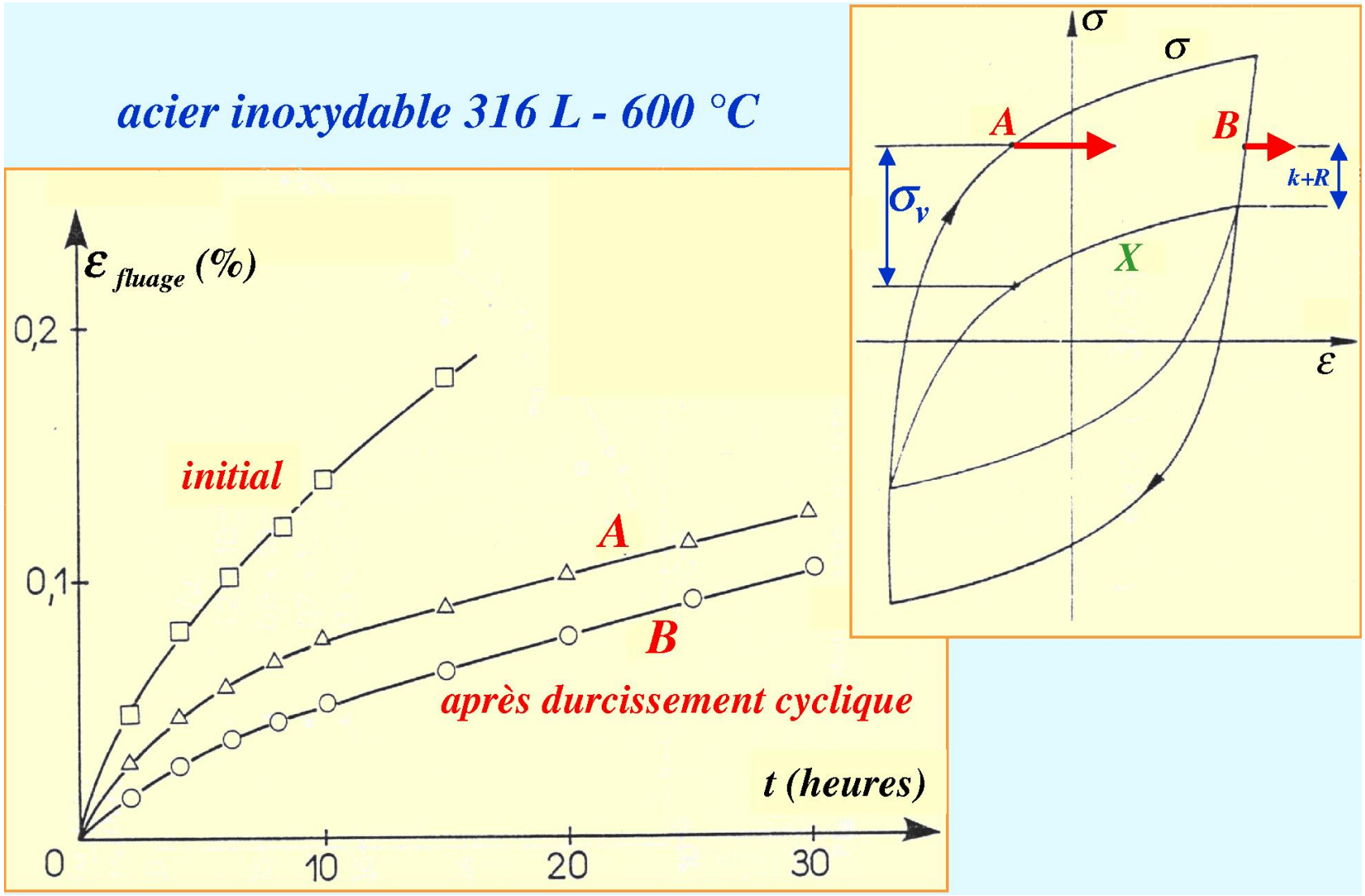
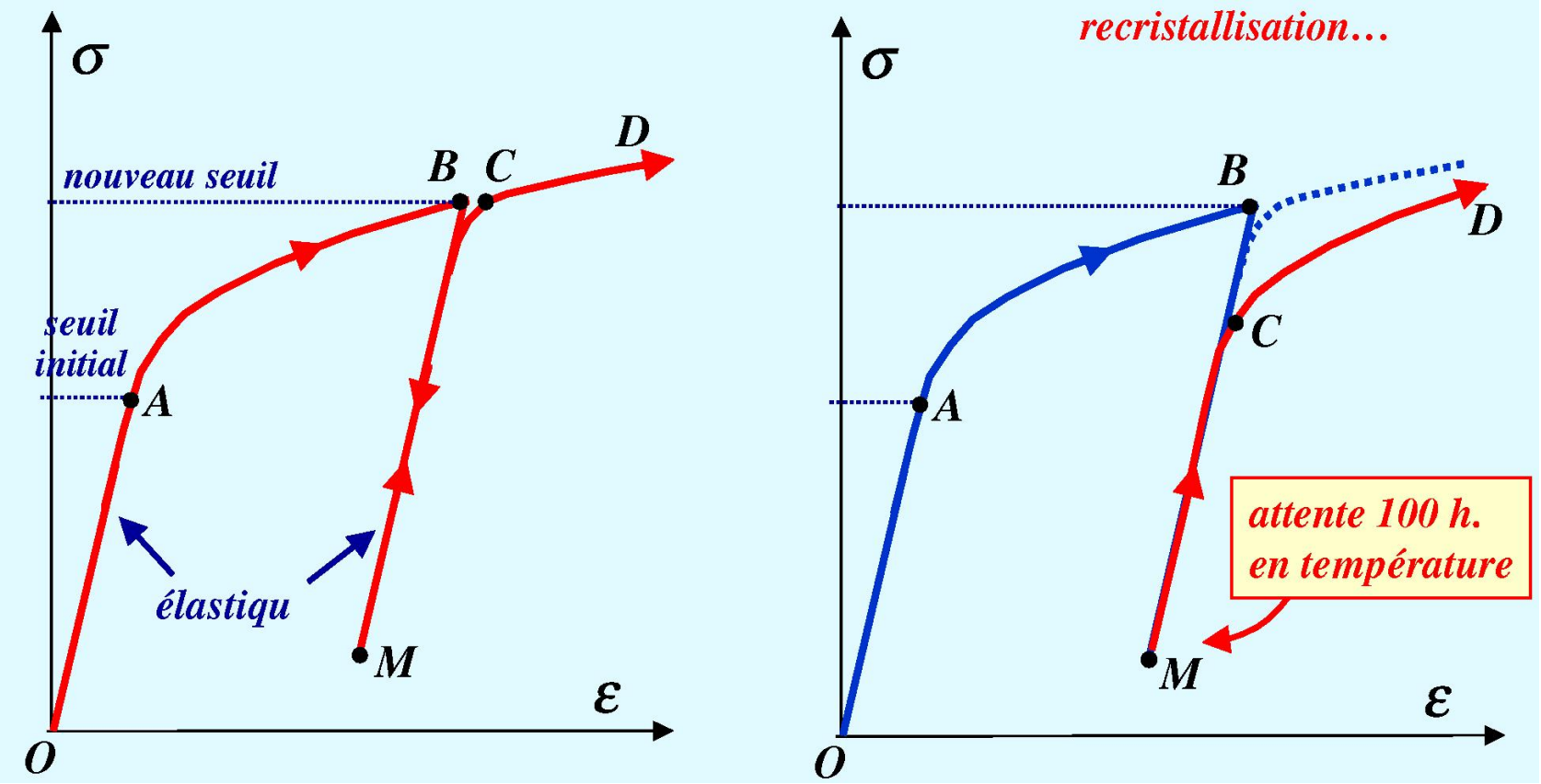


