# Thermomechanical stresses

- Plasticity at variable temperature
- Examples of aging in unstable materials
- Classical tests
- A stand alone setup
- Modeling of the two-bar test
- Life prediction

*–Problems related to thermal gradient and unstable materials–*

# Material model at constant temperature

- 3D expression:

$$f(\underset{\sim}{\boldsymbol{\sigma}}, \underset{\sim}{\boldsymbol{X}}, R) = J(\underset{\sim}{\boldsymbol{\sigma}} - \underset{\sim}{\boldsymbol{X}}) - R - \sigma_y$$

$$R = bQr \qquad \underset{\sim}{\boldsymbol{X}} = \frac{2}{3} C \underset{\sim}{\boldsymbol{\alpha}}$$

$$\dot{r} = \left(1 - \frac{R}{Q}\right) \dot{p} \qquad \dot{\underset{\sim}{\boldsymbol{\alpha}}} = \dot{\underset{\sim}{\boldsymbol{\varepsilon}}}^p - \frac{3D}{2C} \underset{\sim}{\boldsymbol{X}} \dot{p}$$

- 1D expression:

$$f(\sigma, X, R) = |\sigma - X| - R - \sigma_y$$

$$R = bQr \qquad X = C\alpha$$

$$\dot{r} = \left(1 - \frac{R}{Q}\right) \dot{p} \qquad \dot{\alpha} = \dot{\varepsilon}^p - \frac{D}{C} X \dot{p}$$

# Material model at constant temperature

- 3D expression:

$$\dot{\underset{\sim}{\varepsilon}}^p = \dot{p}\,\underset{\sim}{n}$$

Viscoplasticity $\dot{p} = \left\langle \frac{f}{K} \right\rangle^n$

Plasticity $\dot{p} = \frac{\underset{\sim}{n} : \dot{\underset{\sim}{\sigma}}}{H}$ $\quad H = C - D\underset{\sim}{X} : \underset{\sim}{n} + b(Q - R)$

- 1D expression:

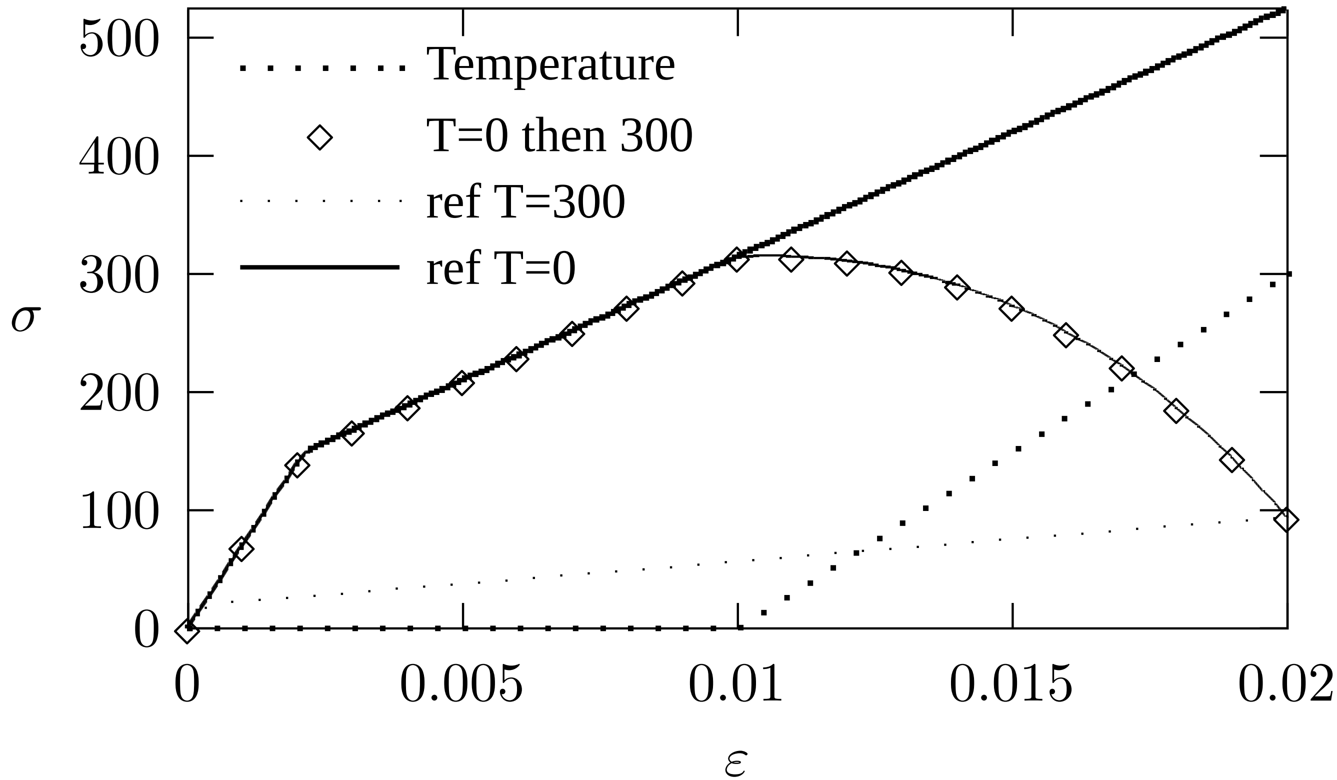
Viscoplasticity $\dot{p} = \left\langle \frac{f}{K} \right\rangle^n$

Plasticity $\dot{p} = \frac{\dot{\sigma}}{H}$ $\quad H = C - DX sign(\sigma - X) + b(Q - R)$

- With:

$$J(\underset{\sim}{\sigma}, \underset{\sim}{X}, R) = \left(\frac{3}{2}\underset{\sim}{s} : \underset{\sim}{s}\right)^{1/2} \qquad \dot{p} = \left(\frac{2}{3}\dot{\underset{\sim}{\varepsilon}}^p : \dot{\underset{\sim}{\varepsilon}}^p\right)^{1/2} \qquad \underset{\sim}{n} = \frac{3}{2}\,\frac{\underset{\sim}{s}}{J}$$

# Tensile test with temperature change (linear kinematic model)




| time | strain | temperature |
|---|---|---|
| 10 | 0.01 | 0 |
| 20 | 0.02 | 300 |

| coefficient | value at 0°C | value at 300°C |
|---|---|---|
| $\sigma_y$ | 150 | 20 |
| $C$ | 30000 | 4000 |

# *Material model at variable temperature*

- One has to change the consistency condition:

$$\dot{f} = 0 \quad \text{writes now (in 1D)} \quad (\dot{\sigma} - \dot{X}) sign(\sigma - X) - \dot{R} + \frac{\partial f}{\partial T}\dot{T} = 0$$

