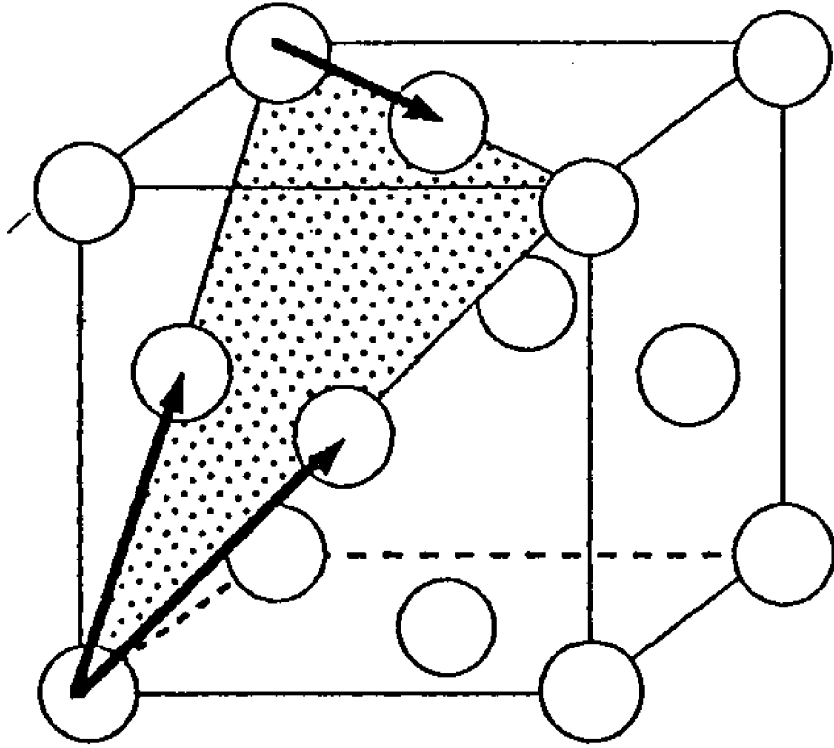


# *Champ de Contraintes et de Déformations en pointe de fissure dans un monocristal*

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# Systemes de glissement-Loi de Schmid



Structure cristalline Cubique à Faces Centrées (CFC)

12 Systèmes **octaédriques**

– plan de glissement  $\{111\}$

– direction de glissement  $\langle 110 \rangle$

Loi de Schmid :  $\tau^s = \underline{\underline{\sigma}} : \underline{\underline{m}}^s = \tau_c$  la cission résolue sur le système s

$$\underline{\underline{m}}^s = \frac{1}{2} (\underline{\underline{m}}^s \otimes \underline{\underline{n}}^s + \underline{\underline{n}}^s \otimes \underline{\underline{m}}^s)$$

$\underline{\underline{m}}^s$  la direction de glissement

$\underline{\underline{n}}^s$  la normale à au plan de glissement

# Modèle monocristallin

décomposition

multiplicative

du gradient des  $\underline{\tilde{F}} = \underline{\tilde{F}}^e \underline{\tilde{F}}^p$

Viscoplastic strain :

$$\dot{\underline{\tilde{F}}}^p \underline{\tilde{F}}^{p-1} = \sum \dot{\gamma}^s \underline{\mathbf{m}}^s \otimes \underline{\mathbf{n}}^s$$

Loi d'écoulement :

$$\dot{\gamma}^s = \left\langle \frac{|\tau^s - x^s| - r^s}{K_N} \right\rangle^n \text{sign}(\tau^s - x^s)$$

Ecrouissage Isotrope :

$$r^s = r_o + Q \sum_{r=1}^N h^{sr} (1 - \exp(-bv^r))$$

Ecrouissage

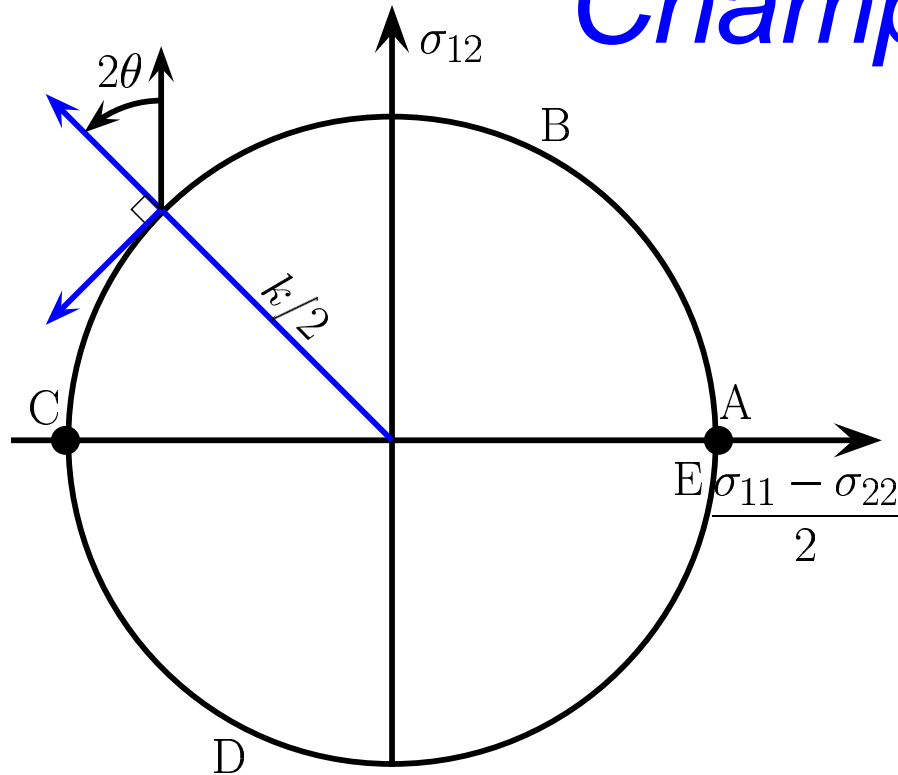
Cinématique :

$$\dot{v}^s = |\dot{\gamma}^s|$$

$$\dot{\alpha}^s = \dot{\gamma}^s - d\dot{v}^s \alpha^s$$

$$x^s = c\alpha^s$$

# Champ de Prandtl

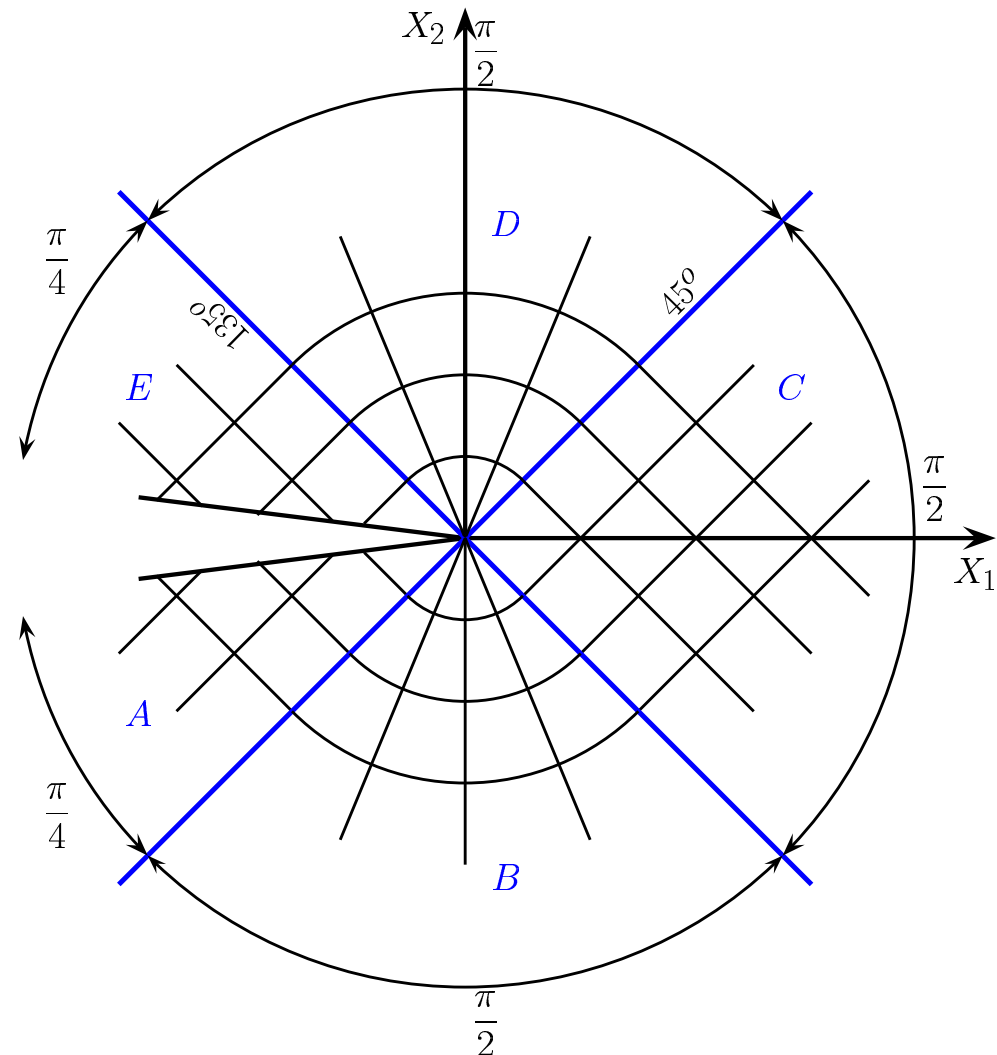


- Déformations planes
- Comportement rigide parfaitement plastique
- Critère isotrope : Tresca, Von Mises