Illustration of the coupling between plasticity and creep



316 stainless steel, tests by Goodall

Hardening/recovery



Hardening

Recovery

Change in hardening law to describe recovery

- Kinematic hardening

$$\dot{\underset{\sim}{\alpha}} = \dot{\underset{\sim}{\varepsilon}}^p - \frac{3C}{2D}\underset{\sim}{X}\dot{p} - \left(\frac{J(\underset{\sim}{X})}{M}\right)^m$$

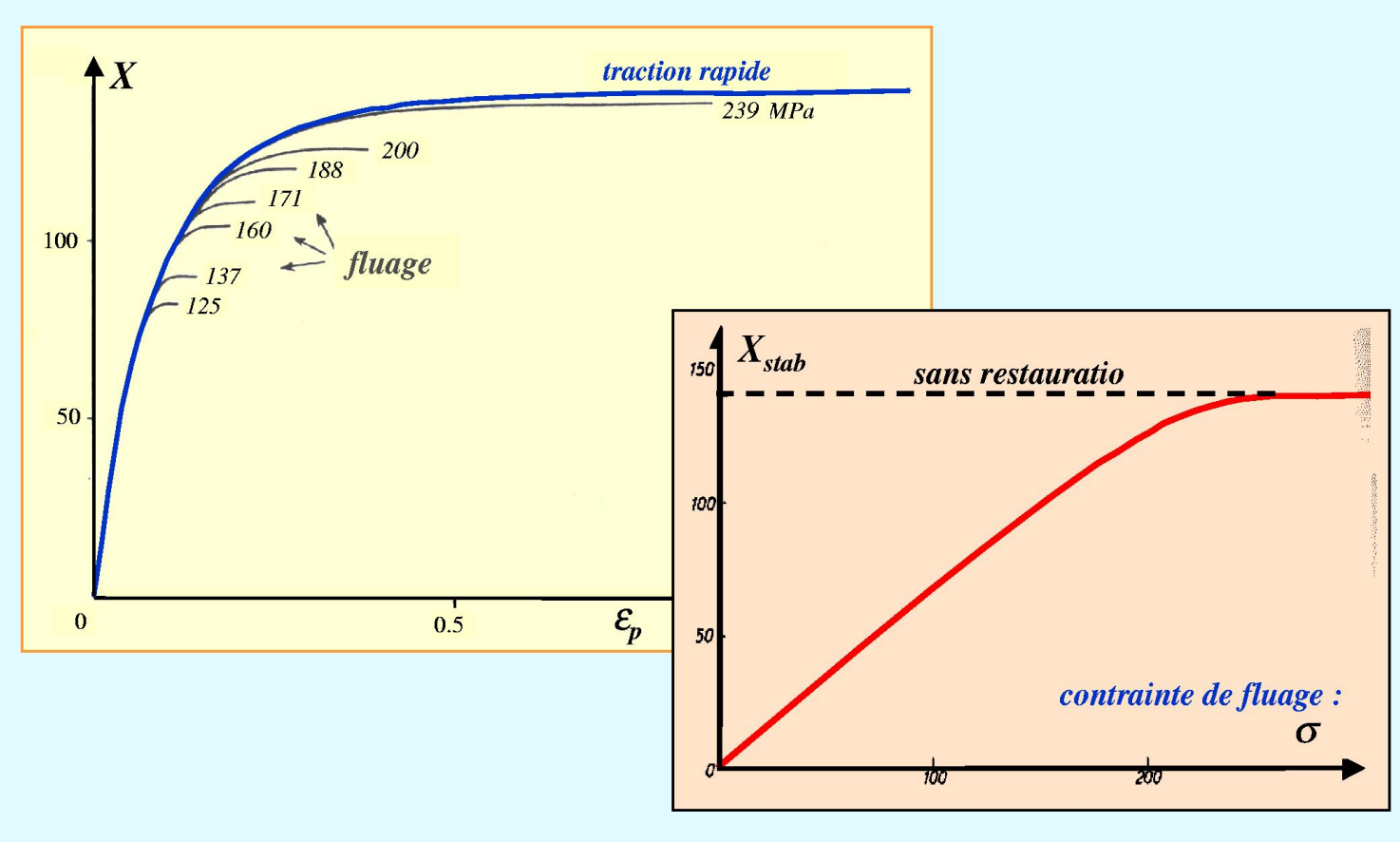