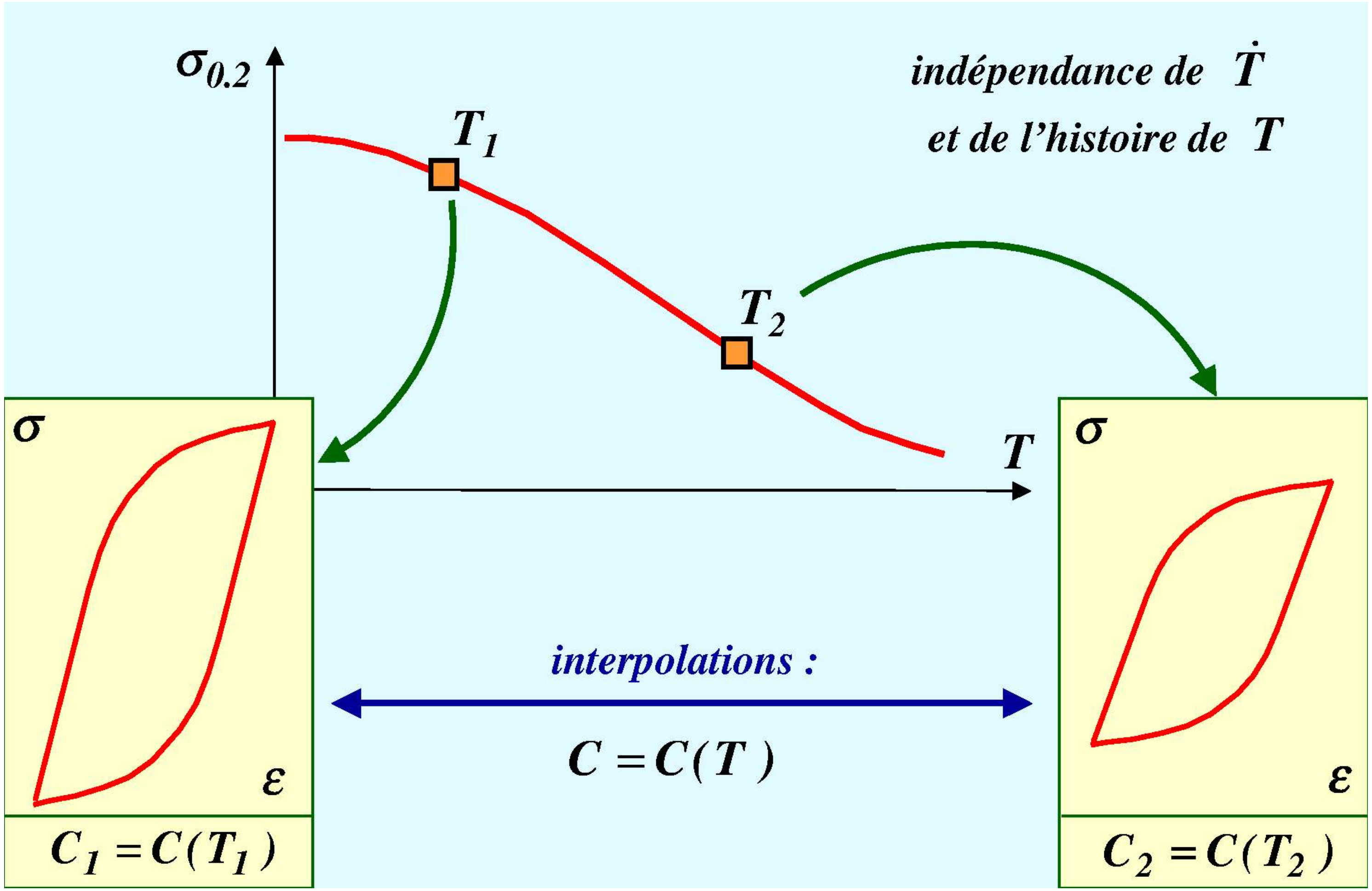
- Additional driving term, for instance in 1D:

$$\dot{p} = \frac{\dot{\sigma} + \frac{\partial f}{\partial T}}{H}$$

- Alternative solution in the algorithmic treatment:

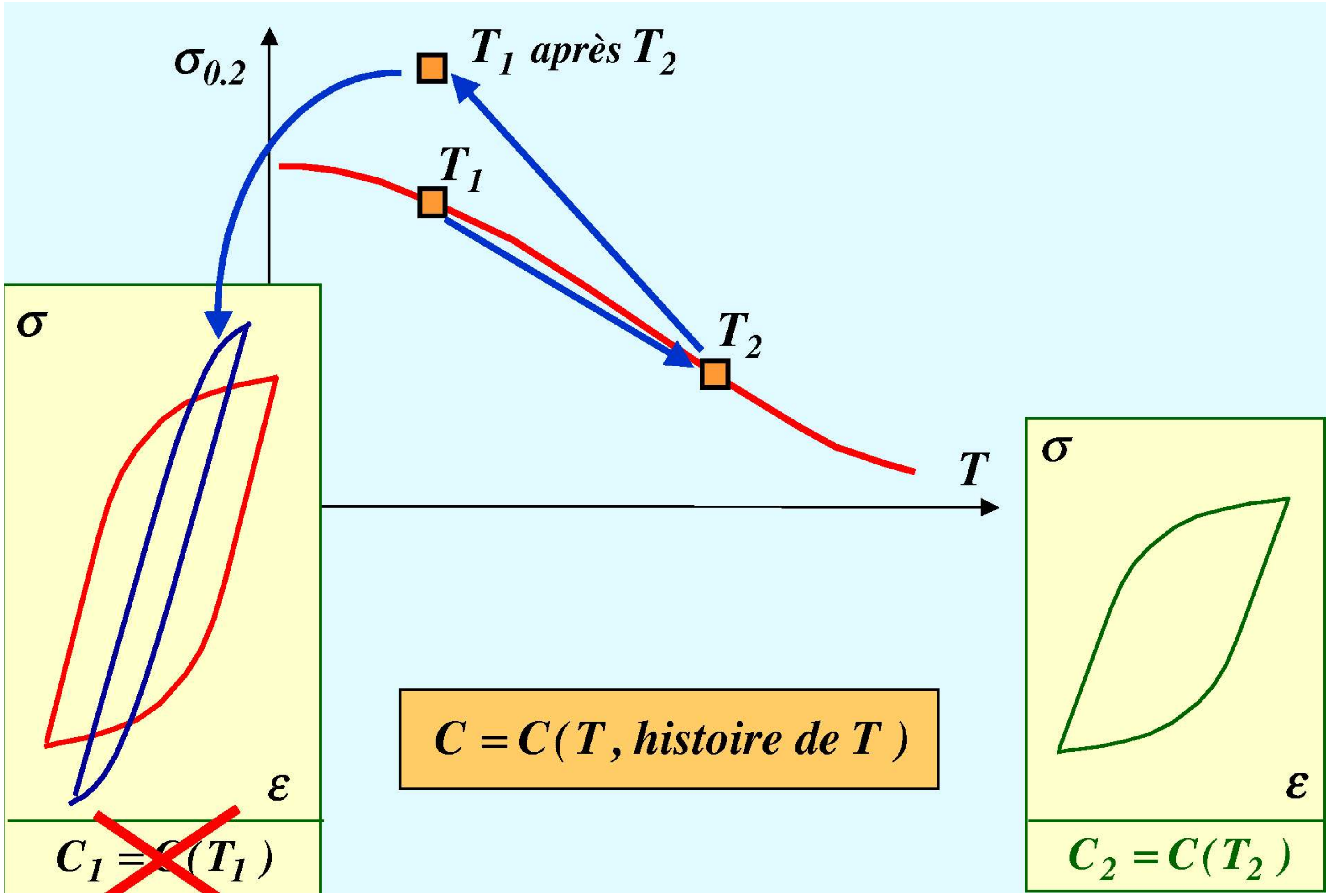
  *write* $f = 0$ *at the end of the increment*