- Méthodes des lignes de glissement
- $\sigma_{ij}$  constantes dans les secteurs A, C, E

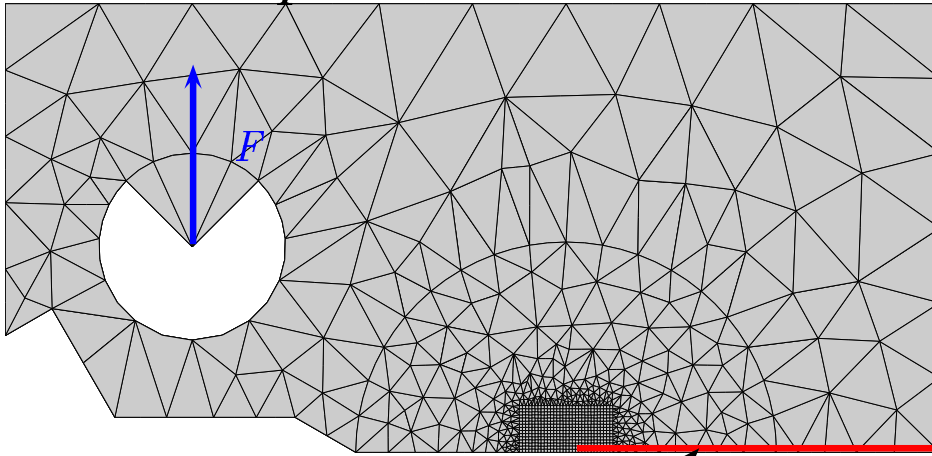
– en D

$$\begin{cases} \sigma_{\theta\theta} = (1 + \frac{3\Pi}{2} - 2\theta)k \\ \sigma_{r\theta} = k \\ \sigma_{rr} = (1 + \frac{3\Pi}{2} - 2\theta)k \end{cases}$$

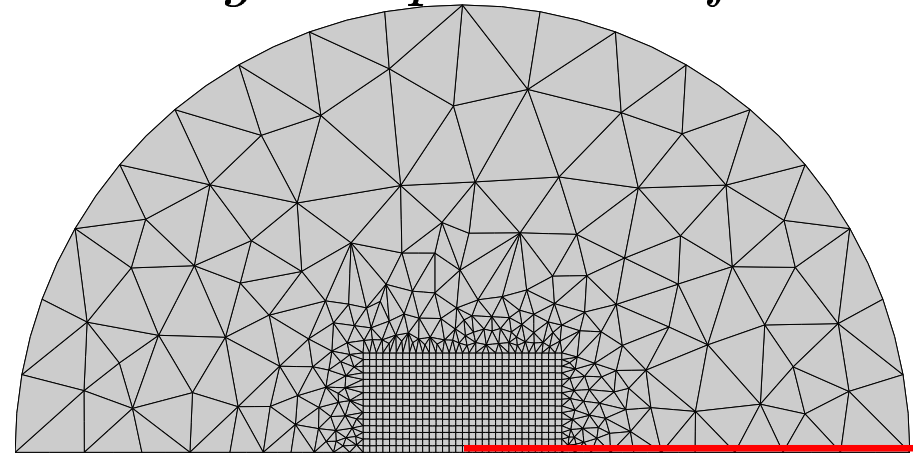


# Maillage et Conditions aux limites

*Eprouvette CT*



*Maillage en pointe de fissure*

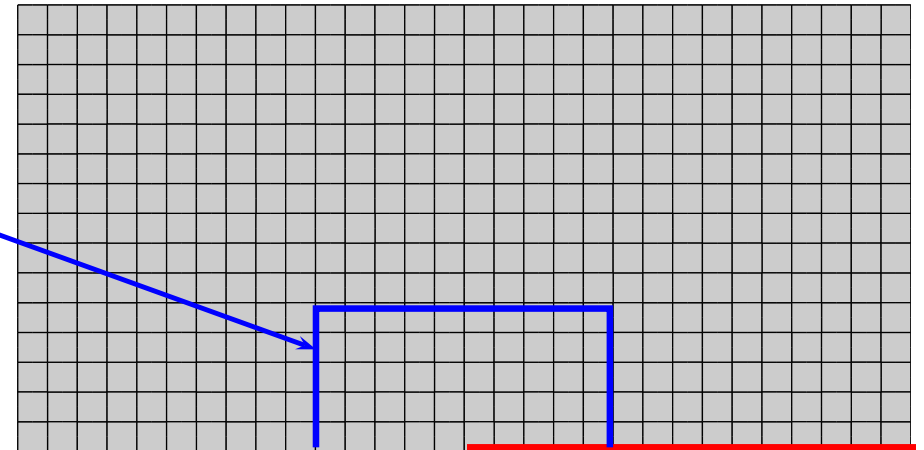


*Ligament*

*Condition de symétrie  $U_y = 0$*

*Dépouillement des résultats*

*Traction monotone*



maillage radial  
maillage regulier

# *Hypothèses et Surface de charge*

[J. Rice, 1987] [J. Rice, D.E. Hawk & R.J. Asaro, 1990]

– *Mode I*

– *Déformations planes :*

$$\begin{pmatrix} \varepsilon_{11} & \varepsilon_{12} & 0 \\ \varepsilon_{12} & \varepsilon_{22} & 0 \\ 0 & 0 & 0 \end{pmatrix}_{X_1, X_2, X_3} \rightarrow \begin{pmatrix} \sigma_{11} & \sigma_{12} & 0 \\ \sigma_{12} & \sigma_{22} & 0 \\ 0 & 0 & \sigma_{33} \end{pmatrix}_{X_1, X_2, X_3}$$

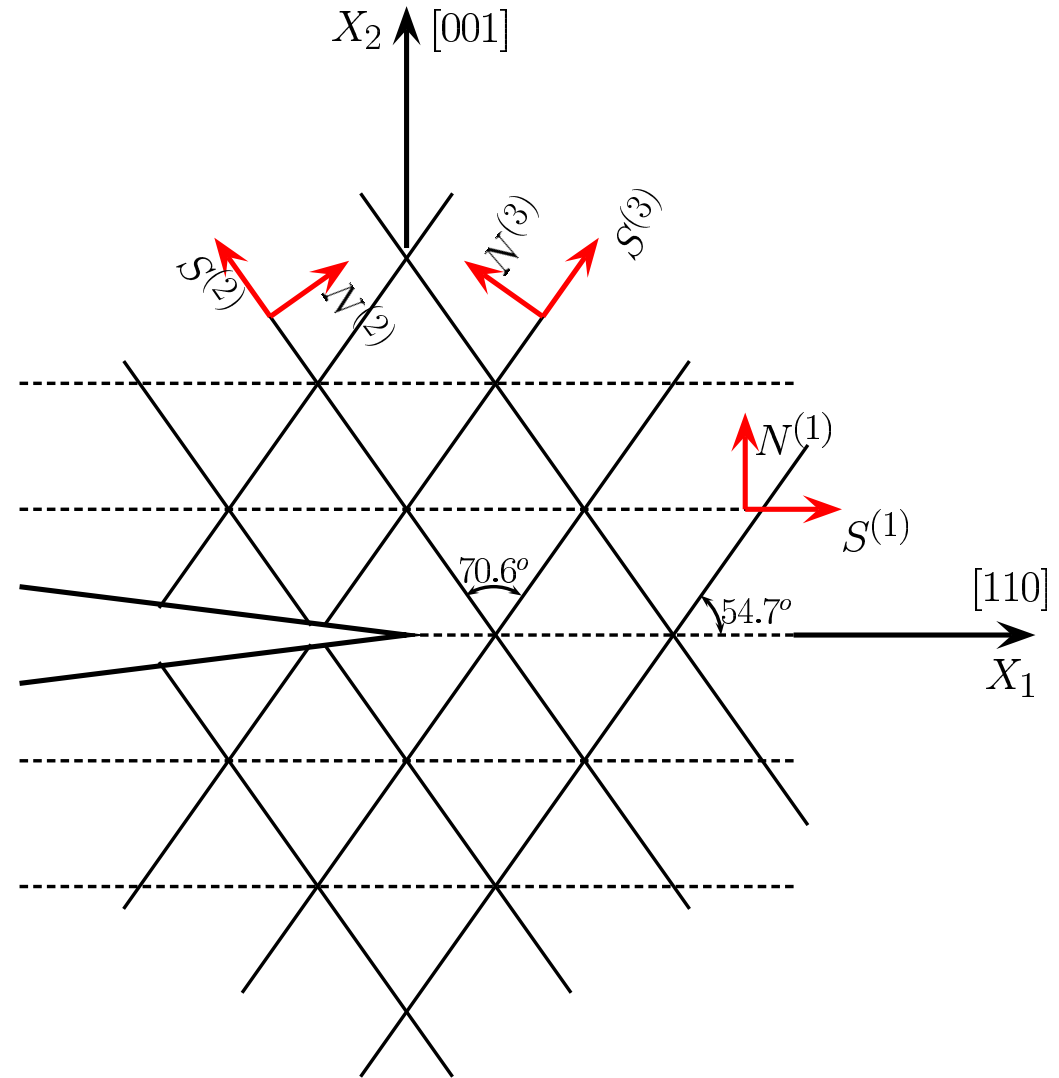
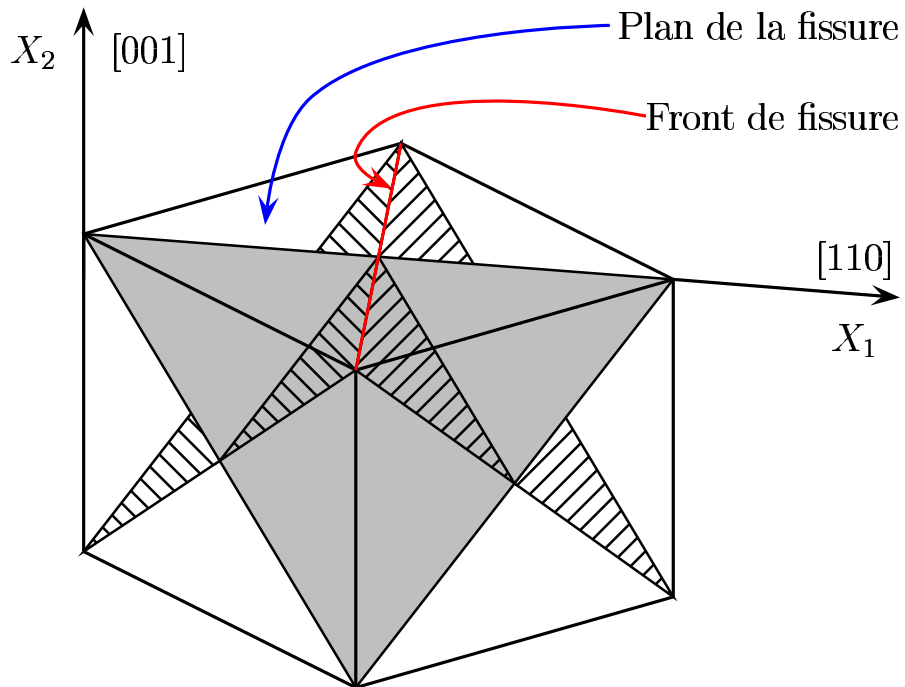
– *Comportement Monocristallin Elastique-parfaitement plastique*