with $J(\underset{\sim}{X}) = \left(\underset{\sim}{X} : \underset{\sim}{X}\right)^{1/2}$

- Isotropic hardening

$$\dot{r} = \left(1 - \frac{R}{Q}\right)\dot{p} - \left(\frac{R}{M'}\right)^{m'}$$

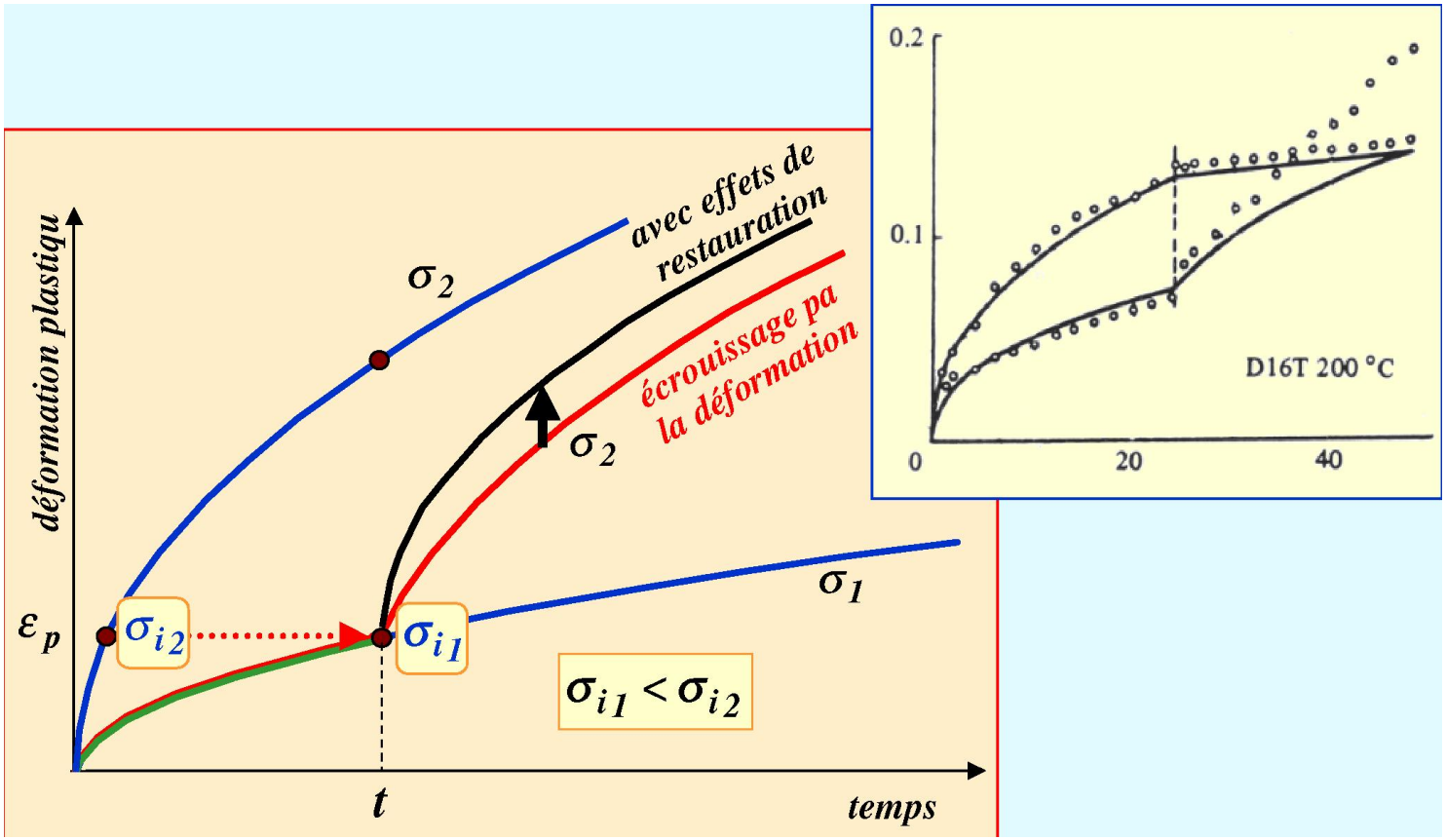
This meets the classical metallurgical models, like Orowan's, using one driving, strain dependent term, for hardening, and a time dependent term for fading memory