# Stable materials



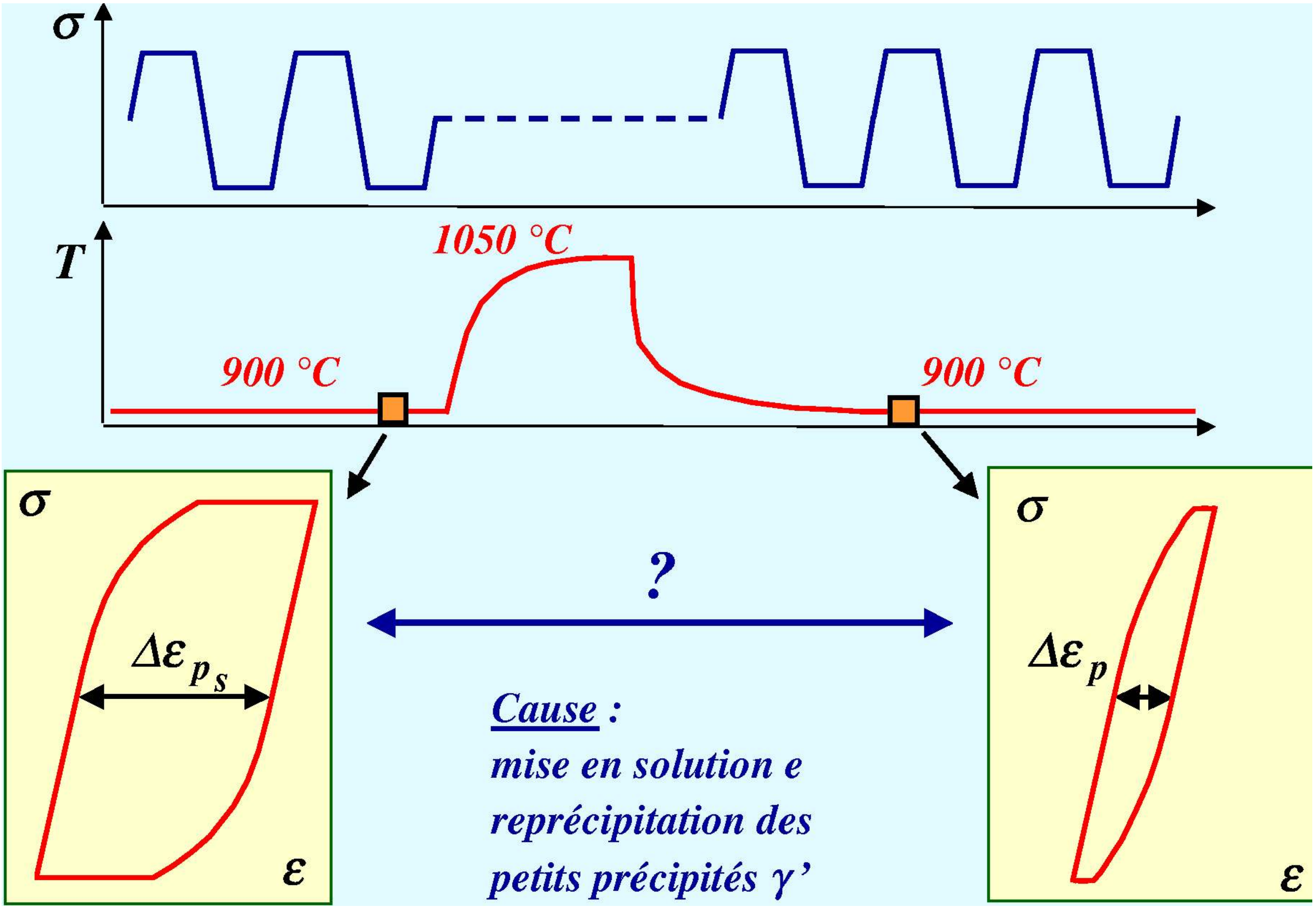

*NO change of the mechanical properties with temperature history*

# Unstable materials



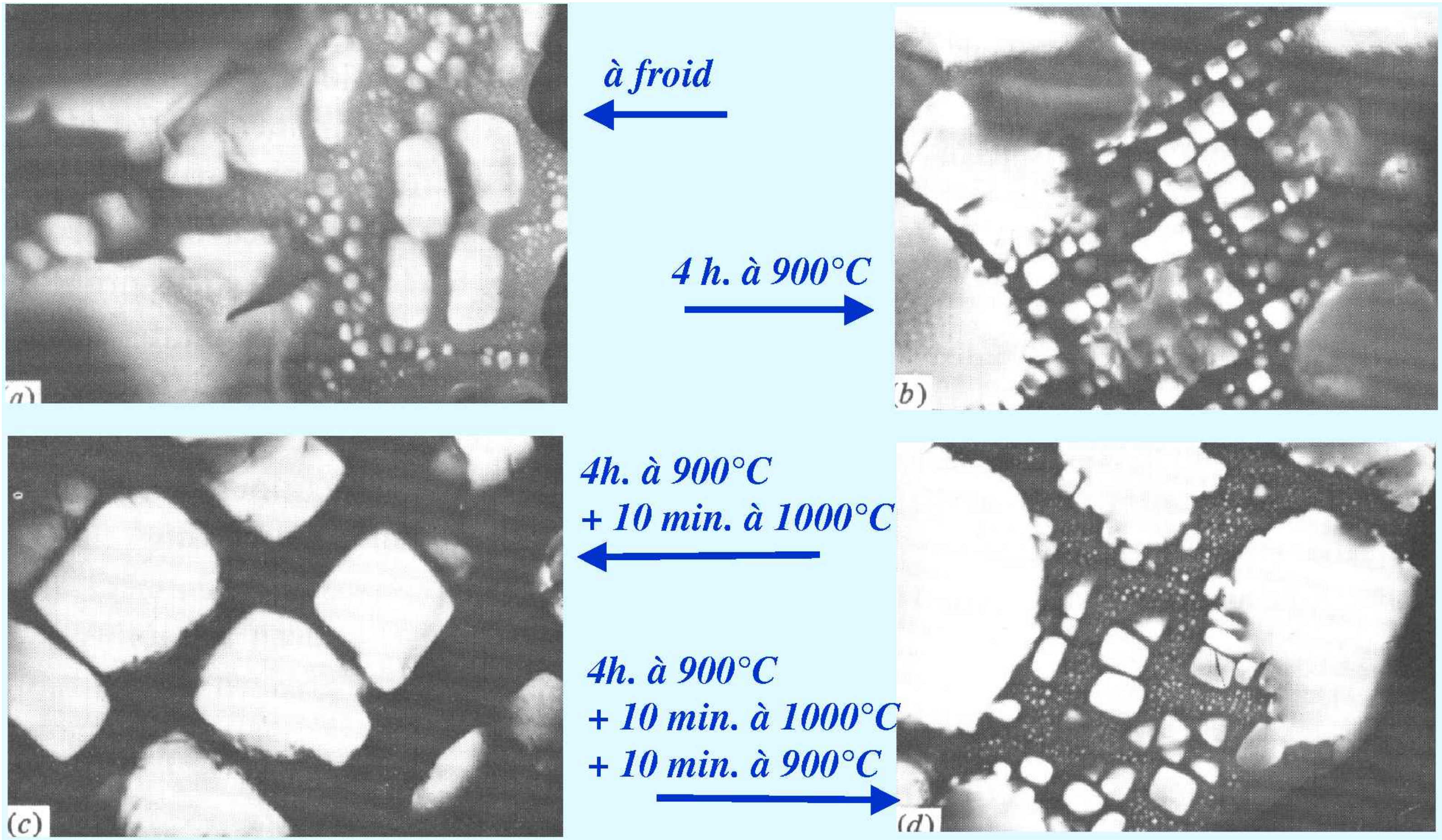

*Influence of the temperature history*

# Aging of IN100 material

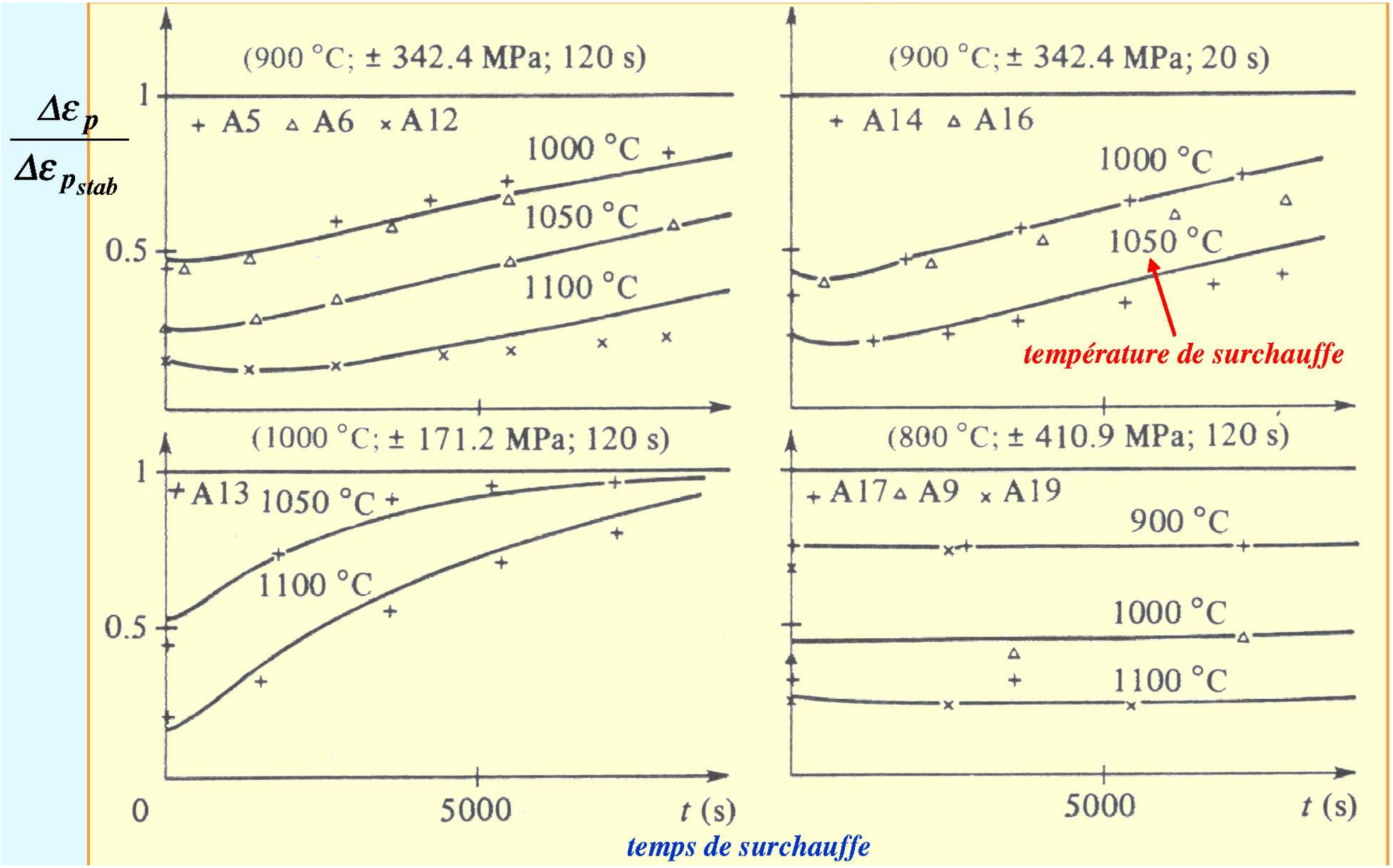



*Precipitate hardened material, dissolution of the precipitates when temperature increases, then cooling produces precipitation*