– *Petites déformations*

– *Prise en compte des systèmes de glissement octaédriques*

– *Loi de Schmid*

# *fissure d'orientation* $(001)_{X_2}[110]_{X_1}$



Combinaison de systèmes de glissement  $\Rightarrow$  déformations planes

# Super Systèmes

$$\text{système (1) : } (001)[110] \left\{ \begin{array}{l} (\bar{1}11)[110] \\ (1\bar{1}1)[110] \end{array} \right.$$

$$\text{système (2) : } (111)[\bar{1}\bar{1}2] \left\{ \begin{array}{l} (111)[\bar{1}01] \\ (111)[0\bar{1}1] \end{array} \right.$$

$$\underline{\mathbf{N}}^1 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}_{\text{cristal}} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}_{\text{éprouvette}}$$

$$\underline{\mathbf{N}}^2 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}_{\text{cristal}} = \frac{1}{\sqrt{3}} \begin{pmatrix} \sqrt{2} \\ 1 \\ 0 \end{pmatrix}_{\text{éprouvette}}$$

$$\underline{\mathbf{S}}^1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}_{\text{cristal}} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}_{\text{éprouvette}}$$

$$\underline{\mathbf{S}}^2 = \frac{1}{\sqrt{6}} \begin{pmatrix} -1 \\ -1 \\ 2 \end{pmatrix}_{\text{cristal}} = \frac{1}{\sqrt{3}} \begin{pmatrix} -1 \\ \sqrt{2} \\ 0 \end{pmatrix}_{\text{éprouvette}}$$

$$(1) (001)[110]$$

$$\tau = \underline{\mathbf{m}}^6 \cdot \underline{\boldsymbol{\sigma}} \cdot \underline{\mathbf{n}}^6 = \underline{\mathbf{m}}^8 \cdot \underline{\boldsymbol{\sigma}} \cdot \underline{\mathbf{n}}^8$$

$$\underline{\mathbf{m}}^6 = \underline{\mathbf{m}}^8 = \underline{\mathbf{S}}^1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}_{\text{cristal}}$$

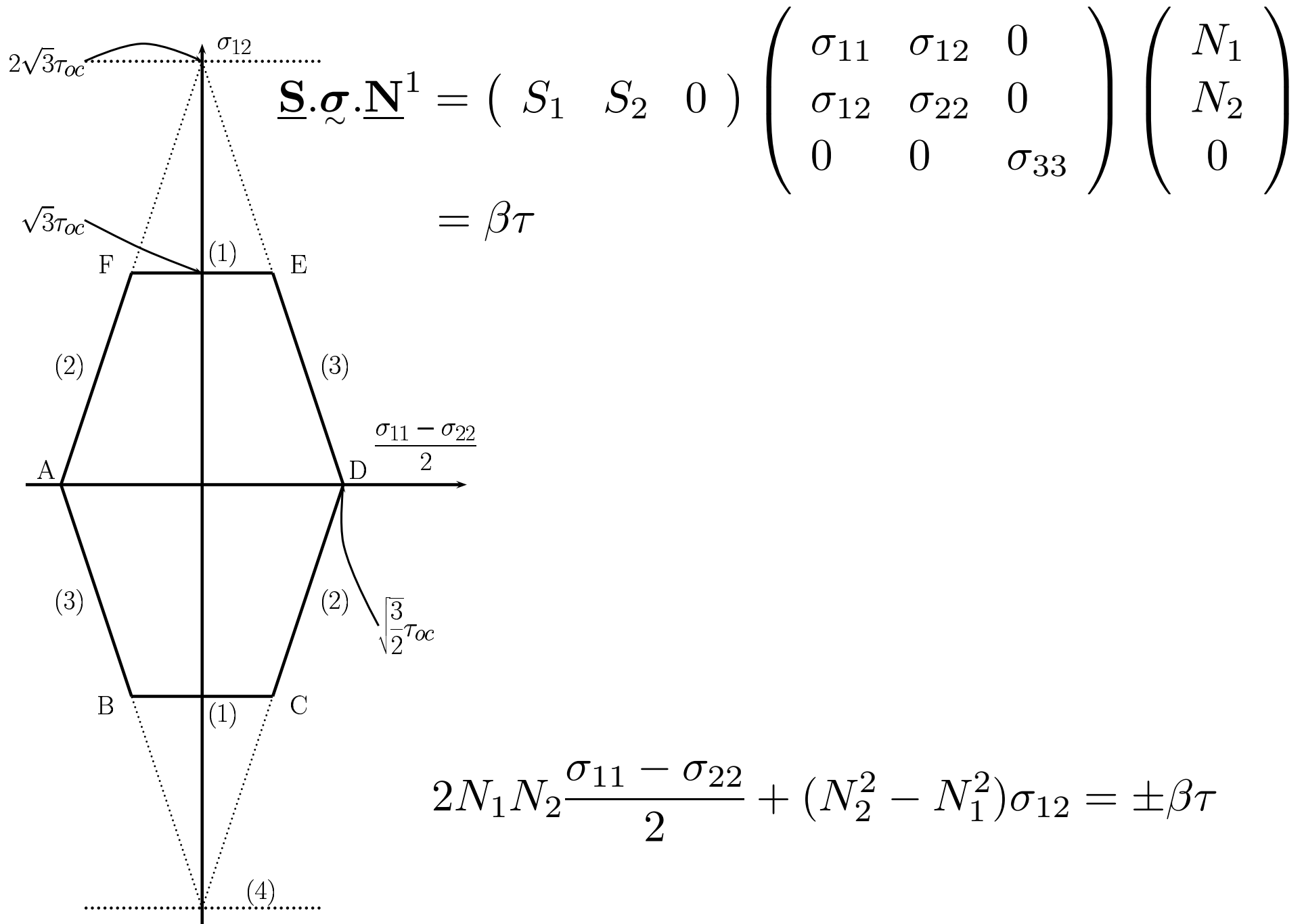
$$\underline{\mathbf{n}}^6 + \underline{\mathbf{n}}^8 = \frac{1}{\sqrt{3}} \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}_{\text{cristal}} + \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}_{\text{cristal}} = \frac{2}{\sqrt{3}} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}_{\text{cristal}} = \frac{2}{\sqrt{3}} \underline{\mathbf{N}}^1$$

$$\tau = \tau_1 = \tau_2 = \frac{1}{2} \underline{\mathbf{m}}^6 \cdot \underline{\boldsymbol{\sigma}} \cdot (\underline{\mathbf{n}}^6 + \underline{\mathbf{n}}^8) = \frac{1}{\sqrt{3}} \underline{\mathbf{S}}^1 \cdot \underline{\boldsymbol{\sigma}} \cdot \underline{\mathbf{N}}^1$$

$$\text{Loi de Schmid dans le plan} \Rightarrow N_{\alpha} \sigma_{\alpha\lambda} S_{\lambda} = \pm \beta \tau$$