Resulting internal stress in creep



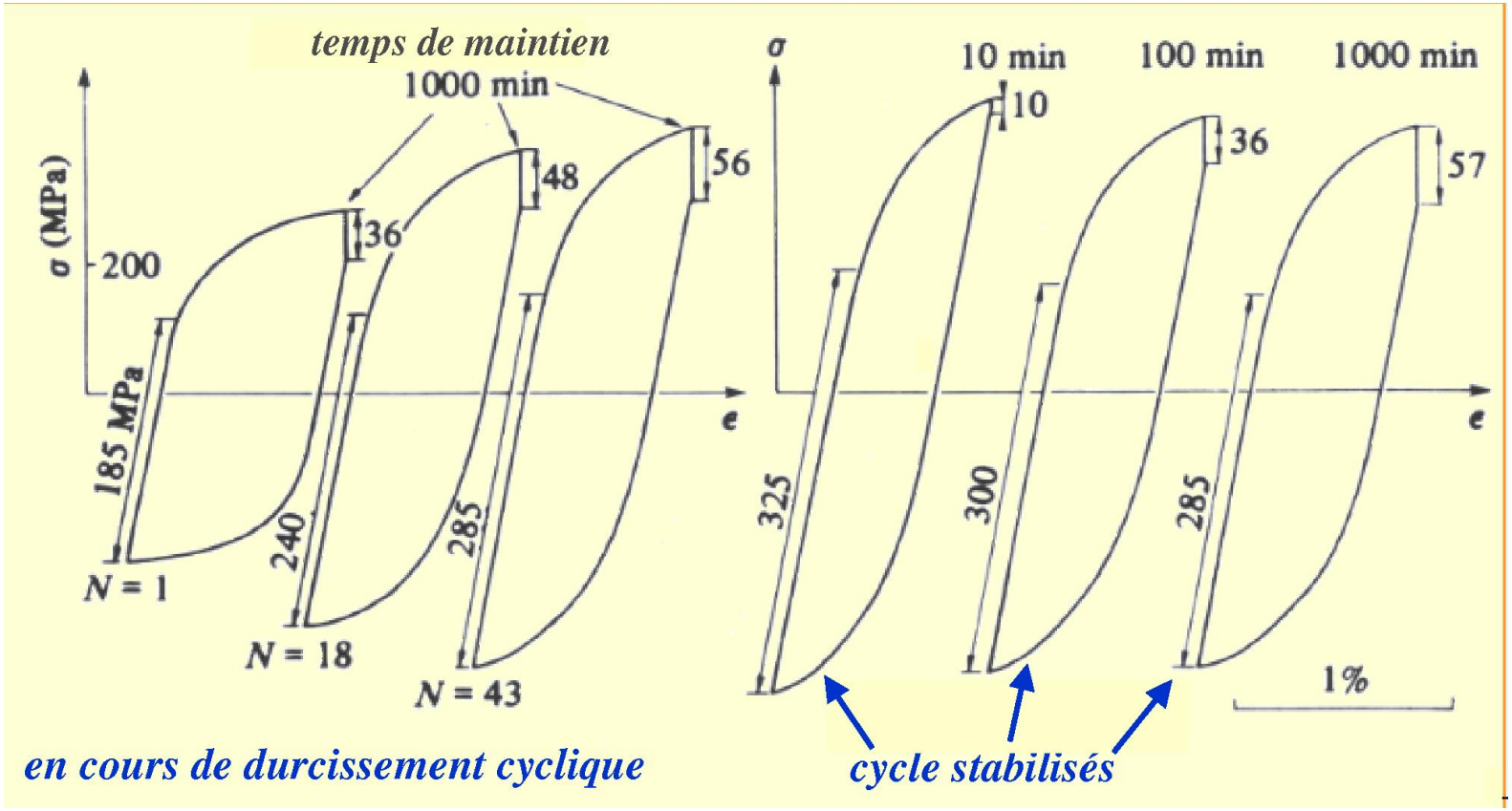
This allows creep for low stress levels

Resulting internal stress in creep (2)