# Aging of IN100 material



# Aging of IN100 material, variation of plastic strain range



# Aging of IN100 material, mechanical modeling

- Additional term in the yield function:

$$f(\sigma, X, R, R^*) == |\sigma - X| - R - R^* - \sigma_y$$

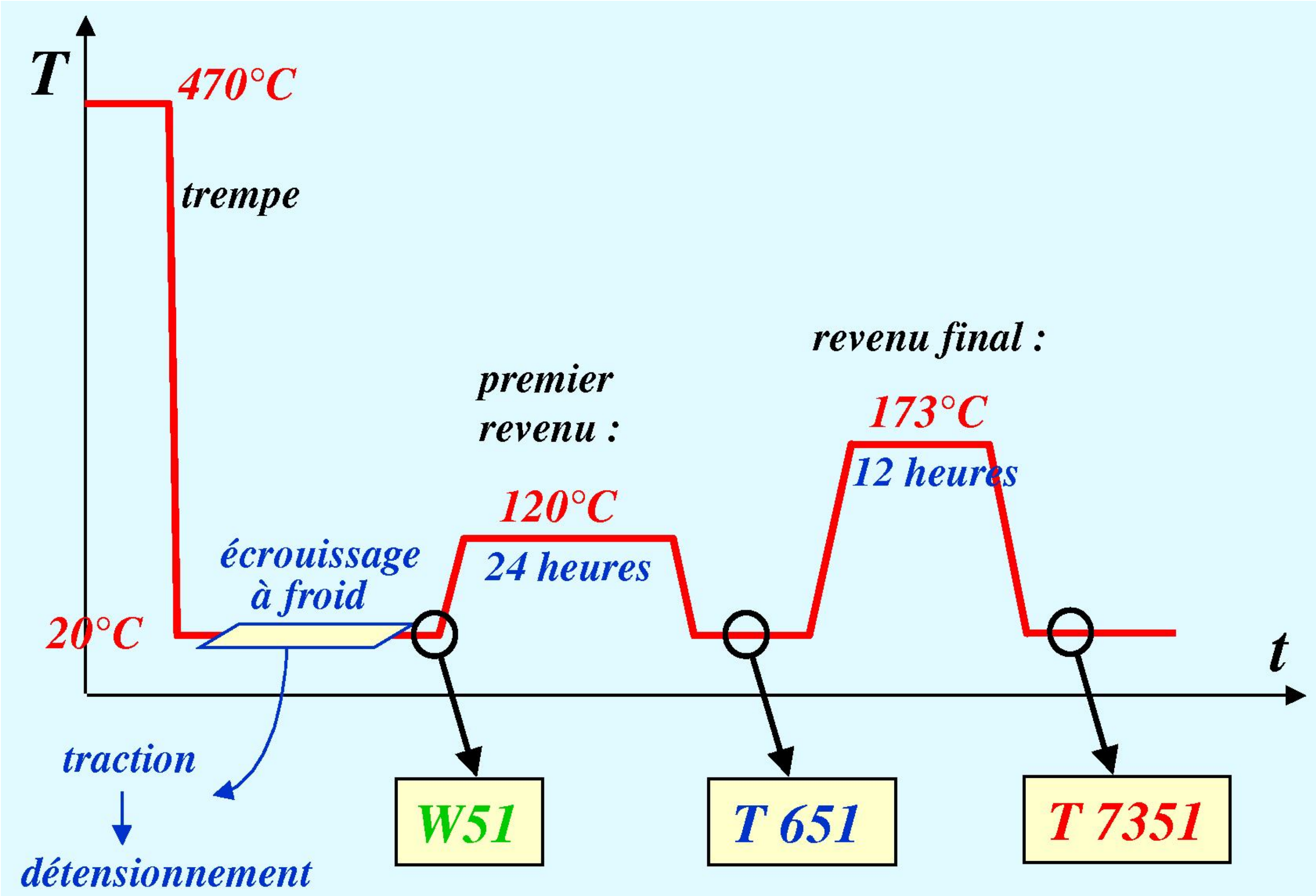
- Introduction of a new, temperature like variable, to keep track of the history:

$$\dot{T}_f = \frac{T - T_f}{\tau}$$

- Mechanism of hardening/recovery on the additional variable:

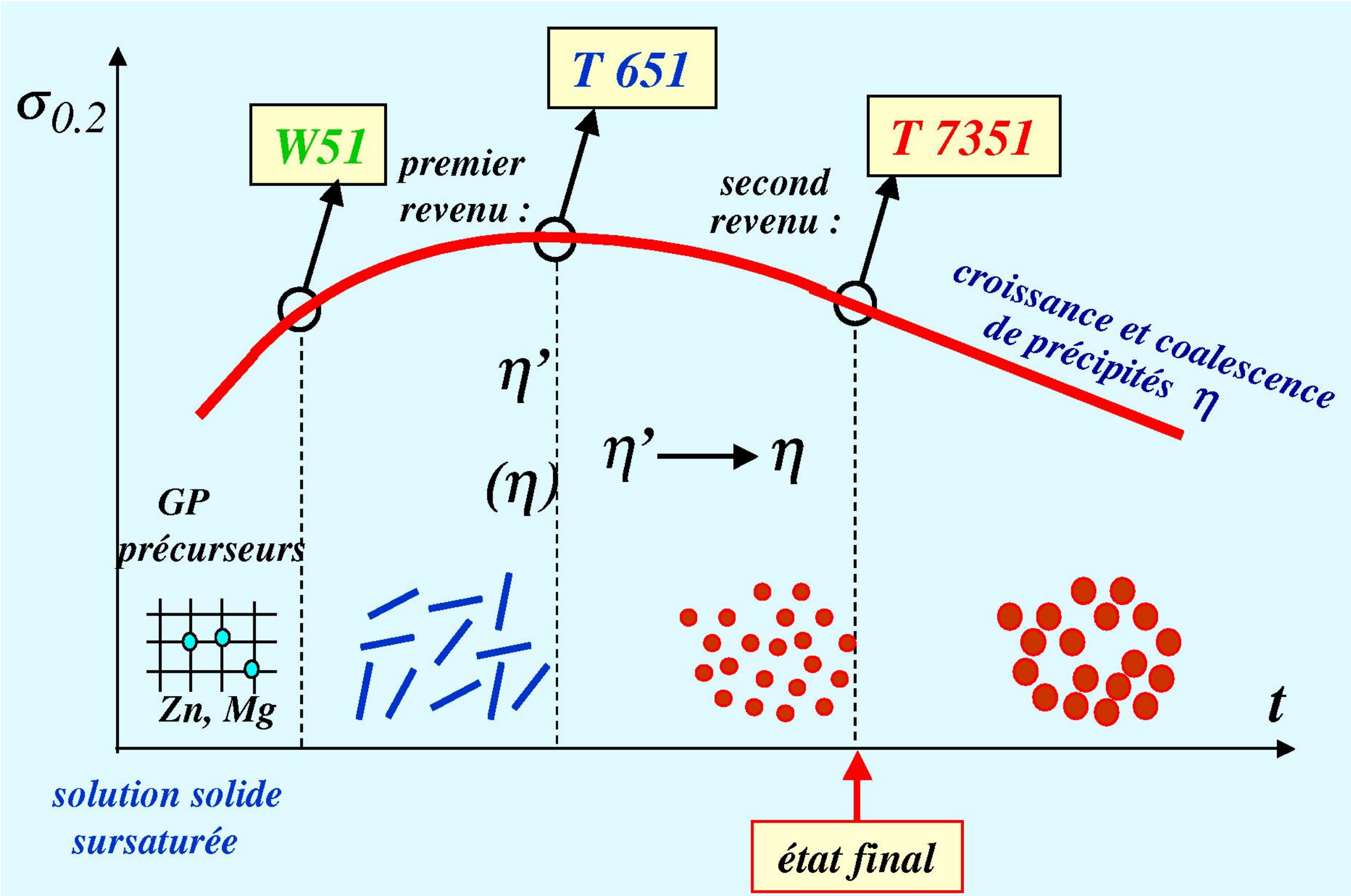
$$\dot{R}^* = a \langle T_f - T \rangle - \frac{R^*}{A}$$

# Thermomechanical treatment of aluminium alloys




*7000 serie*

# Thermomechanical treatment of aluminium alloys (2)




*Precipitates evolution*

# *A model for modeling aluminium aging after the peak*

- Additional term in the yield function:

$$f(\sigma, X, R, R^*) == |\sigma - X| - R - R^* - \sigma_y$$

- One just has to represent softening. One aging variable, $a$, two additional coefficients, $a_\infty$ and $\tau$:

$$\dot{a} = \left\langle \frac{a_\infty - a}{\tau} \right\rangle$$

- Softening on $R^*$:

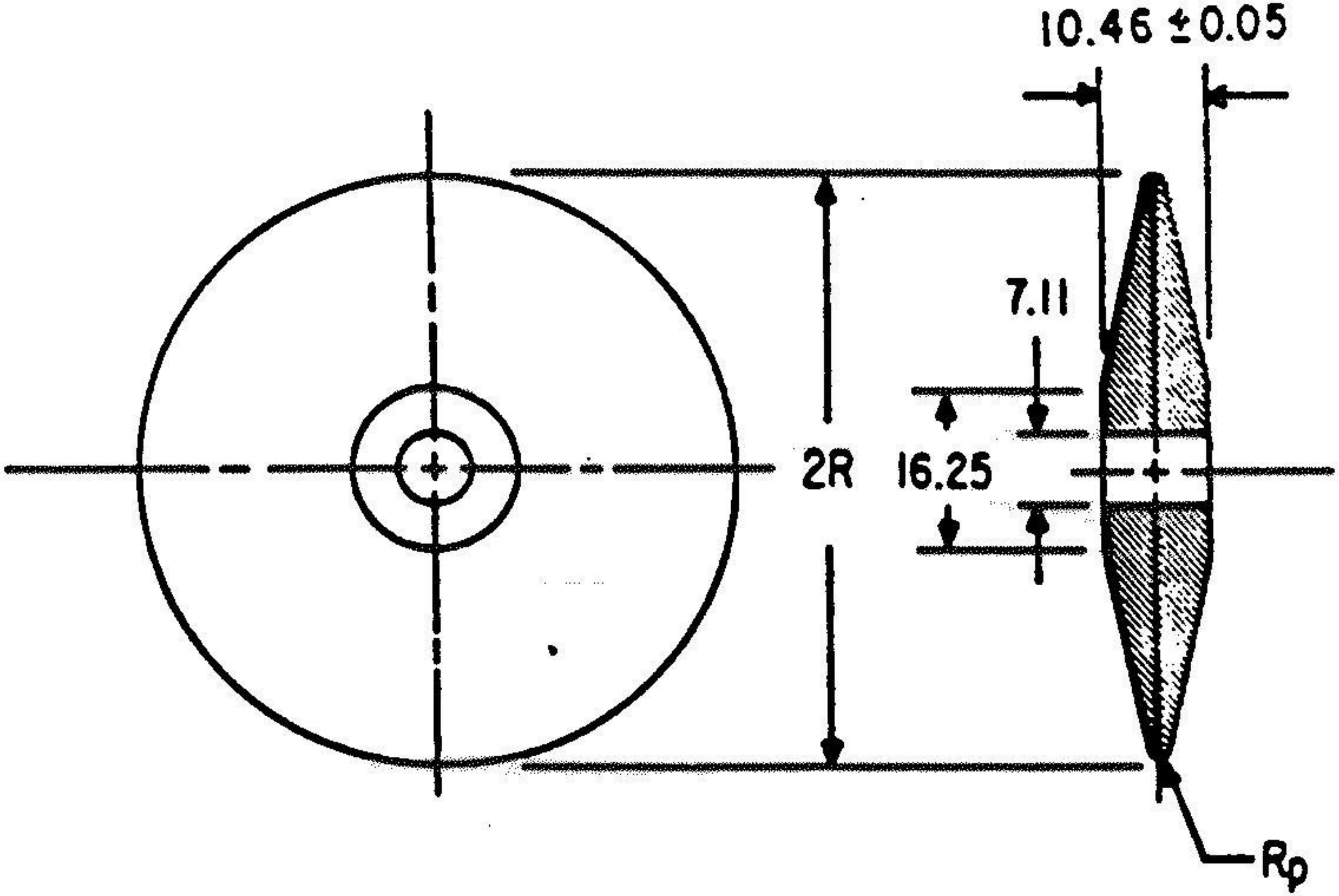
$$R^* = R_0^*(1 - a)$$

- Softening on $X$:

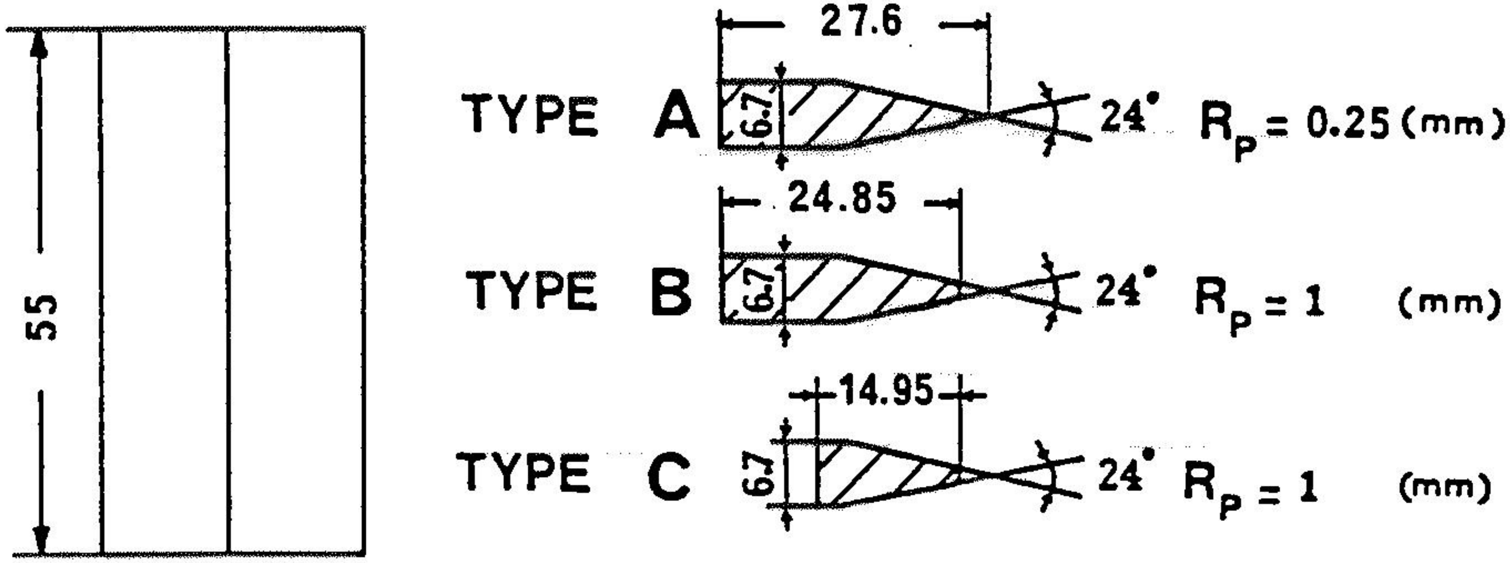
$$X = X_1 + X_2$$

$$X_1 \text{ unchanged} \qquad X_2 = C_2(1 - a)\alpha_2$$

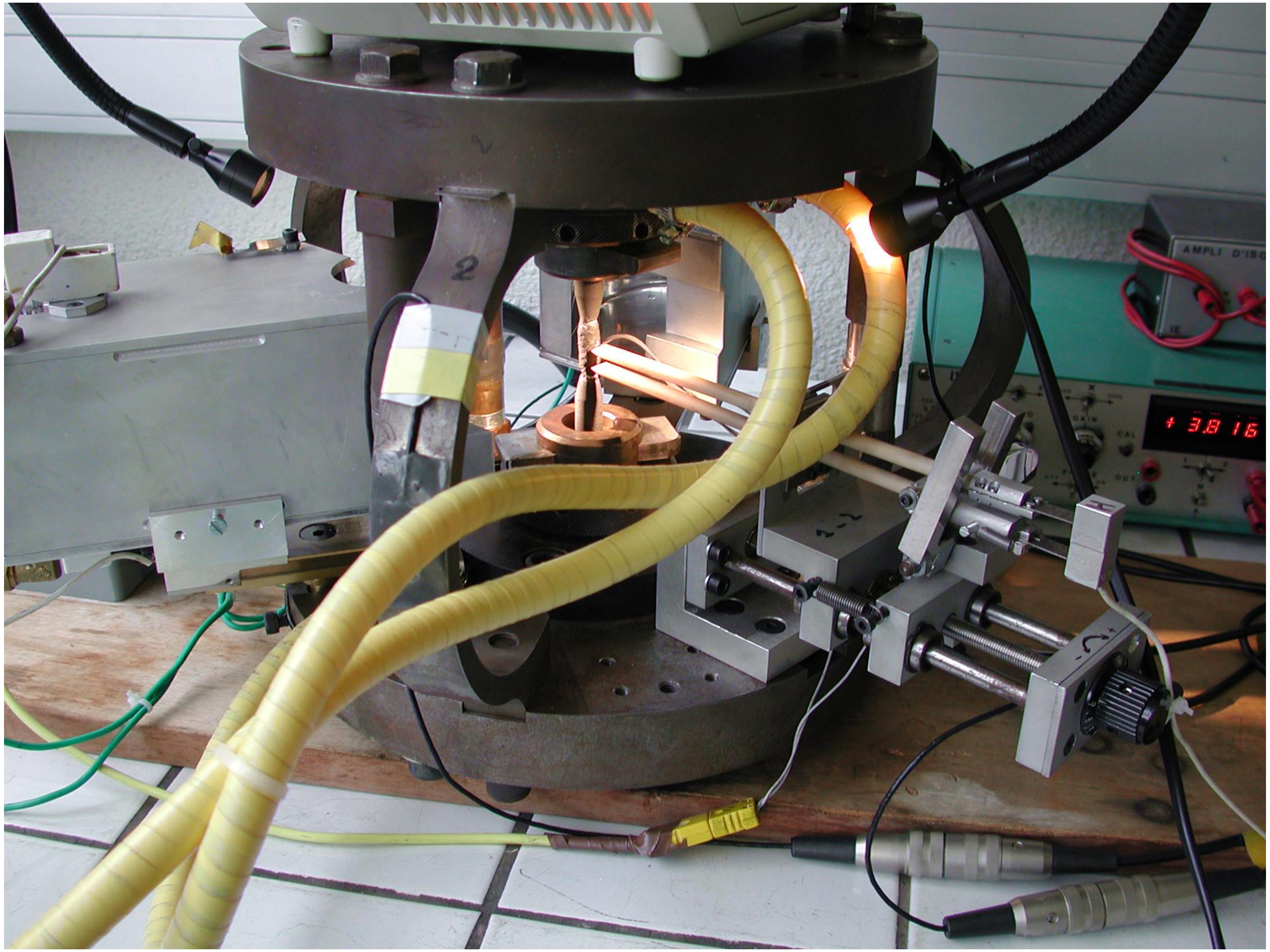
# Glenny test



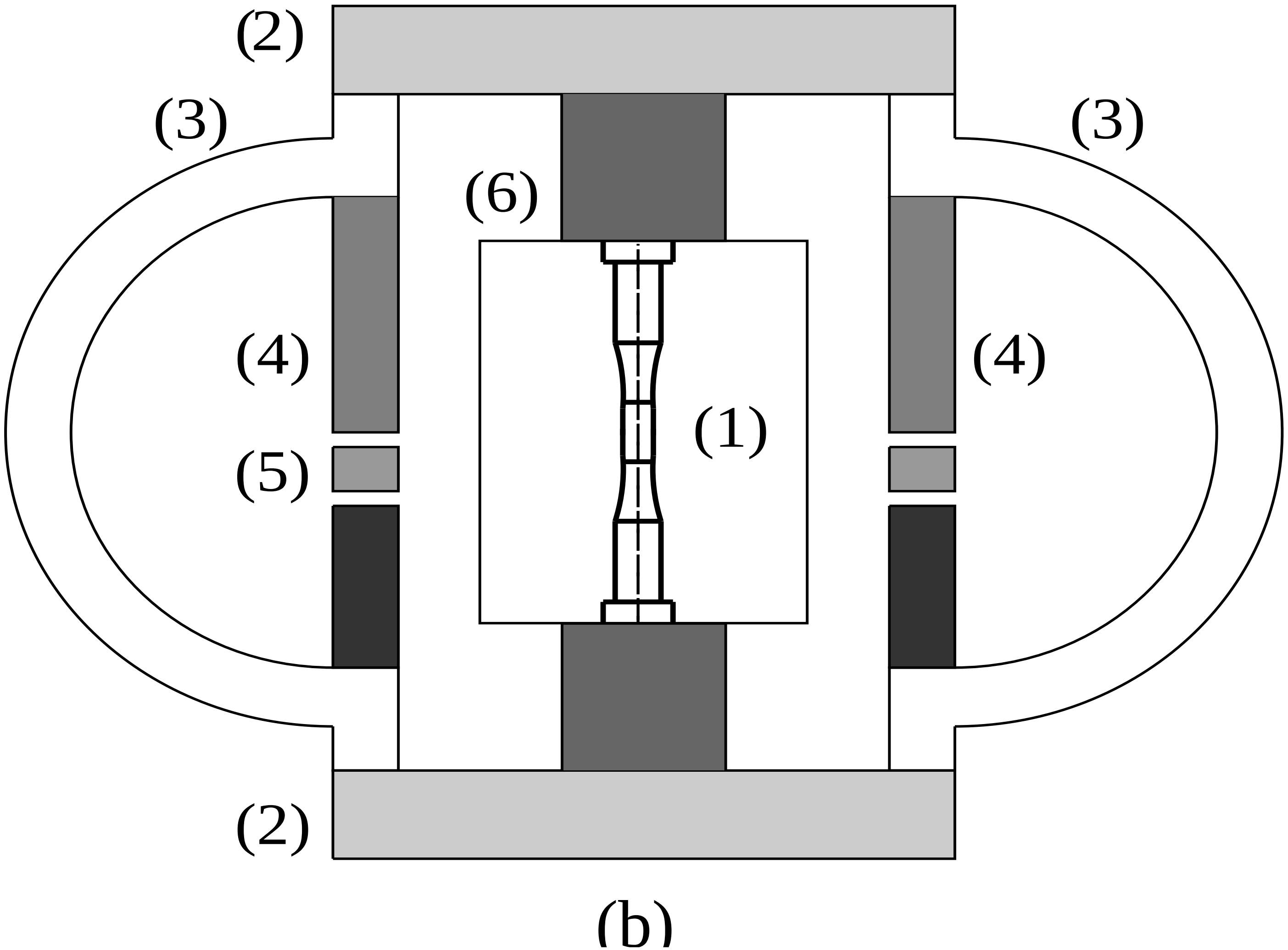
# Setup using torchs



# A "three bar" type setup

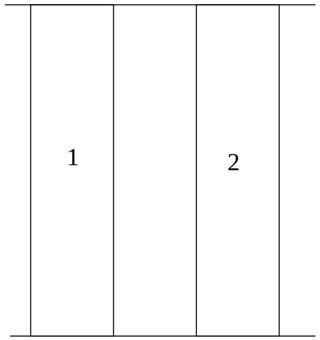


# *Elements of the "three bar" type setup*




(1) Specimen (2) Plateaux (3) Springs (4) Columns (5) Cylinders (6) Furnace

# A simplified model for a three bar system




Hot bar Cold bar

$$\varepsilon = \frac{\sigma}{E_1} + \varepsilon^p + \alpha T \qquad \varepsilon = \frac{\sigma_2}{E_2} = -\frac{\sigma_1}{E_2}$$

$$\frac{\sigma}{E^*} + \varepsilon^p + \alpha T = 0 \quad \text{with} \quad \frac{1}{E^*} = \frac{1}{E_1} + \frac{1}{E_2}$$

Constitutive equations:

$$\dot{\varepsilon}^p = \left\langle \frac{|\sigma - X| - R - \sigma_y}{K} \right\rangle^n$$