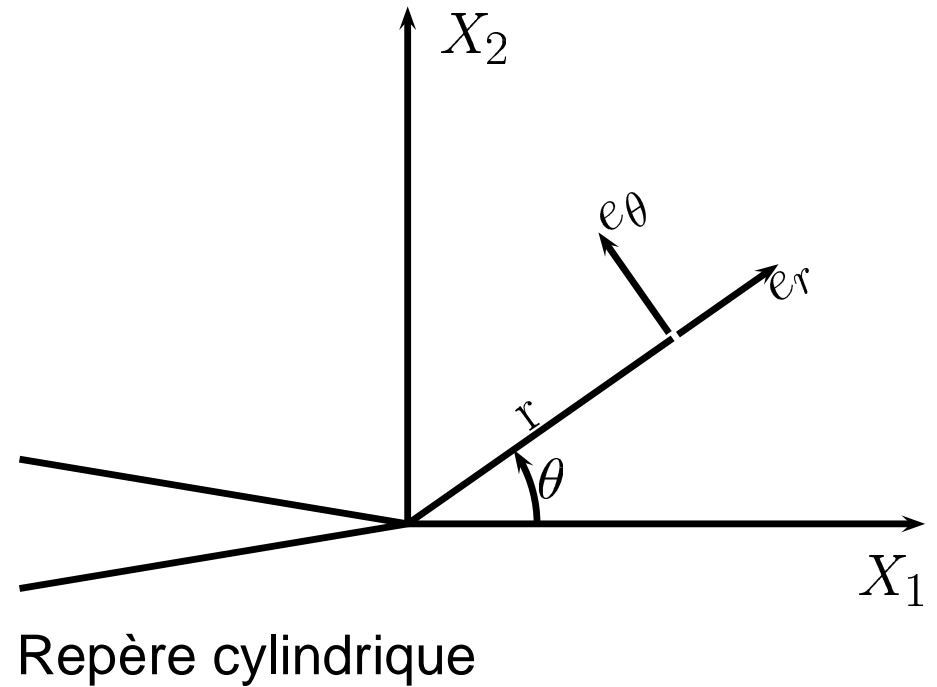
# Surface de charge



# Equations d'équilibre

$$\operatorname{div} \underline{\underline{\sigma}} = \frac{\partial \sigma_{ij}}{\partial x_i} = \frac{\partial \sigma_{ij}}{\partial r} \frac{\partial r}{\partial x_i} + \frac{\partial \sigma_{ij}}{\partial \theta} \frac{\partial \theta}{\partial x_i} = 0$$

$$\begin{cases} \frac{\partial \sigma_{11}}{\partial x_1} + \frac{\partial \sigma_{12}}{\partial x_2} = 0 \\ \frac{\partial \sigma_{12}}{\partial x_1} + \frac{\partial \sigma_{22}}{\partial x_2} = 0 \end{cases}$$



Expression en fonction des coordonnées cylindriques :

$$\begin{cases} \frac{\partial \sigma_{11}}{\partial x_1} + \frac{\partial \sigma_{12}}{\partial x_2} = \frac{\partial \sigma_{11}}{\partial r} \cos \theta - \frac{\partial \sigma_{11}}{\partial \theta} \frac{\sin \theta}{r} + \frac{\partial \sigma_{12}}{\partial r} \sin \theta + \frac{\partial \sigma_{12}}{\partial \theta} \frac{\cos \theta}{r} = 0 \\ \frac{\partial \sigma_{12}}{\partial x_1} + \frac{\partial \sigma_{22}}{\partial x_2} = \frac{\partial \sigma_{12}}{\partial r} \cos \theta - \frac{\partial \sigma_{12}}{\partial \theta} \frac{\sin \theta}{r} + \frac{\partial \sigma_{22}}{\partial r} \sin \theta + \frac{\partial \sigma_{22}}{\partial \theta} \frac{\cos \theta}{r} = 0 \end{cases}$$

# Analyse Asymptotique

$\times r$   
 passage à la limite  $\lim_{r \rightarrow 0}$   
 avec  $\lim_{r \rightarrow 0} r \frac{\partial \sigma_{ij}}{\partial r} = 0$

$$\left\{ \begin{array}{l} - \lim_{r \rightarrow 0} \left( \frac{\partial \sigma_{11}}{\partial \theta} \right) \sin \theta + \lim_{r \rightarrow 0} \left( \frac{\partial \sigma_{12}}{\partial \theta} \right) \cos \theta = 0 \\ - \lim_{r \rightarrow 0} \left( \frac{\partial \sigma_{12}}{\partial \theta} \right) \sin \theta + \lim_{r \rightarrow 0} \left( \frac{\partial \sigma_{22}}{\partial \theta} \right) \cos \theta = 0 \end{array} \right.$$

Le critère de plasticité - derivation de l'équation de la surface de charge :

$$\frac{\partial \sigma_{12}}{\partial \theta} = - \frac{N_1 N_2}{N_2^2 - N_1^2} \frac{\partial (\sigma_{11} - \sigma_{22})}{\partial \theta}$$

$$\left\{ \begin{array}{l} \frac{N_1 N_2}{N_2^2 - N_1^2} \lim_{r \rightarrow 0} \frac{\partial (\sigma_{11} - \sigma_{22})}{\partial \theta} \sin \theta + \lim_{r \rightarrow 0} \frac{\partial \sigma_{22}}{\partial \theta} \cos \theta = 0 \\ \lim_{r \rightarrow 0} \frac{\partial \sigma_{11}}{\partial \theta} \sin \theta + \frac{N_1 N_2}{N_2^2 - N_1^2} \lim_{r \rightarrow 0} \frac{\partial (\sigma_{11} - \sigma_{22})}{\partial \theta} \cos \theta = 0 \end{array} \right.$$

$$\left\{ \begin{array}{l} a = \lim_{r \rightarrow 0} \frac{\partial (\sigma_{11} + \sigma_{22})}{\partial \theta} \\ b = \lim_{r \rightarrow 0} \frac{\partial (\sigma_{11} - \sigma_{22})}{\partial \theta} \end{array} \right. \quad \left\{ \begin{array}{l} a \sin \theta + b \left( 2 \frac{N_1 N_2}{N_2^2 - N_1^2} \cos \theta + \sin \theta \right) = 0 \\ a \cos \theta + b \left( 2 \frac{N_1 N_2}{N_2^2 - N_1^2} \sin \theta - \cos \theta \right) = 0 \end{array} \right.$$

# Analyse Asymptotique

$$\begin{cases} a \sin \theta + b \left( 2 \frac{N_1 N_2}{N_2^2 - N_1^2} \cos \theta + \sin \theta \right) = 0 \\ a \cos \theta + b \left( 2 \frac{N_1 N_2}{N_2^2 - N_1^2} \sin \theta - \cos \theta \right) = 0 \end{cases}$$

Système linéaire homogène à deux inconnues.