Modeling of two-level creep tests is improved with a recovery term

Rôle of recovery term for cyclic loading

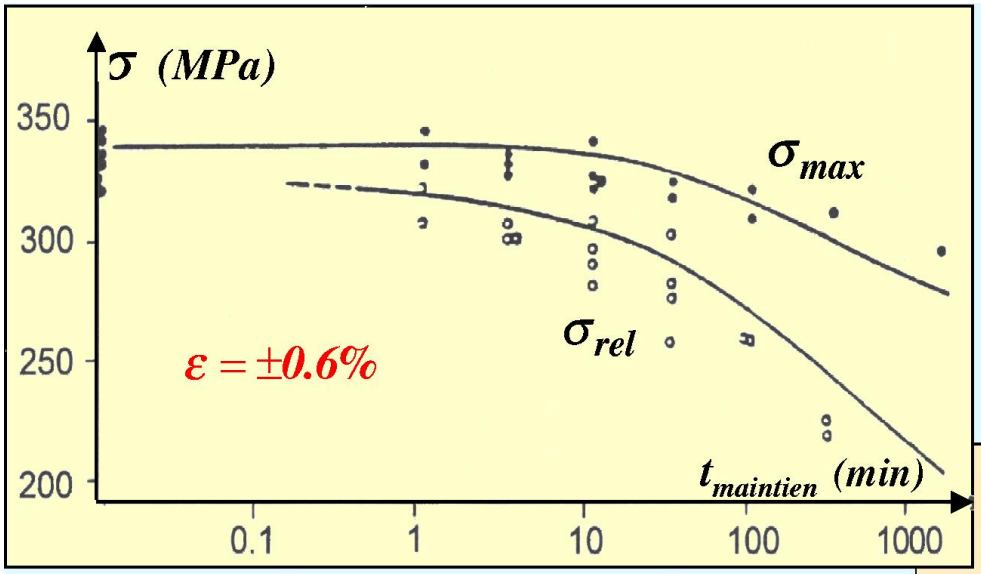


First cycles

Stabilized cycles

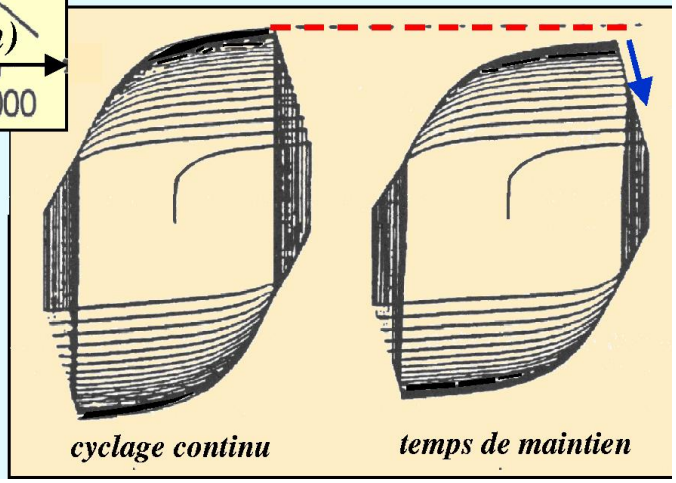
Tests on 316 stainless steel

Rôle of recovery term for cyclic relaxation tests



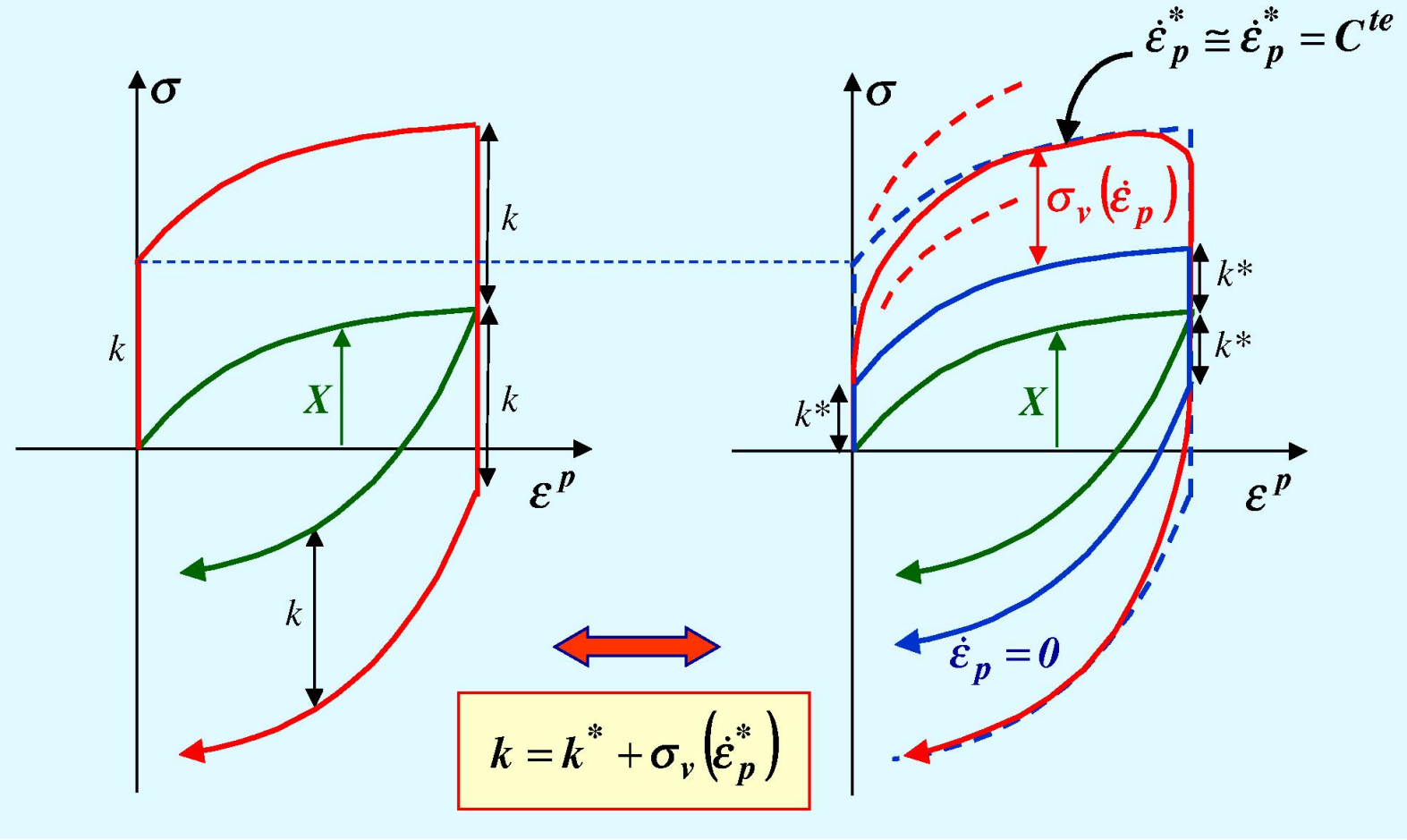
acier inoxydable 316 SPH