$$X = C\alpha \quad \dot{\alpha} = \dot{\varepsilon}^p - \frac{D}{C} X \dot{p} \qquad R = bQr \quad \dot{r} = (1 - \frac{R}{Q})\dot{p}$$

Integrated variables: $\varepsilon^p$, $\alpha$, $r$ Conjugated variables: $\sigma$, $X$, $R$

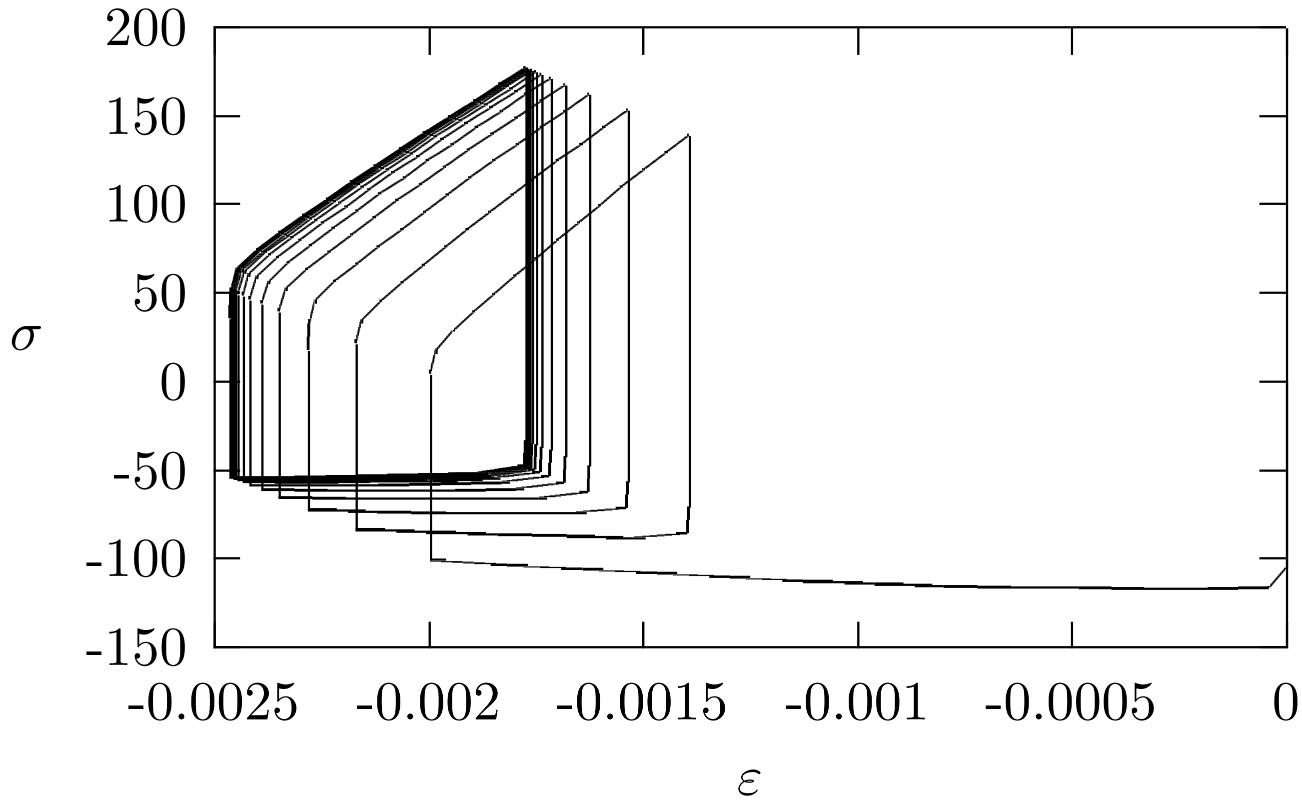
# Numerical implementation

```
@Class twobar : BASIC_NL_BEHAVIOR, BASIC_SIMULATOR {
   @Coefs    Estar,alpha,sigy,Q,b,C,D,K,n;
   @sVarInt  epl,alfa,r;
   @sVarAux  sigma,X,R;
};

@Derivative {
  double dp,f;
  int rt  = EXTERNAL_PARAM::rank_of("temperature");
  double T= (*curr_mat_data->param_set())[rt];

  sigma = -(alpha*T+epl)*Estar;
      X = C*alfa;
      R = sigy + b*Q*r;

      f = abs(sigma-X)-R;

  if(f>0){
     dp = pow(f/K,n);
   depl = sign(sigma-X)*dp;
  dalfa = depl-D*alfa*dp;
     dr = (1-b*r)*dp;}

  else {depl=dalfa=dr=0.;}
}
```