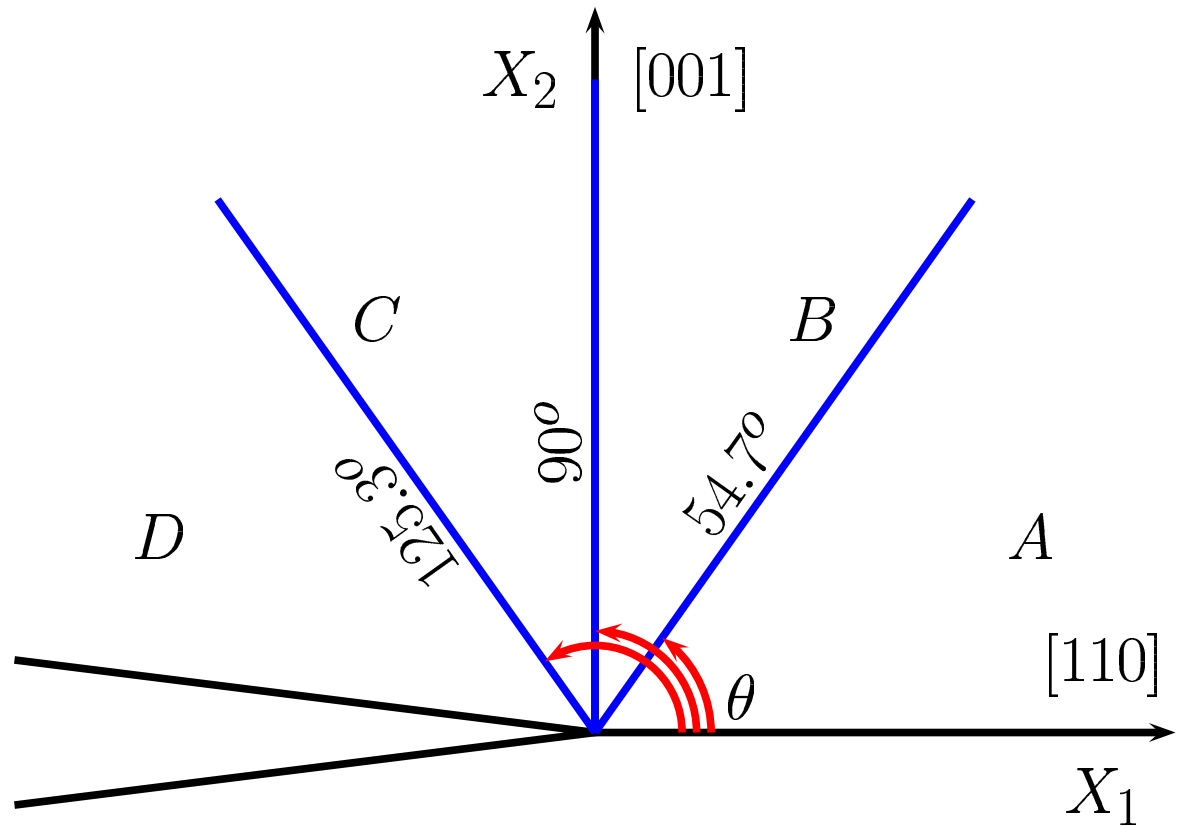
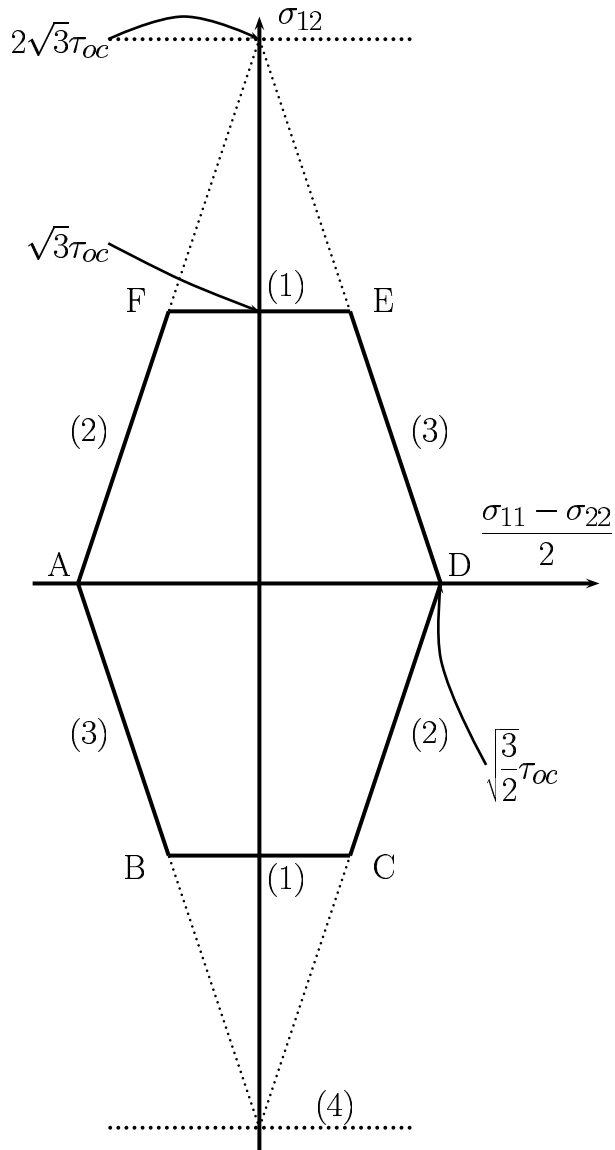
$$\begin{cases} \lim_{r \rightarrow 0} \frac{\partial \sigma_{11}}{\partial \theta} = 0 \\ \lim_{r \rightarrow 0} \frac{\partial \sigma_{22}}{\partial \theta} = 0 \end{cases} \quad \begin{cases} \left( 2 \frac{N_1 N_2}{N_2^2 - N_1^2} (\sin^2 \theta - \cos^2 \theta) - 2 \sin \theta \cos \theta \right) = 0 \\ \tan 2\theta = \frac{2 N_1 N_2}{N_2^2 - N_1^2} \end{cases}$$

⇒ Composantes cartésiennes constantes par secteur. Six discontinuités possibles.

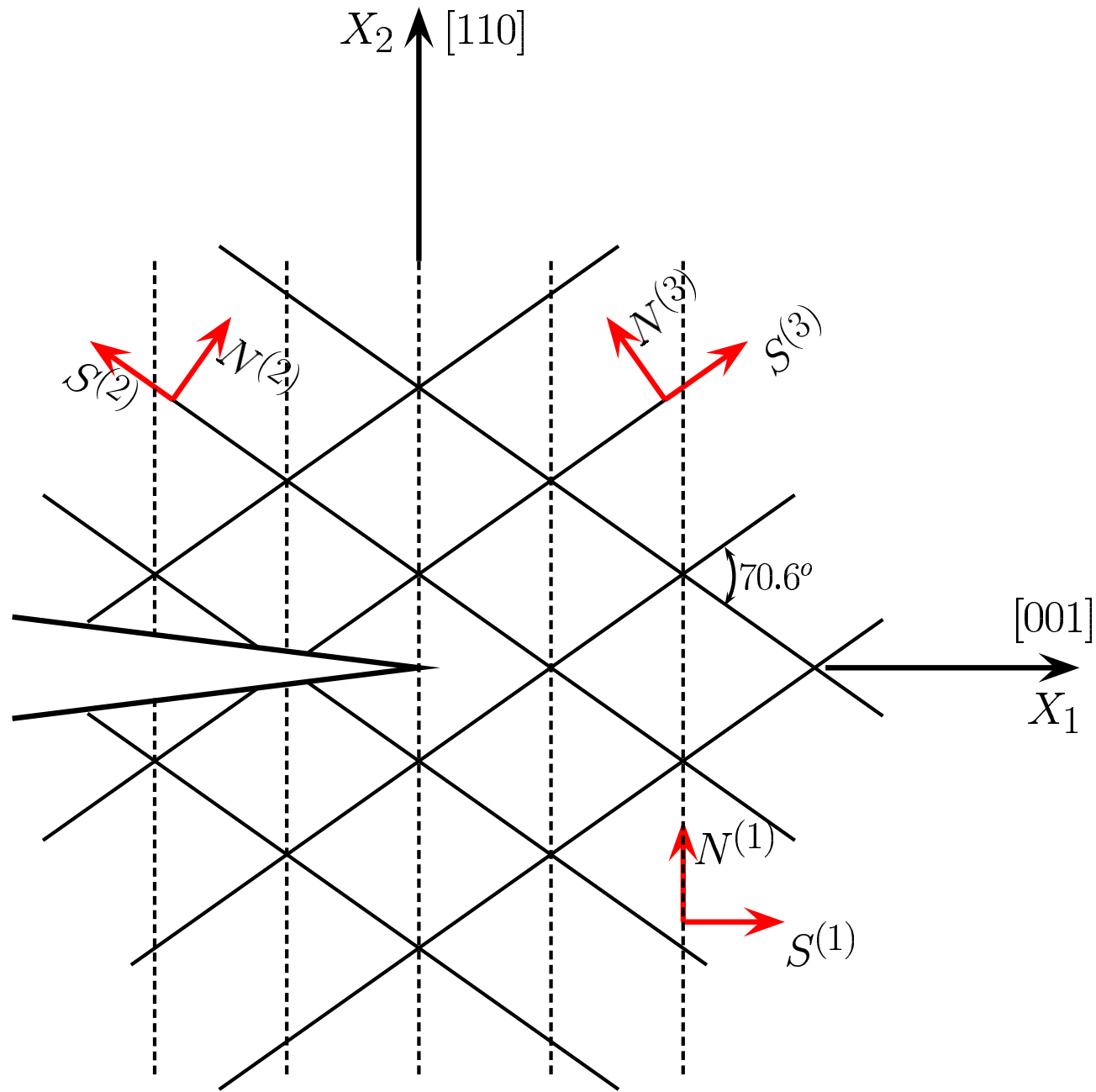
La vecteur contrainte doit être continu au passage de la ligne de discontinuité

$$[\![\underline{\sigma}]\!] \cdot \underline{\mathbf{e}}_\theta = 0 \text{ avec } \underline{\mathbf{e}}_\theta = \begin{pmatrix} -\sin \theta \\ \cos \theta \end{pmatrix}$$

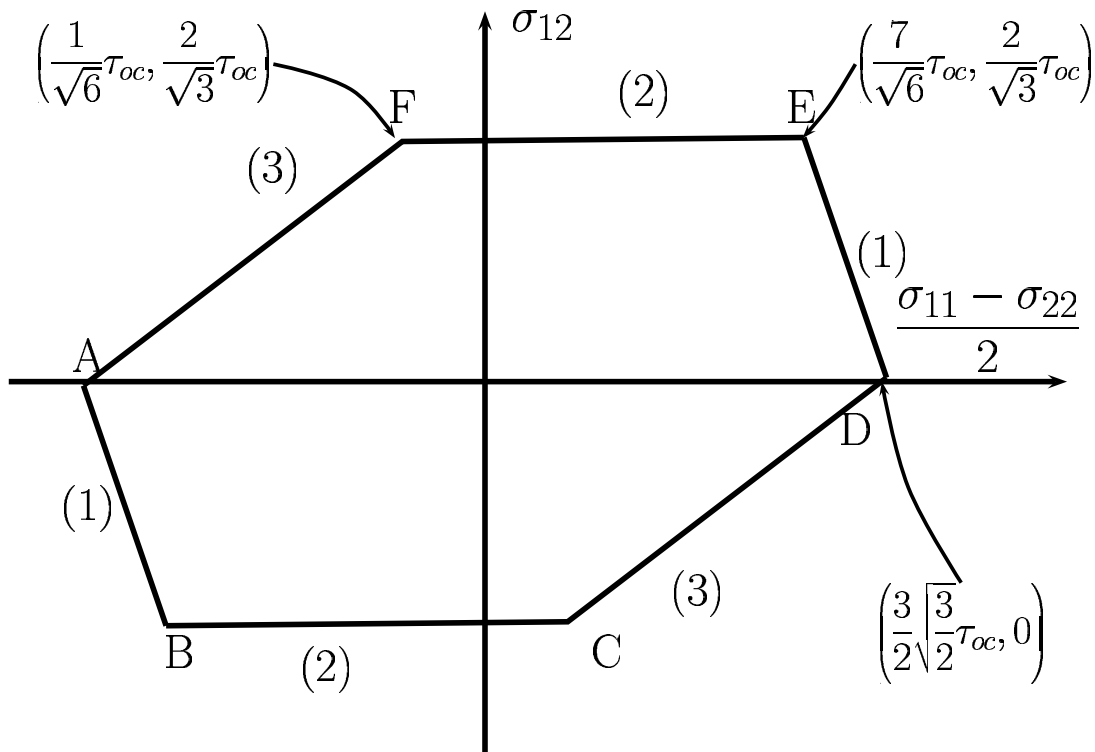
# *Fissure d'orientation* $(001)_{X_2}[110]_{X_1}$