600°C

effet de restauration
par le temps



Tests on 316 stainless steel

Equivalence Time independent plasticity–Viscoplasticity



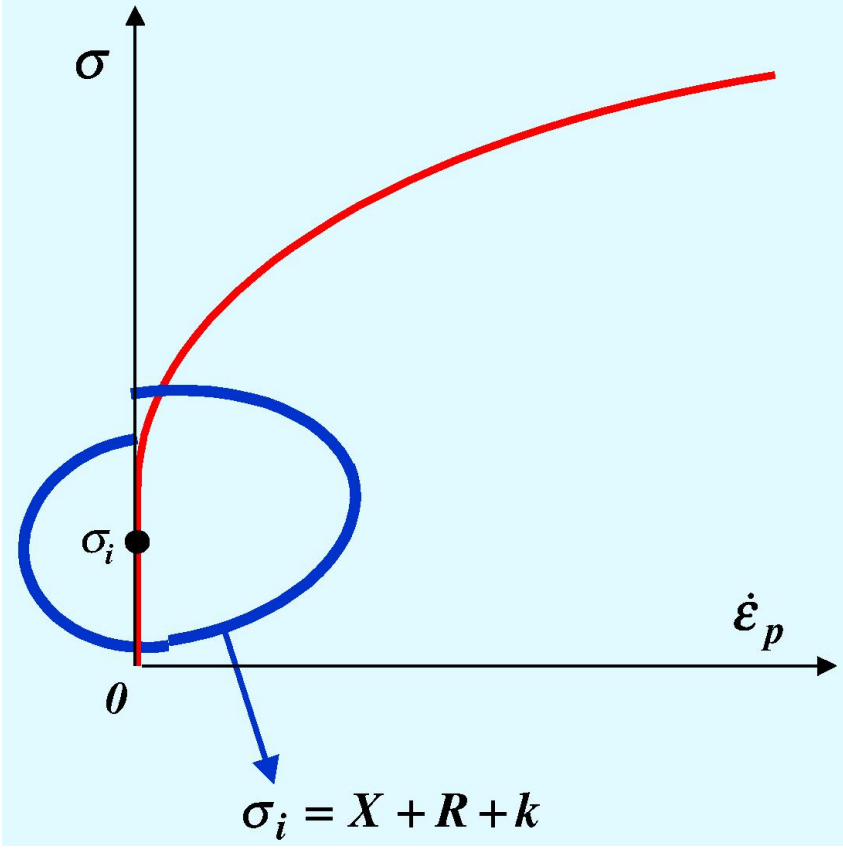
Time independent plasticity

Viscoplasticity

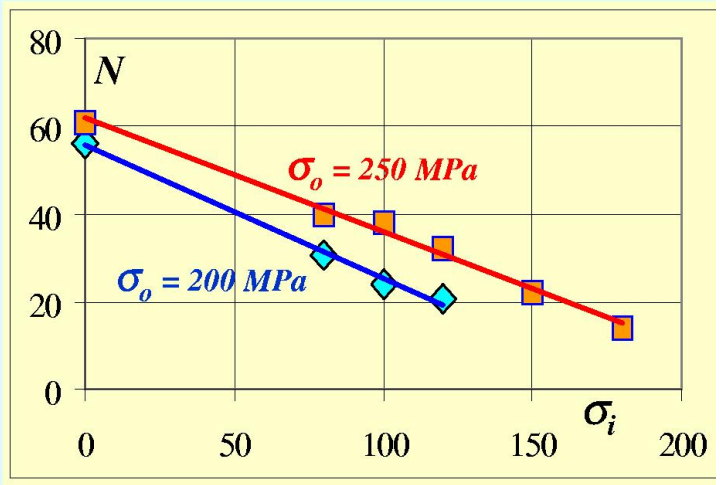
Identification problems

N = exposant de la fonction puissance