# Data file

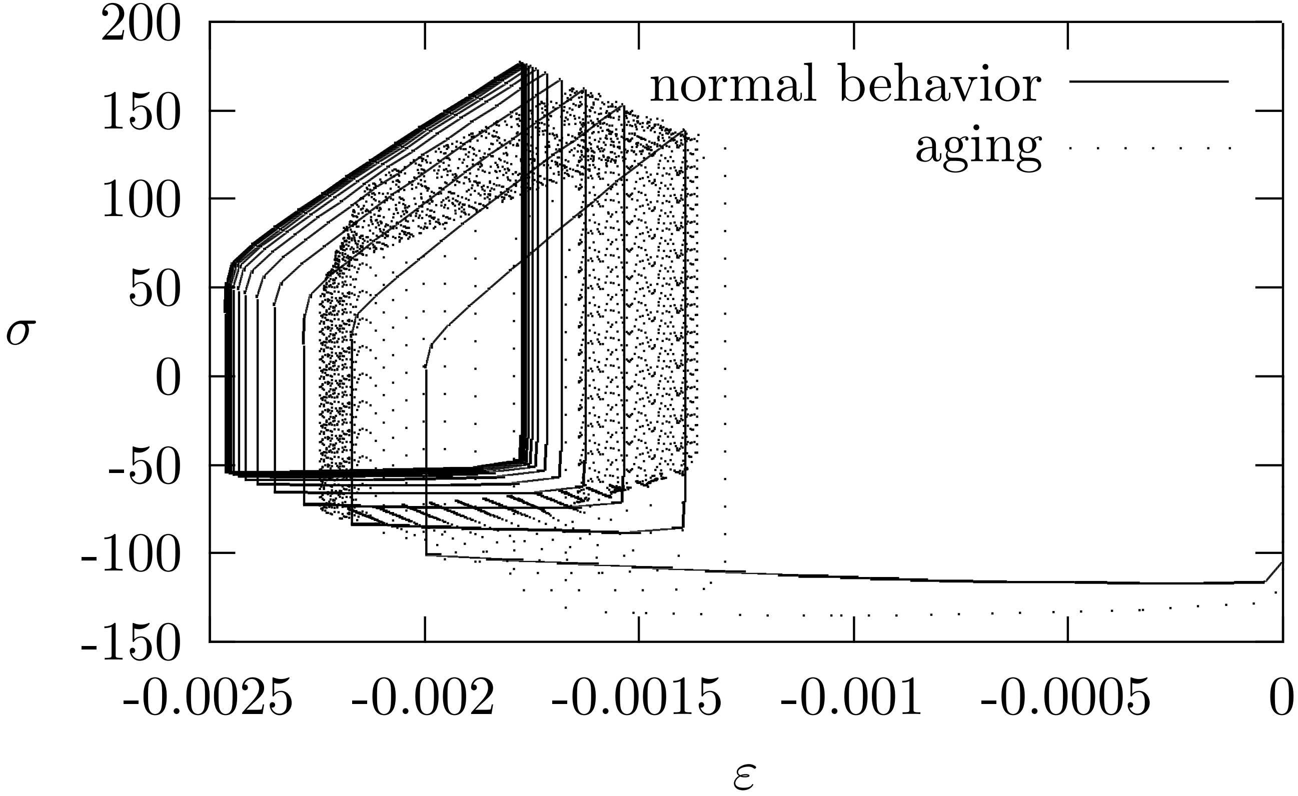
```
****simulate
***test visco1a
**load  cycle 100
 *segment 20
  time   param:temperature
  0.      0.
 10.    300.
 20.      0.
**model
  *file visco.inp
**output
  time sigma epl X R temperature
****return

***behavior twobar
**model_coef
  Estar  100000.
  alpha  10.e-06
  sigy   function 150.-130.*temperature/300.;
  Q     0.
  b    10.
  C 50000.
  D   500.
  K   100.
  n     5.
***return
```

# *Result*

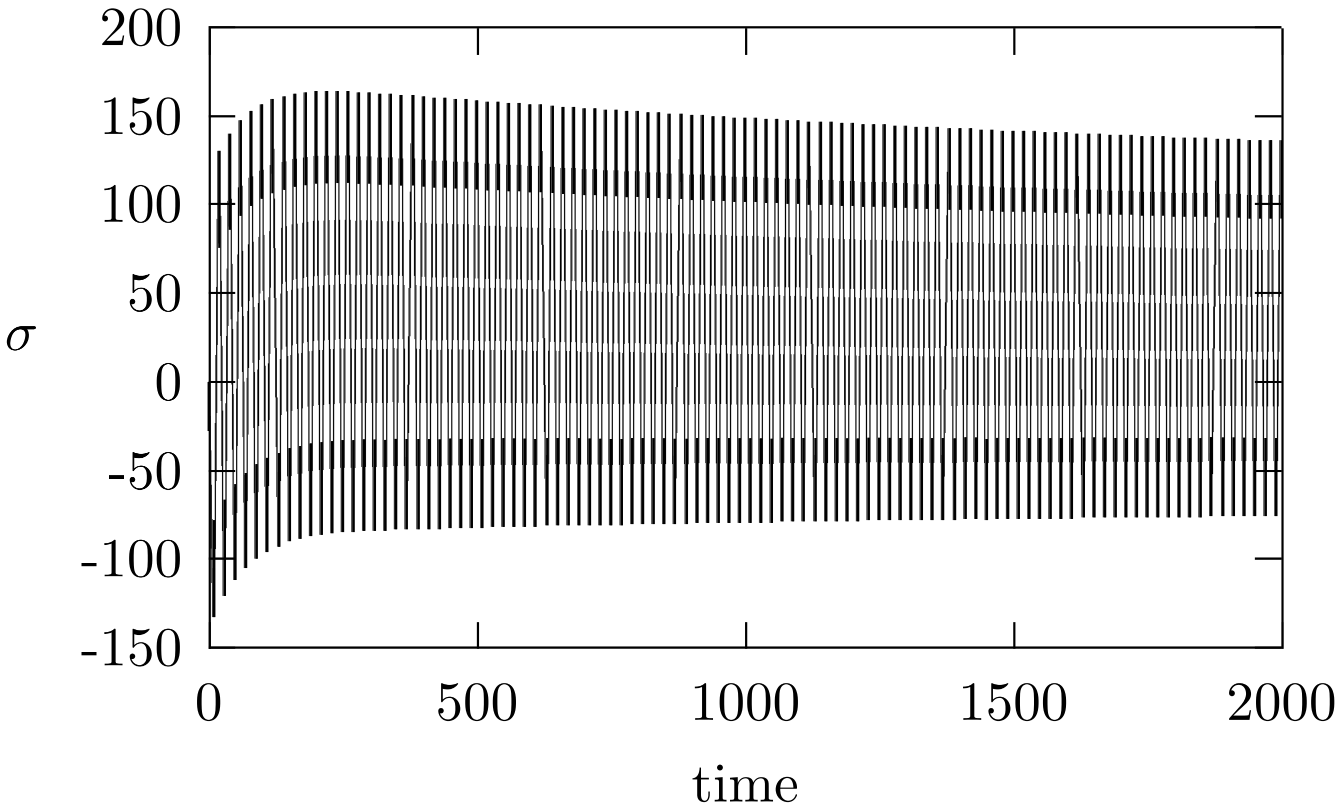



*Typical stress-strain loop reaching non symmetrical steady state*

# Influence of aging



# *Influence of aging*




*Stress evolution* versus *time for the model with aging*

# *Thermomechanical stresses*

- **Strong formulations in terms of state variables are needed for variable temperature**
- **Material aging induces problems in stress redistribution**
- **Compute the whole life of the component**

*–Problems related to thermal gradient and unstable materials–*