# *Fissure d'orientation* $(110)_{X_2}[001]_{X_1}$

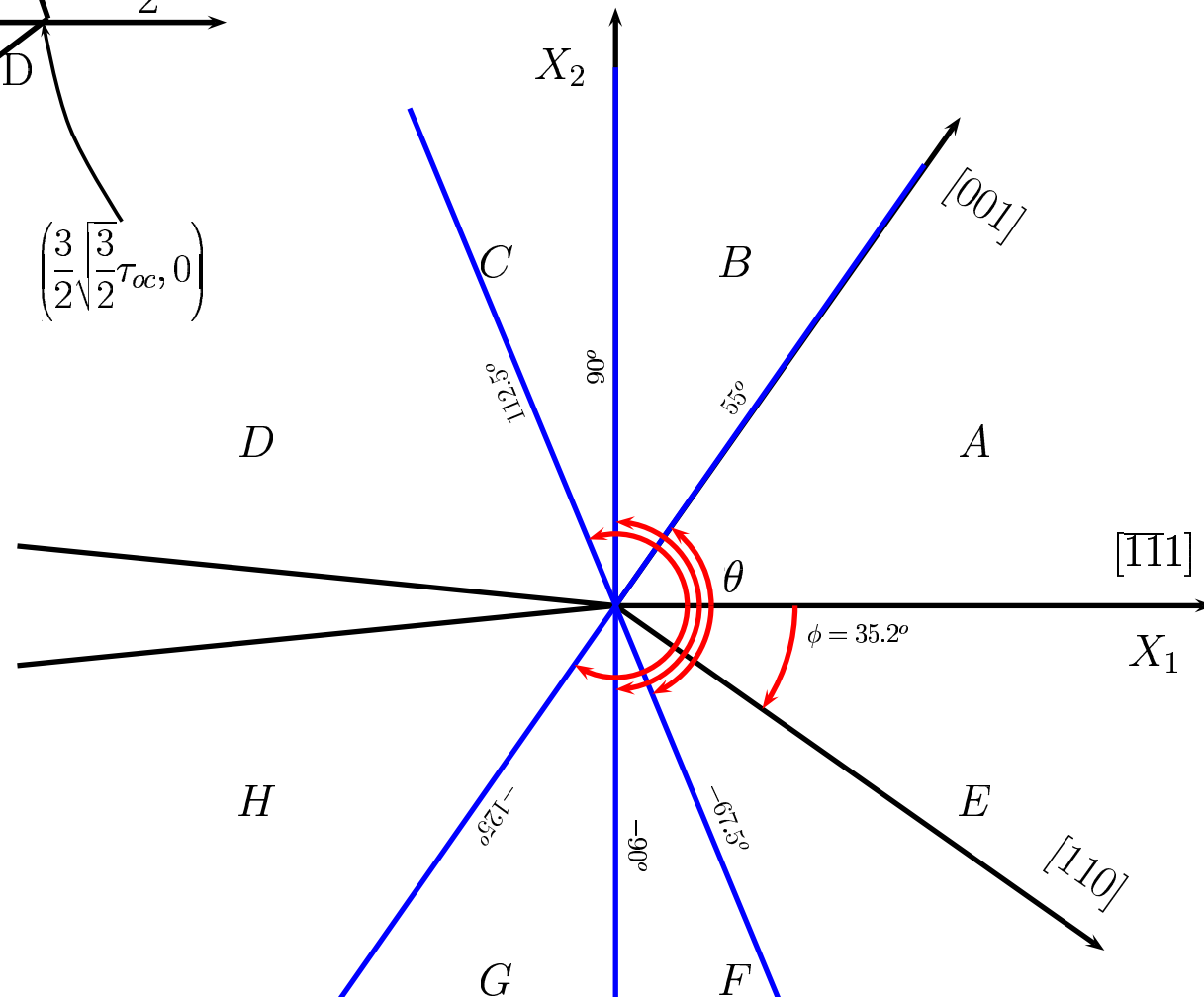


# Fissure d'orientation $(\bar{1}\bar{1}2)_{X_2}[111]_{X_1}$

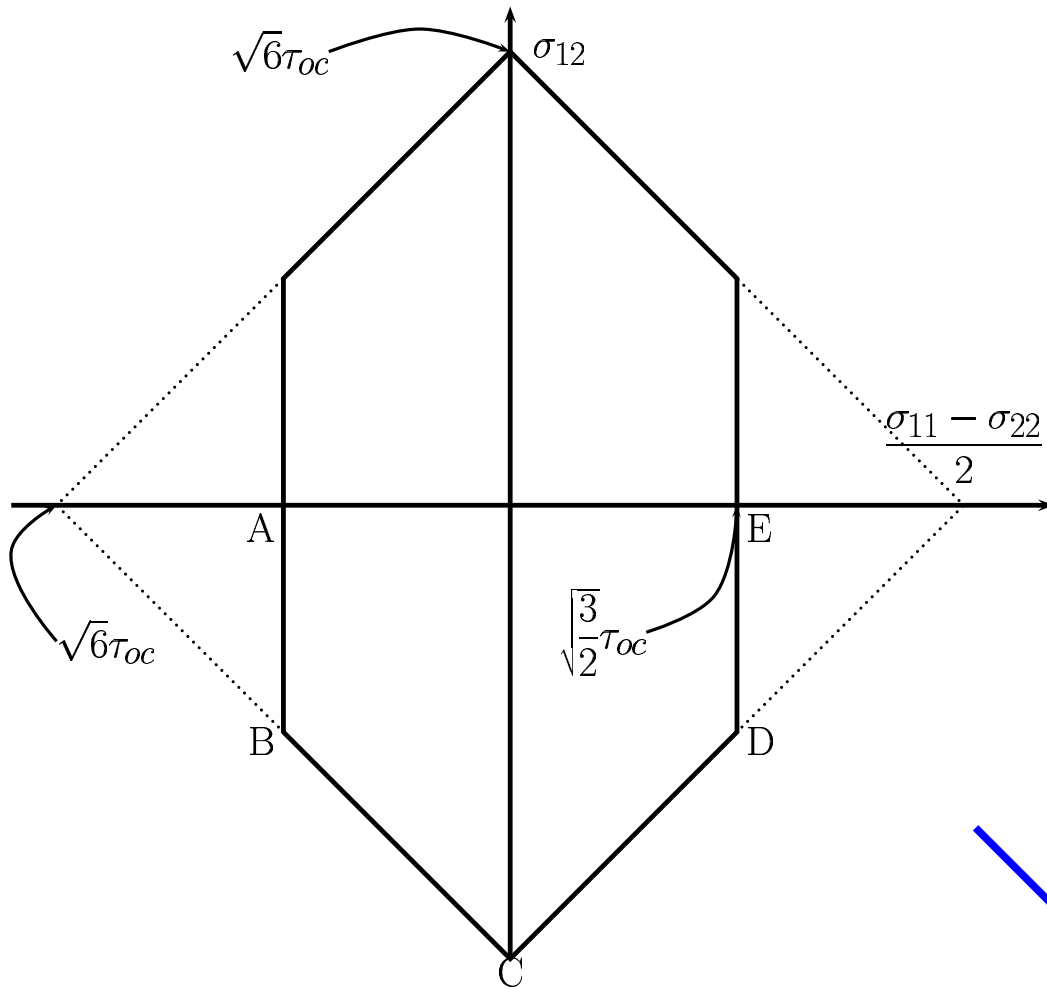


Zrun CT16\_160\_112\_111\_radial.inp  
Zrun CT16\_160\_112\_111\_regulier.inp