$$\dot{\varepsilon}_p = \left(\frac{\sigma - \sigma_i}{K} \right)^N$$

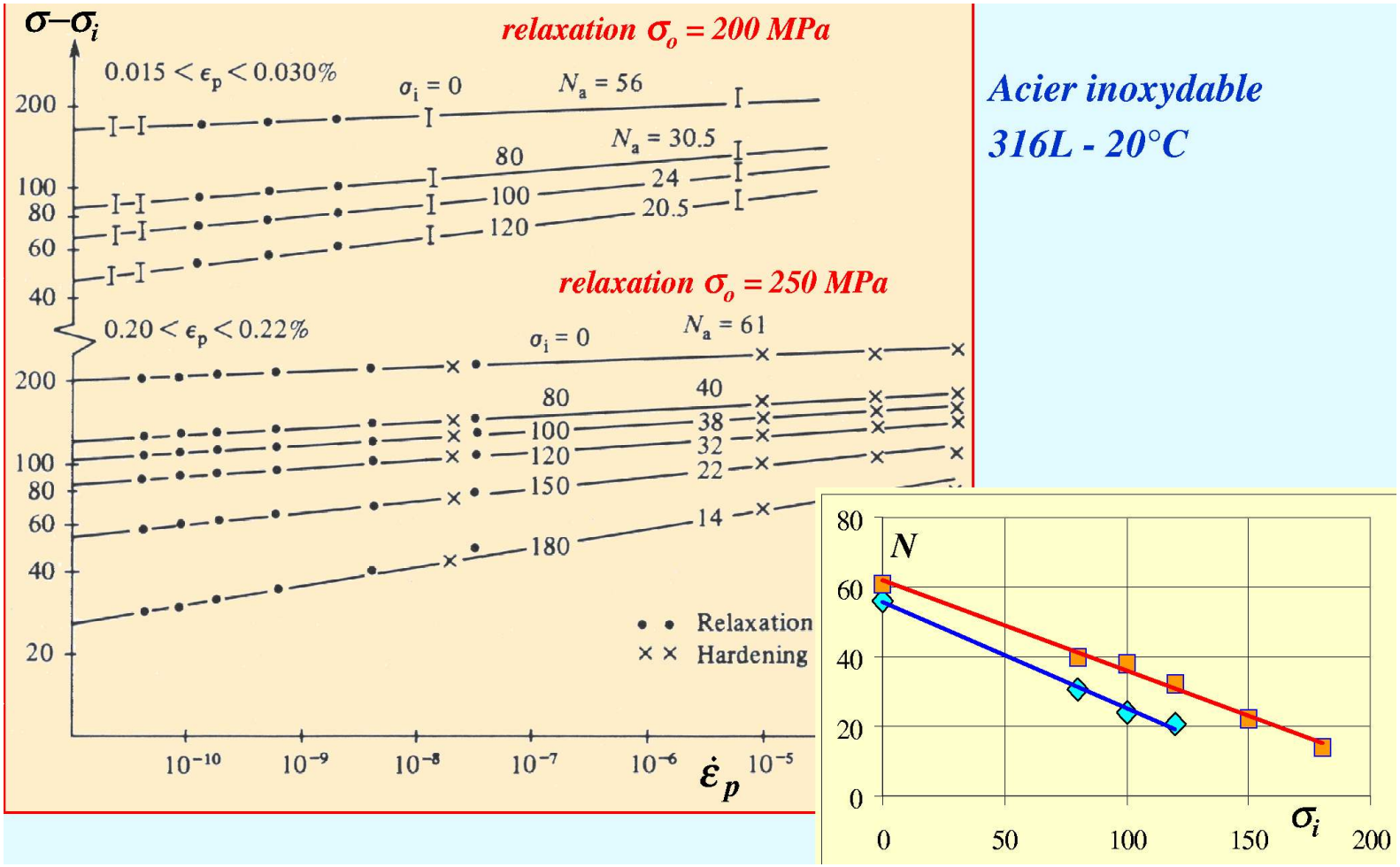


Acier inoxydable
316L - 20°C



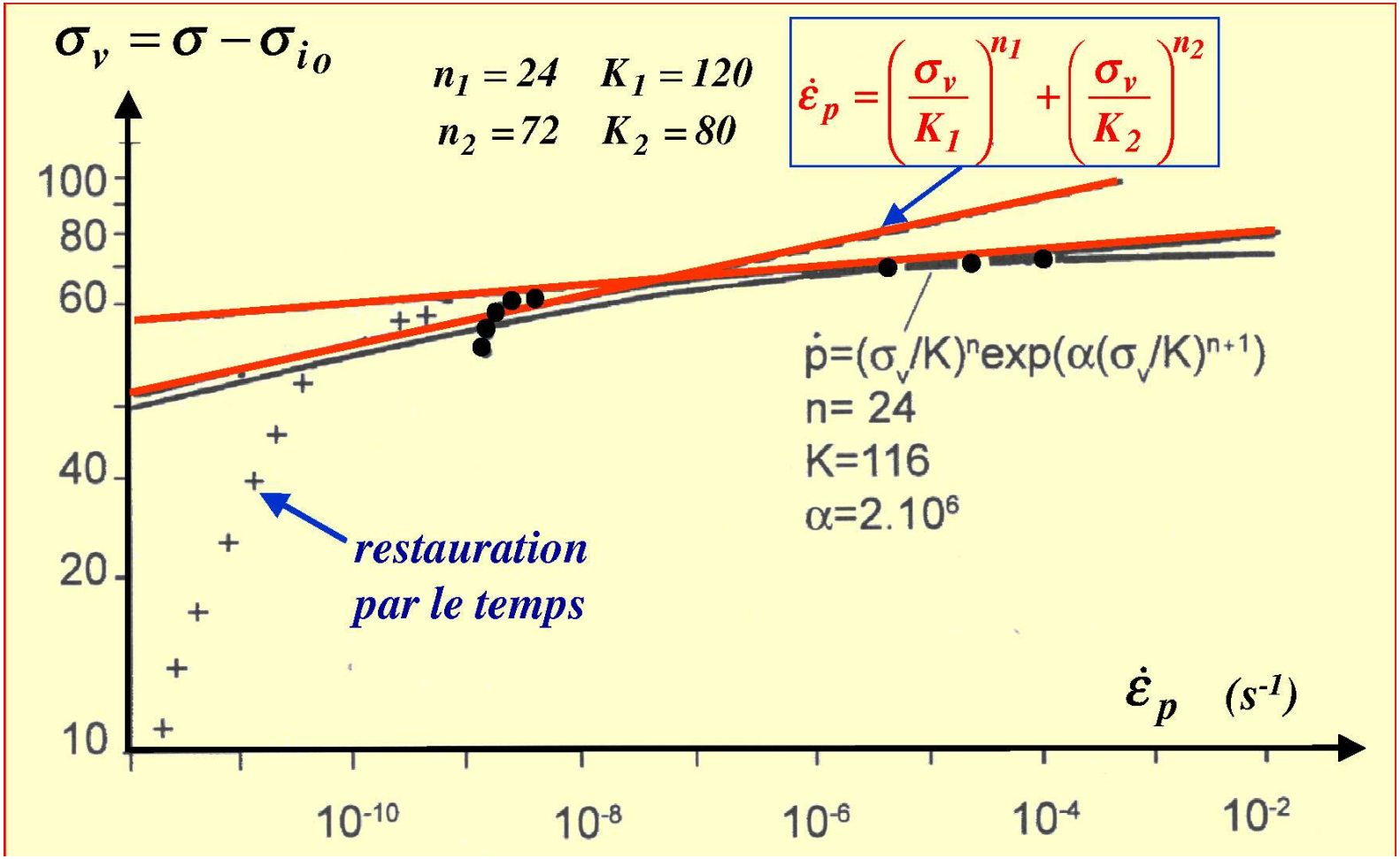
Several possible sets of internal stress and exponent (σ_i–N)

Identification problems (2)



The problem is solved by adding time sensitive tests like relaxation

Two-slope viscosity



Various unified models and their identification

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Various unified models and their identification

- **Isotropic and nonlinear kinematic hardening needed for cyclic loadings**
- **Many models in the literature**
- **Much more models in the nature than in the literature**
- **MORE on GS cast iron**

–Browse behaviors of real world material–