Zmaster CT16\_160\_112\_111\_radial  
Zmaster CT16\_160\_112\_111\_regulier

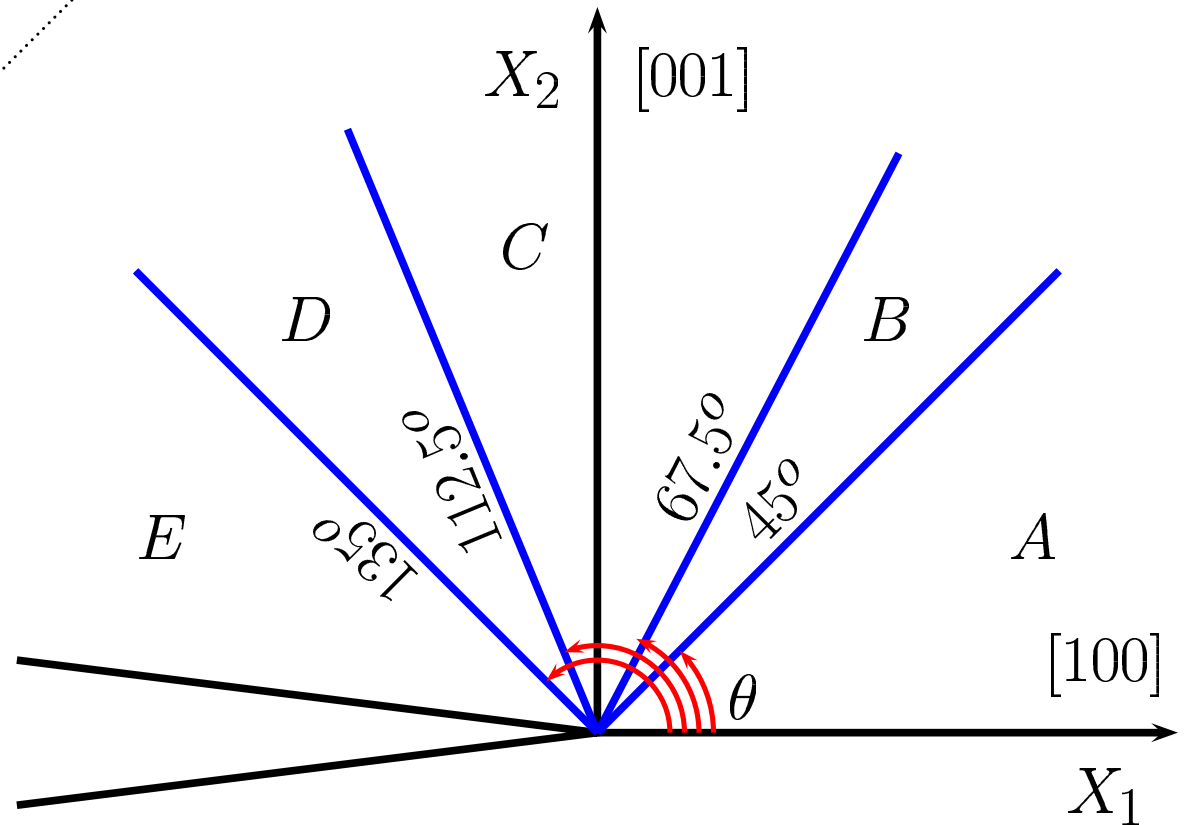


# *Fissure d'orientation* $(001)_{X_2}[100]_{X_1}$



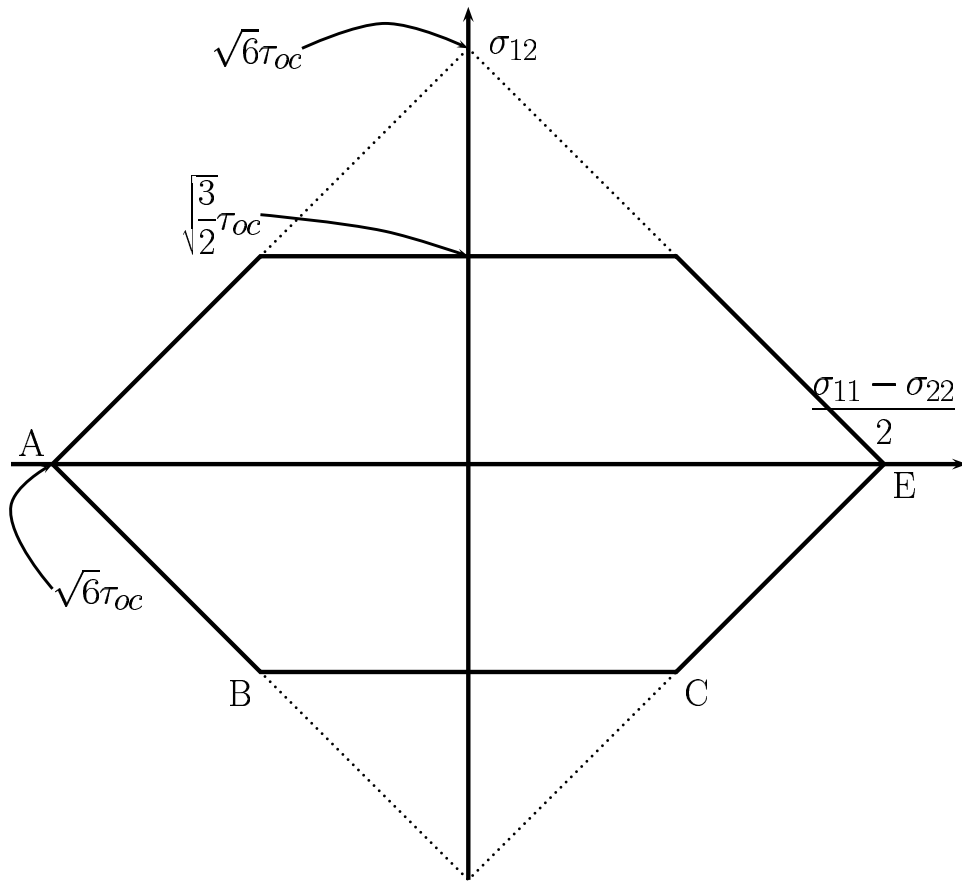
Zrun CT16\_160\_001\_100\_radial.inp  
Zrun CT16\_160\_001\_100\_regulier.inp

Zmaster CT16\_160\_001\_100\_radial  
Zmaster CT16\_160\_001\_100\_regulier



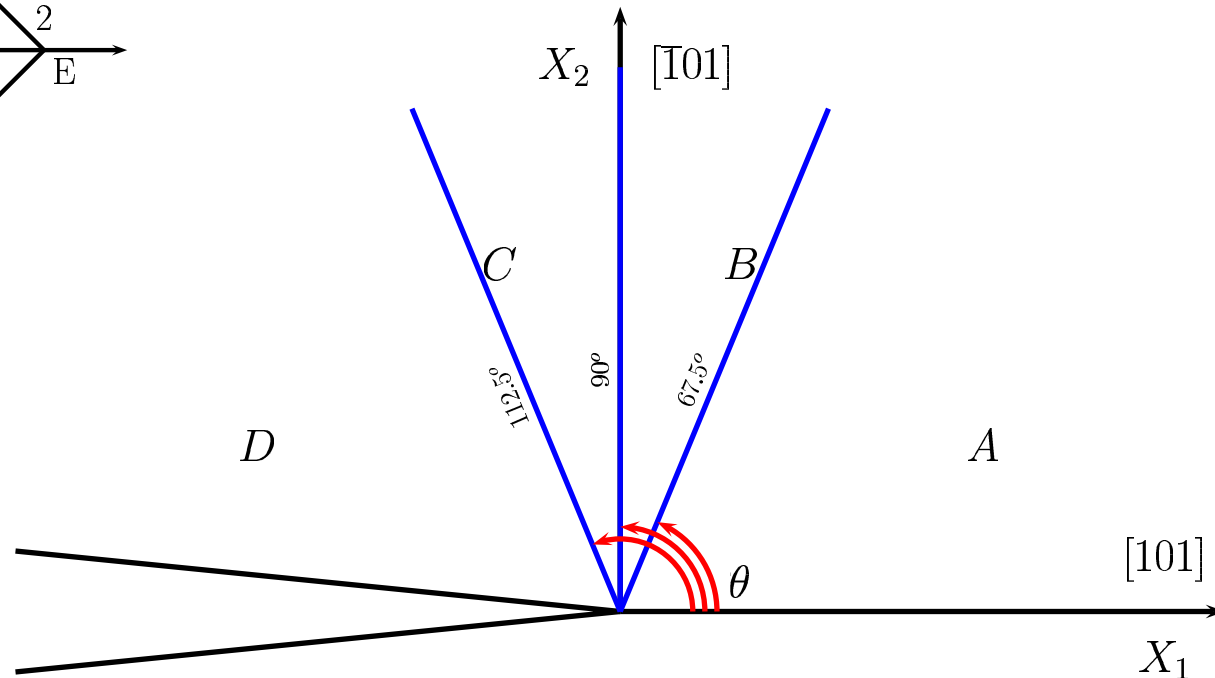


# Fissure d'orientation $(\bar{1}01)_{X_2}[101]_{X_1}$

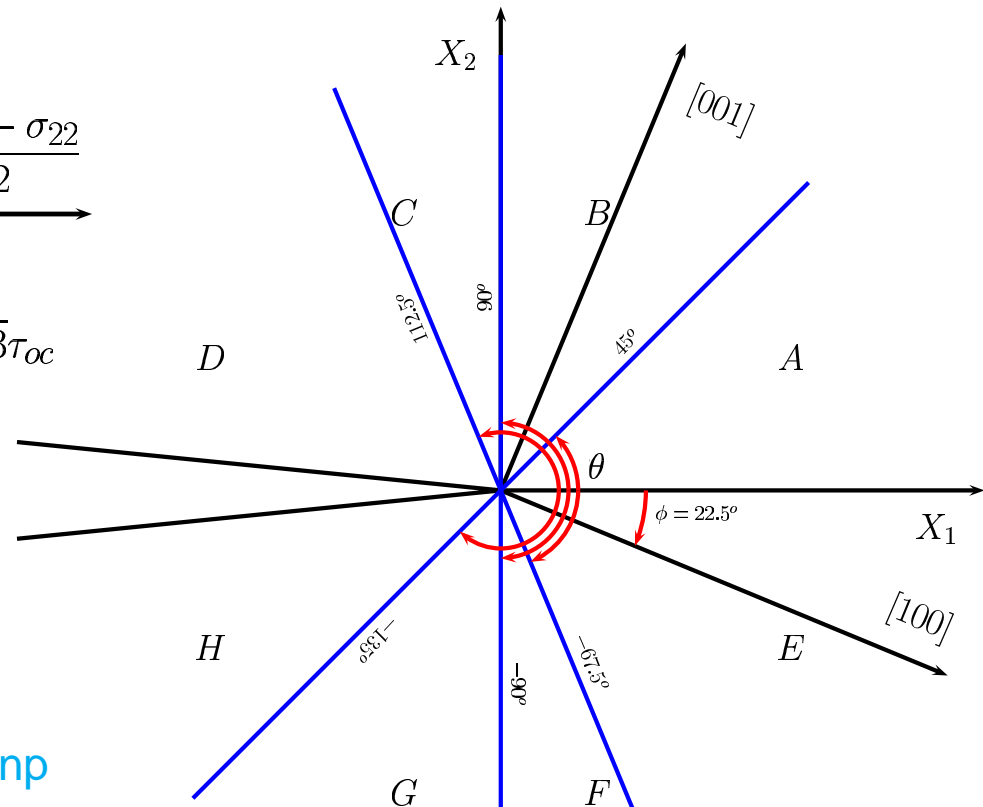
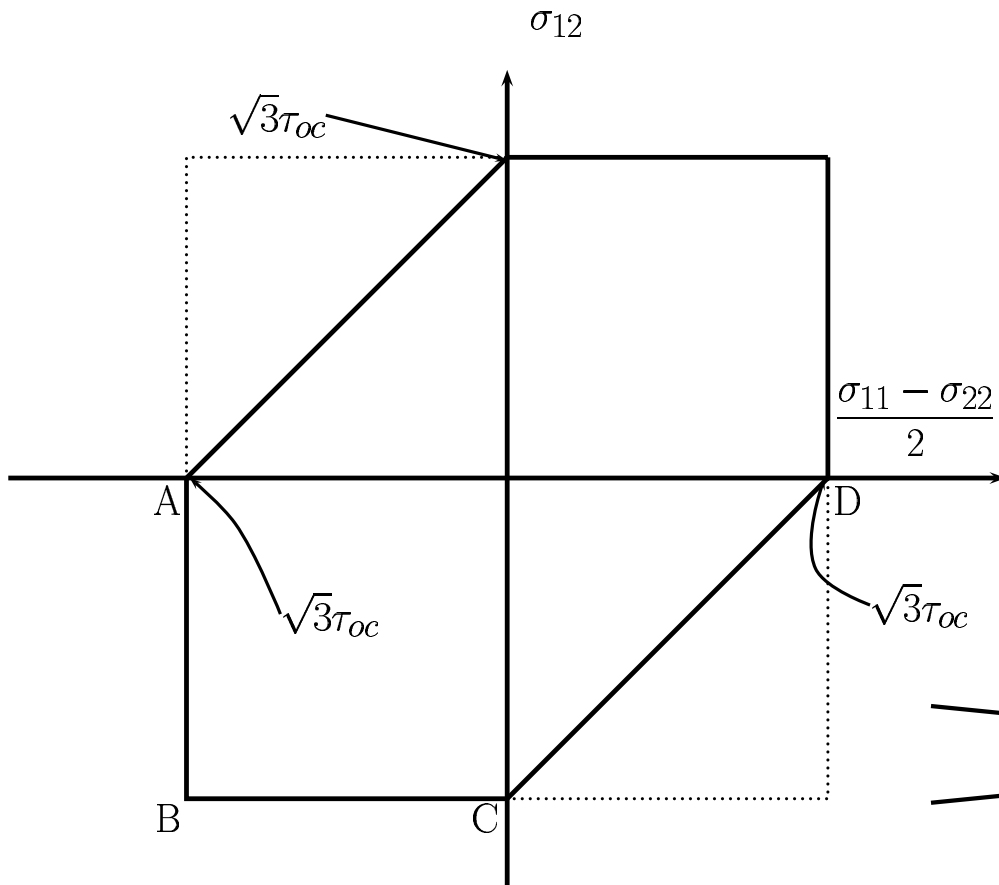


Zrun CT16\_160\_101\_101\_radial.inp  
Zrun CT16\_160\_101\_101\_regulier.inp

Zmaster CT16\_160\_101\_101\_radial  
Zmaster CT16\_160\_101\_101\_regulier

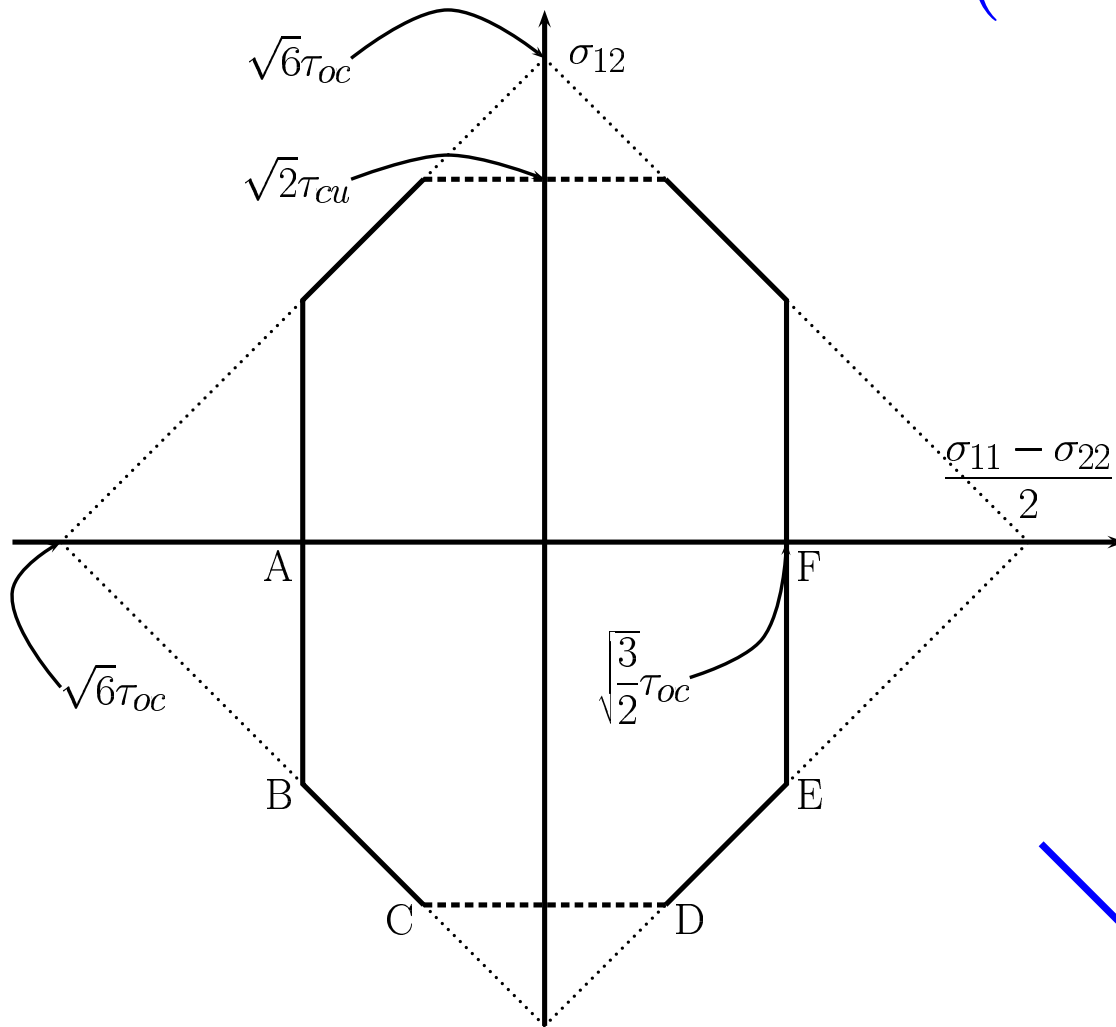


# Fissure d'orientation $(001)_{X_2}[100]_{X_1}$ désorienté de $22.5^\circ$



Zrun CT16\_160\_001\_100\_22degree\_radial.inp  
 Zrun CT16\_160\_001\_100\_22degree\_regulier.inp  
 Zmaster CT16\_160\_001\_22degree\_100\_radial  
 Zmaster CT16\_160\_001\_22degree\_100\_regulier

# *Fissure d'orientation (001)<sub>X<sub>2</sub></sub>[100]<sub>X<sub>1</sub></sub> Cubique*



Zrun CT16\_160\_001\_100\_cub2\_radial.inp  
Zrun CT16\_160\_001\_100\_cub2\_regulier.inp

Zmaster CT16\_160\_001\_100\_cub2\_radial  
Zmaster CT16\_160\_001\_100\_cub2\_regulier

